NON CONSERVATIVE ULTIMATE BOUND ESTIMATION IN LTI PERTURBED SYSTEMS

Ernesto Kofman

Laboratorio de Sistemas Dinamicos. FCEIA - Universidad Nacional de Rosario. Riobamba 245 bis - (2000) Rosario

Abstract: A closed formula for ultimate bound estimation in LTI systems with non-vanishing perturbations is presented. The formula, based on geometrical properties of the system, gives a non conservative alternative to the classic Lyapunov's second method based study. Although it is not proven that the bound obtained is always less conservative than the Lyapunov's bound, some examples are introduced were the estimations are significantly improved. Finally, the application of the methodology to the study of quantization effects in control systems is discussed.

Keywords: Perturbed Systems, Lyapunov Analysis, Ultimate Bound.

1. INTRODUCTION

Several problems related to the analysis of dynamical systems include the effects of perturbations. These perturbations can result from modeling errors, aging or uncertanties and disturbances which exist in any realistic problem (Khalil, 1996).

In some cases, the perturbations disappear when the system approaches the equilibrium point. Those disturbances are called *vanishing perturbations* and –under certain conditions– the asymptotic stability of the perturbed system can be ensured.

Other problems include perturbation terms which does not disappear. These signals are called *nonvanishing perturbations* and now the asymptotic stability is no longer possible. However, under certain conditions, the *ultimately boundedness* of the trajectories can be guaranteed.

Nonvanishing perturbations can represent, for instance, effects of quantization in A/D and D/A converters (Kofman, 2001), unknown disturbance signals (see for instance Koskouej and Zinober (2000)), unmodeled dynamics (Lee et al., 1998), errors in networked control systems (Walsh et al., 2001) and errors in numerical methods (Kofman and Junco, 2001).

In all those problems, it is always important having an estimation for the ultimate bound since it constitutes a measure of the undesirable effect of the perturbation. The tool to estimate this bound is based on the use of a Lyapunov function of the non perturbed system (also called nominal system). Then the sign of its derivative along the perturbed system trajectories is studied and the bound can be estimated as the smallest level surface where the mentioned sign is negative.

The problem with this Lyapunov-based technique is that the bound obtained may be very conservative. The reason can be found in the fact that the structure of the system is lost when the Lyapunov analysis is performed.

A different approach is performed in (Kofman, 2002a). There, the author studies the particular case of LTI systems with perturbations in the state variables resulting from the use of quantization-based numerical methods. The analysis is performed using the diagonalized system and the perturbation is assumed to be bounded in each component. As a result, an ultimate bound is obtained for each state variable avoiding the Lyapunov analysis.

In this paper, we generalize the mentioned study for general LTI systems with bounded input and state perturbations. The ultimate bound is then obtained, not only in terms of the p-norm, but also expressed as a separated bound for each state variable.

The paper is organized as follows. In Section 2 the problem is stated and the notation is presented. Further, in Section 3 the classic Lyapunov based bound is deduced using norm 2 and norm ∞. Section 4 introduces the main result, the non conservative study for the ultimate bound estimation. Fnally, in Section 5 some examples are discussed showing the advantages of the new methodology over the classic Lyapunov based study.

This work is complemented with the application of the results to the design of asynchronous digital controllers in the companion paper (Kofman, 2002b).

2. PRELIMINARIES

2.1 Perturbed Systems

Consider the system

$$\dot{x}(t) = f(t,x) + g(t,x) \tag{1}$$

where $f : [0,\infty)\times D \to \mathbb{R}^n$ and $g : [0,\infty)\times D \to \mathbb{R}^n$ are piecewise continuous in t and locally Lipschitz in x on $[0,\infty)\times D$, and $D \in \mathbb{R}^n$ is a domain that contains the origin $x = 0$.

The system (1) can be seen as a perturbation of the nominal system

$$\dot{x}(t) = f(t,x) \tag{2}$$

Let us consider, without loss of generality, that $f(t,0) = 0$ (i.e. the origin is an equilibrium point of the nominal system). In a typical situation, we do not know $g(t,x)$ but we know some information about it.

When the function g satisfies $g(t,0) = 0$, the origin is also an equilibrium point of the perturbed system (1). In this case, $g(t,x)$ is defined as a *vanishing perturbation*.

