# A Mixed Modeling Approach for Efficient Simulation of PWM Switching Mode Power Supplies.

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***Abstract*—This work introduces a novel modeling approach that allows to obtain fast simulations of PWM DC–DC Switched Mode Power Supplies (SMPS). The proposed methodology combines the use of precise switched models during transient evolutions and averaged models during steady state or slowly varying conditions. In that way, the resulting mixed modeling approach enables to obtain the detailed switching behavior of SMPS in the context of long term simulations. The commutation between models is automatically performed in runtime by an algorithm that detects the transient or slowly varying condition according to the evolution of some model variables. When the precise switched model is used, the mentioned algorithm also adjusts the averaged model parameters so that it results accurate irrespective of the operating point. The article also describes the implementation of the methodology in the Modelica language and reports simulation experiments showing that the results are as accurate as those obtained using precise switched models, but several times faster.**

***Index Terms*—Hybrid Systems, Switched Mode Power Supplies, Modelica.**

## I. Introduction

SMPS impose several difficulties to ordinary differential equations (ODEs) solvers due to the presence of commutations. Since the numerical algorithms cannot integrate across discontinuous functions, when a discontinuity is detected they must stop the simulation, find the exact time point at which the discontinuity occurred (event location), advance the simulation until that point, process the discontinuity (event handling) and restart the simulation from that point [1]. This process, also known as *zero crossing detection and handling*, is computationally expensive and it also imposes an upper limit to the simulation step size, that is always less than the time between two consecutive discontinuities. Taking into account that SMPS switches commutate at a frequency that is higher than the continuous dynamics [2], the step size becomes limited to very small values.

There are several approaches that allow to mitigate the costs of the event location and handling [3], [4]. However, they do not escape from the limitations on the step size. Other strategies propose to use fixed steps with a compensation for the occurrence of events [5], [6]. A different solution for the efficient simulation of discontinuous ODEs is provided by the Quantized State System (QSS) methods [7], [1], that can predict the occurrence of events (simplifying the event location issue) and do not need to restart the whole simulation after each discontinuity [8].

Although the mentioned approaches can help to reduce computational costs, there are several situations that require the simulation of several hours or days of evolution what would lead to unacceptable simulation times. To avoid these problems, the precise switched models are usually replaced by averaged models that do not contain discontinuities [9], [10], [11]. These models can be simulated very fast, but at the expense of sacrificing detail (like voltage and current ripple). In addition, most simple averaged models may result inaccurate in some transient situations or during discontinuous conduction mode (anyway, there are large signal averaged models that work fine in most situations [12]). There are also methods that propose to include ripple information by superimpose