Otherwise, when the property $g(t,0) = 0$ cannot be ensured, the term $g(t,x)$ is defined as a *non-vanishing perturbation*. In this case, the origin of (1) is no longer an equilibrium point and its stability cannot be studied. All what can be expected is that if the perturbation is small in some sense, then $x(t)$ will be ultimately bounded by a small bound (Khalil, 1996).

Definition 1. The solutions of (1) are said to be uniformly ultimately bounded if there exist positive constants μ and c, and for every $\alpha \in (0,c)$, there is a positive constant $T = T(\alpha)$ such that

$$\|x(t_0)\| < \alpha \Rightarrow \|x(t)\| \leq \mu,\ \forall t \geq t_0 + T \tag{3}$$

In this case, the constant μ will be called the ultimate bound.

2.2 LTI systems with nonvanishing perturbations

Consider the LTI nominal system

$$\dot{x}(t) = f(t,x) = Ax(t) + Bu(t) \tag{4}$$

where $A \in \mathbb{R}^{n\times n}$ is a Hurwitz matrix, $u \in \mathbb{R}^m$ and $B \in \mathbb{R}^{n\times m}$.

Taking into account that the goal is studying ultimate boundedness, the presence of the input $u(t)$ is not necessary. Then, in presence of general input and state perturbations, and using the fact that $u(t) = 0$, the system (4) can be writen as

$$\dot{x}(t) = A[x(t) + \Delta x(t)] + B\Delta u(t) \tag{5}$$

It will be assumed that the perturbation components satisfy

$$|\Delta x_i(t)| \leq \Delta x_{max_i} \quad 1 \leq i \leq n \tag{6}$$

and

$$|\Delta u_j(t)| \leq \Delta u_{max_j} \quad 1 \leq j \leq m \tag{7}$$

where Δx_{max_i} and Δu_{max_j} are non-negative constants.

Comparing (1) and (5) it results that

$$g(t,x) = A\Delta x(t) + B\Delta u(t) \tag{8}$$

Since the perturbations do not depend on the state x, the property $g(t,x) = 0$ cannot be ensured and the perturbation is nonvanishing.

2.3 Notation

The symbol $|\cdot|$ will indicate the componentwise module of a matrix or vector. If x is a vector with components $x_1,\ldots,x_n$, then $|x|$ will be a new vector of the same size than x with components $|x_1|,\ldots,|x_n|$.

For vectors having the same dimension, the vector inequality $x \leq y$ implies that $x_i \leq y_i$ for every component of x and y.

According to these definitions, the following property will be satisfied:

$$|T\cdot x| \leq |T|\cdot|x|$$

and using the notation defined, the inequalities (6) and (7) can be rewriten as

$$|\Delta x(t)| \leq \Delta x_{max} \tag{9}$$

and

$$|\Delta u(t)| \leq \Delta u_{max} \tag{10}$$

Thus, the problem will be finding an ultimate bound for the system (5) where the perturbations satisfy (9) and (10).

3. LYAPUNOV-BASED ANALYSIS

Before introucing the new approach, the ultimate bound of system (5) will be obtained following a Lyapunov's analysis. The perturbations are considered to be bounded by (9) and (10).

The study will be done for norm 2 and norm ∞. Although the norm ∞ of the bound can be obtained from the norm 2 –using the fact that the first one is always less or equal than the second one– this bound could result even more conservative.

Let $U(x) = x^T P x$ where $P = P^T > 0$ satisfies

$$A^T P + PA = -Q \tag{11}$$

with $Q = Q^T > 0$. Then

$$\begin{aligned}\dot{U}(x) &= \dot{x}^T P x + x^T P \dot{x} \\ &= [A(x + \Delta x) + B\Delta u]^T P x + \\ &\quad + x^T P[A(x + \Delta x) + B\Delta u]\end{aligned}$$

and

$$\dot{U}(x) = -x^T Q x + 2x^T PA\Delta x + 2x^T PB\Delta u \tag{12}$$

we know that

$$\begin{aligned}2x^T PA\Delta x &\leq 2\|x\| \cdot \|PA\| \cdot \|\Delta x\| \\ &\leq 2\|x\| \cdot \|PA\| \cdot \|\Delta x_{max}\|\end{aligned} \tag{13}$$

and similarly

$$\begin{aligned}2x^T PB\Delta u &\leq 2\|x\| \cdot \|PB\| \cdot \|\Delta u\| \\ &\leq 2\|x\| \cdot \|PB\| \cdot \|\Delta u_{max}\|\end{aligned} \tag{14}$$

Up to here, the analysis was the same to both norms. Now, the study should continue using particular properties of each norm.

3.1 Ultimate Bound in Norm 2

In norm 2, we know that:

$$x^T Q x \geq \|x\|_2^2 \lambda_{min}(Q) \tag{15}$$

Then, if

$$\begin{aligned}\|x\|_2 \geq \rho \triangleq \frac{2}{\lambda_{min}(Q)}(&\|PA\|_2 \cdot \|\Delta x_{max}\|_2 + \\ &+ \|PB\|_2 \cdot \|\Delta u_{max}\|_2)\end{aligned} \tag{16}$$

it results from (13) and (14) that

$$x^T Q x \geq \|x\|_2^2 \lambda_{min}(Q) \geq 2x^T PA\Delta x + 2x^T PB\Delta u$$

and from (12) we have

$$\dot{U}(x) < 0$$

Let

$$c \triangleq \max_{\|x\|_2 = \rho} U(x) = \rho^2 \lambda_{max}(P) \tag{17}$$

Then, it can be ensured that the trajectories will finish inside the level surface given by

$$U(x) = c = \rho^2 \lambda_{max}(P)$$

and the ultimate bound will be μ_2 so that

$$\min_{\|x\|_2 = \mu_2} U(x) = \mu_2^2 \lambda_{min}(P) = c$$

this is

$$\mu_2 = \sqrt{\frac{c}{\lambda_{min}(P)}}$$

using (17) and (16) in the last equation, we have

$$\begin{aligned}\mu_2 = \sqrt{\frac{\lambda_{max}(P)}{\lambda_{min}(P)}} \cdot \frac{2}{\lambda_{min}(Q)} \cdot (&\|PA\|_2 \cdot \\ &\cdot \|\Delta x_{max}\|_2 + \|PB\|_2 \cdot \|\Delta u_{max}\|_2)\end{aligned} \tag{18}$$

3.2 Ultimate Bound in Norm ∞

The analysis in norm ∞ is almost identical to the norm 2. The only difference is that the lower bound given by (15) does not stand for norm ∞. To obtain a similar inequality, we will need the following result (Luenberger, 1989).

Lemma 1. (Constrained Quadratic) Let $U(x) = x^t Q x$, where $Q = Q^T > 0$, and let C be a row vector in $\mathbb{R}^n$ and r be a nonzero scalar. Then the minimum of $U(\cdot)$ along a hyperplane $\{x | Cx = r\}$ is given by: $r^2 / CQ^{-1}C^T$.

Using this lemma, the minimum of the quadratic function when the component $x_i = r$ is $r^2/e_i Q^{-1} e_i^T = r^2/(Q^{-1})_{i,i}$. Thus, it results that

$$\min_{\|x\|_\infty = r} x^T Q x = \frac{r^2}{\max_i (Q^{-1})_{i,i}}$$

and the equation (15) can be replaced by

$$x^T Q x \geq \frac{\|x\|_\infty^2}{\max_i (Q^{-1})_{i,i}} \tag{19}$$

Then, following a similar idea to the analysis in norm 2, the bound obtained is

$$\begin{aligned}\mu_\infty = 2b_q \sqrt{b_p \|P\|_\infty} \cdot (&\|PA\|_\infty \|\Delta x_{max}\|_\infty + \\ &+ \|PB\|_\infty \|\Delta u_{max}\|_\infty)\end{aligned} \tag{20}$$

where

$$b_q \triangleq \max_{1 \leq i \leq n} (Q^{-1})_{i,i}$$

and

$$b_p \triangleq \max_{1 \leq i \leq n} (P^{-1})_{i,i}$$

4. NON CONSERVATIVE ULTIMATE BOUND ESTIMATION

Based on the study of a decoupled system (Lemma 2 and Corollary 1) Theorem 1 is deduced. This theorem calculates a bound for the trajectories of a LTI perturbed system assuming that the trajectories start from the origin. Then, based on the mentioned result, Theorem 2 gives an estimation of the ultimate bound by components and in terms of a norm p for a LTI perturbed system with arbitrary initial conditions.

4.1 Trajectories Starting from the Origin

Lemma 2. Consider the following first order equation with complex coefficient

$$\dot{x} = a(x + \Delta x) + B\Delta u \tag{21}$$

where a, x, $\Delta x \in \mathbb{C}$, $\Delta u \in \mathbb{C}^k$ and $B \in \mathbb{C}^{1\times k}$. Assume also that $\mathbb{R}\mathrm{e}(a) < 0$, $|\Delta x| \leq \Delta x_{max}$ and $|\Delta u| \leq \Delta u_{max}$.

Let $x(t)$ be a solution of (21) from the initial condition $x(t_0) = 0$. Then, for all $t \geq t_0$ it results that

$$|x(t)| \leq \left|\frac{a}{\mathbb{R}\mathrm{e}(a)}\right| \Delta x_{max} + \left|\frac{B}{\mathbb{R}\mathrm{e}(a)}\right| \Delta u_{max}$$

PROOF. Let $x = \rho \cdot e^{j\theta}$ with ρ, $\theta \in \mathbb{R}$. With this, equation (21) becomes

$$\dot{\rho} \cdot e^{j\theta} + j\rho \cdot e^{j\theta} \cdot \dot{\theta} = a(\rho \cdot e^{j\theta} + \Delta x) + B\Delta u$$

Then, we have

$$\dot{\rho} + j\rho \cdot \dot{\theta} = a(\rho + \Delta x \cdot e^{-j\theta}) + B\Delta u \cdot e^{-j\theta}$$

Taking the real part of the equation above it results that

$$\dot{\rho} = \mathbb{R}\mathrm{e}(a)\rho + \mathbb{R}\mathrm{e}(a\Delta x \cdot e^{-j\theta}) + \mathbb{R}\mathrm{e}(B\Delta u \cdot e^{-j\theta})$$

and

$$\dot{\rho} \leq \mathbb{R}\mathrm{e}(a)\rho + |a|\Delta x_{max} + |B|\Delta u_{max}$$

Thus, when

$$\rho = |x(t)| = \frac{|a|\Delta x_{max} + |B|\Delta u_{max}}{|\mathbb{R}\mathrm{e}(a)|}$$

it results that $\dot{\rho} \leq 0$ and $|x(t)|$ cannot become greater than the given bound.

Applying Lemma 2 to each component of a decoupled system, the following corollary is obtained

Corollary 1. Consider the system (5) where x, $\Delta x \in \mathbb{C}^n$, $A \in \mathbb{C}^{n\times n}$, $\Delta u \in \mathbb{C}^k$ and $B \in \mathbb{C}^{n\times k}$. Assume further than A is a diagonal matrix with $\mathbb{R}\mathrm{e}(A_{i,i}) < 0$ and consider the inequalities (9) and (10). Let $x(t)$ be a solution of (5) from $x(t_0) = 0$. Then, for all $t \geq t_0$ we have

$$|x(t)| \leq |\mathbb{R}\mathrm{e}(A)^{-1}A|\Delta x_{max} + |\mathbb{R}\mathrm{e}(A)^{-1}B|\Delta u_{max}$$

With this corollary, the following theorem can be derived

Theorem 1. Consider system (5) where x, $\Delta x \in \mathbb{R}^n$, $A \in \mathbb{R}^{n\times n}$, $\Delta u \in \mathbb{R}^k$ and $B \in \mathbb{R}^{n\times k}$. Assume that A is a diagonalizable Hurwitz matrix and suposse that the perturbation terms satisfy (9) and (10). Let $x(t)$ be a solution of the system starting from $x(t_0) = 0$. Then, for all $t \geq t_0$

$$\begin{aligned}|x(t)| \leq |V| \cdot (&|\mathbb{R}\mathrm{e}(\Lambda)^{-1}\Lambda||V^{-1}|\Delta x_{max} + \\ &+ |\mathbb{R}\mathrm{e}(\Lambda)^{-1}V^{-1}B|\Delta u_{max})\end{aligned} \tag{22}$$

where Λ is a diagonal eigenvalues matrix of A and V is an associated matrix of eigenvectors, that is

$$V^{-1}AV = \Lambda \tag{23}$$

PROOF. Let $x = Vz$. It results from (5) that

$$V\dot{z} = A(Vz + \Delta x) + B\Delta u$$

Then,

$$\dot{z} = \Lambda(z + V^{-1}\Delta x) + V^{-1}B\Delta u \tag{24}$$

Taking into account the restrictions in $|\Delta x|$ and $|\Delta u|$, we have

$$|V^{-1}\Delta x| \leq |V^{-1}|\Delta x_{max}$$

and

$$|V^{-1}B\Delta u| \leq |V^{-1}B|\Delta u_{max}$$

Since Λ is a diagonal matrix with $\mathbb{R}\mathrm{e}(\Lambda_{i,i}) < 0$ (since A is Hurwitz) and taking into account the last inequalities, system (24) satisfies the hypothesis of Corollary 1. Then, for all $t \geq t_0$ we can ensure that

$$\begin{aligned}|z(t)| \leq &|\mathbb{R}\mathrm{e}(\Lambda)^{-1}\Lambda||V^{-1}|\Delta x_{max} + \\ &+ |\mathbb{R}\mathrm{e}(\Lambda)^{-1}V^{-1}B|\Delta u_{max}\end{aligned}$$

and finally, we have

$$\begin{aligned}|x(t)| &= |Vz(t)| \leq |V||z(t)| \leq \\ &\leq |V|(|\mathbb{R}\mathrm{e}(\Lambda)^{-1}\Lambda||V^{-1}|\Delta x_{max} + \\ &+ |\mathbb{R}\mathrm{e}(\Lambda)^{-1}V^{-1}B|\Delta u_{max})\end{aligned}$$

which completes the proof.

4.2 Ultimate Bound of a Perturbed LTI System

Theorem 1 shows that the solutions of the perturbed system (5) which start in the origin cannot abandon the region given by inequality (22). The following theorem, based on the previous one, shows that the trajectories which start in arbitrary intial conditions finish inside the mentioned region. This property allows estimating the ultimate bound.

Theorem 2. The system (5), under the hypothesis of Theorem 1 is globally ultimately bounded with ultimate bound

$$\mu = \| |V| \cdot (|\mathbb{R}e(\Lambda)^{-1}\Lambda| \cdot |V^{-1}|\Delta x_{max} + \\ + |\mathbb{R}e(\Lambda)^{-1}V^{-1}B|\Delta u_{max})\| \quad (25)$$

Moreover, it exists a finite time $t_1 = t_1(c, x_0)$ so that for each positive constant c the solutions satisfy

$$|x(t)| \leq (1+c)|V| \cdot (|\mathbb{R}e(\Lambda)^{-1}\Lambda| \cdot |V^{-1}|\Delta x_{max} + \\ + |\mathbb{R}e(\Lambda)^{-1}V^{-1}B|\Delta u_{max}) \quad (26)$$

for all $t \geq t_1$ and for an arbitrary initial condition x_0.

PROOF. Let $x(t)$ be a solution of (5) starting from an arbitrary initial condition $x(0) = x_0$, and let $\tilde{x}(t)$ be a solution of the nominal system

$$\dot{\tilde{x}} = A\tilde{x} \quad (27)$$

starting from the same initial condition.

Let $e(t) = x(t) - \tilde{x}(t)$. Then, it results that $e(0) = 0$ and $e(t)$ satisfies the equation (5), which implies that it satisfies the hypothesis of Theorem 1. Then,

$$|e(t)| \leq |V| \cdot (|\mathbb{R}e(\Lambda)^{-1}\Lambda| \cdot |V^{-1}|\Delta x_{max} + \\ +|\mathbb{R}e(\Lambda)^{-1}V^{-1}B|\Delta u_{max}) \quad (28)$$

Since A is Hurwitz, the nominal system (27) is exponencially stable. Then, for each positive constant c a finite time t_1 exists so that for all $t > t_1$ we have

$$|\tilde{x}(t)| \leq c|V| \cdot (|\mathbb{R}e(\Lambda)^{-1}\Lambda| \cdot |V^{-1}|\Delta x_{max} + \\ +|\mathbb{R}e(\Lambda)^{-1}V^{-1}B|\Delta u_{max}) \quad (29)$$

Then, for $t > t_1$,

$$x(t) = e(t) + \tilde{x}(t)$$
$$|x(t)| \leq |e(t)| + |\tilde{x}(t)|$$

and replacing with (28) and (29) we arrive to (26), which completes the proof.

5. EXAMPLES AND APPLICATIONS

Consider the perturbed system

$$\dot{x}(t) = \begin{bmatrix} 0 & 100 \\ -100 & -10001 \end{bmatrix} \cdot [x(t) + \Delta x(t)]$$

where the perturbation components are bounded by

$$|\Delta x_1(t)| \leq 0.01; \;\; |\Delta x_2(t)| \leq 0.0001$$

This example, taken from (Kofman, 2002a), represents the dynamics of the error in the simulation of a stiff system with the QSS method (Kofman and Junco, 2001).

The use of the Lyapunov-based formula (18) with $Q = I$ gives the ultimate bound $\mu_2 = 200.115$, while (20) gives $\mu_\infty = 202.06$ (in this case the analysis in norm ∞ is useless).

In the other hand, our approach –Theorem 2 and the equation (25)– gives $\tilde{\mu}_2 = 0.0100085$ (bound in norm 2) and $\tilde{\mu}_\infty = 0.01$. The estimation is about 20000 times less conservative.

Moreover, Theorem 2 concludes that according to (26), after certain time t_1 we will have

$$|x_1(t)| < (1+c)0.01; \; |x_2(t)| < (1+c)0.0003$$

for any given positive constant c. If the goal were knowing about the second component, this result is even much better than the obtained with (25).

The calculations were made using the eigenvectors and eigenvalues matrices

$$V = \begin{bmatrix} 1 & -0.01 \\ -0.01 & 1 \end{bmatrix}; \; \Lambda = \begin{bmatrix} -1 & 0 \\ 0 & -10000 \end{bmatrix}$$

The reason for the poor performance of the Lyapunov analysis is, in part, due to the big difference between the system eigenvalues (the system is stiff). Thus, the eigenvalues of matrix P are also very different which implies that the level surfaces of $U(x)$ are very flat ellipses. Hence, the radius ρ which defines the ball where the Lyapunov function derivative is negative differs significantly from the radius μ_2, which defines the ball cointaining all the level surfaces that passes by the ball with radius ρ (see Figure 1).

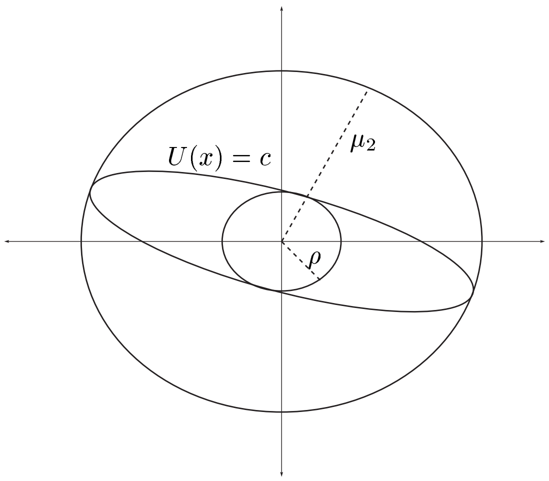

Fig. 1. Lyapunov Bound in Norm 2

However, the bad performance is also due to the loss of the problem structure when the Lyapunov function is used. In this way, the analysis cannot obtain any profit from the fact that the perturbation term acts in each direction with different amplitude. Another reason for the conservative result is related to the choice of matrix Q. It was taken equal to I just for simplicity, but a different choice might yield in a better bound.

Consider now the same example with an input perturbation.

$$\dot{x}(t) = \begin{bmatrix} 0 & 100 \\ -100 & -10001 \end{bmatrix} \cdot x(t) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} \cdot d(t)$$

where $|d(t)| \leq 1$. Using the Lyapunov bound given by (18) with $Q = I$, we obtain $\mu_2 = 100.055$, while (20) gives $\mu_\infty = 100.54$ (which again does not say anything new).

The use of (25) gives $\tilde{\mu}_2 = 1.00015$ and $\tilde{\mu}_\infty = 1.0001$. Then, the results with the new approach are about 100 times better. The Theorem 2 also ensures that, according to (26), after some time it will be true that

$$|x_1(t)| < 1.0001; \;\; |x_2(t)| < (1+c)0.01$$

Finally, consider the system

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 \\ -1 & -1 \end{bmatrix} \cdot x(t) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} \cdot d(t)$$

where $|d(t)| \leq 1$.

This is a very simple non-stiff second order system with complex eigenvalues in presence of input perturbations. Here, the bounds obtained via Lyapunov's analysis are $\mu_2 = 5.11667$ and $\mu_\infty = 4.64758$ (here the bound with norm ∞ is useful). In the other hand, the bounds obtained with the new approach are $\tilde{\mu}_2 = 3.266$ and $\tilde{\mu}_\infty = 2.3094$, which still are less conservative.

The eigenvectors and eigenvalues matrices used here were now

$$V = \begin{bmatrix} 0.612 - j0.354 & 0.612 + j0.354 \\ j0.707 & j0.707 \end{bmatrix}$$

and

$$\Lambda = \begin{bmatrix} -0.5 + j0.866 & 0 \\ 0 & -0.5 - j0.866 \end{bmatrix}$$

In this last example, the advantage of the method is only due to the conservation of the problem structure.

Further examples of the use of Theorem 1 applied to the design and the study of quantization effects in asynchronous digital controllers can be found in (Kofman, 2002b).

6. CONCLUSIONS

A new approach for the estimation of the ultimate bound in perturbed LTI systems was introduced. This methodology, based on the system and perturbation structure, give an alternative to the classic Lyapunov-based analysis.

The main advantage of the new approach is related to the fact that the bound obtained can be much less conservative than the one obtained with a Lyapunov analysis. Other benefit consists in the possibility of obtaining separated bounds for each state variable. The mentioned advantages were illustrated through the analysis of some simple examples showing in some cases an amazing improvement of the Lapunov's bound.

The results were also applied to some practical problems of asynchronous digital control (Kofman, 2002b) and used in the estimation of the error in discrete event simulation methods (Kofman, 2002a).

Future work should generalize the results including the case of non-diagonalizable Hurwitz matrices. A theoretical comparision between the bounds given by (18), (20) and (25) would be interesting since it could establish in which cases the new approach ensures a better result than the classic ones.

REFERENCES

H. Khalil. *Nonlinear Systems*. Prentice-Hall, New Jersey, 2nd edition, 1996.

E. Kofman. Quantized-State Control. A Method for Discrete Event Control of Continuous Systems. Part I. In *Proceedings of RPIC'01*, pages 103–108, 2001.

E. Kofman. A Second Order Approximation for DEVS Simulation of Continuous Systems. *Simulation*, 78(2), 2002a.

E. Kofman. Quantized-State Control of Linear Systems. Technical Report LSD0202, LSD-UNR, 2002b. Submitted to AADECA'02. Available at www.eie.fceia.unr.edu.ar/∼ekofman/.

E. Kofman and S. Junco. Quantized State Systems. A DEVS Approach for Continuous System Simulation. *Transactions of SCS*, 18(3):123–132, 2001.

A.J. Koskouej and A.S. Zinober. Sliding Mode Controller-Observer Design for Multivariable Linear Systems with Unmatched Uncertainty. *Cybernetics*, 36(1):95–115, 2000.

J.S. Lee, H. Shim, J. Byun, and J.H. Seo. Robust Stabilization of a class of MIMO nonlinear systems: Sliding mode control & Passification Approach. In *Proceedings of ACC98*, 1998.

D.G. Luenberger. *Linear and Nonlinear Programming, 2nd Ed.* Addison-Wesley, 1989.

G. Walsh, O. Beldiman, and L. Bushnell. Error Encoding Algorithms for Networked Control Systems. *Automatica*, 2001.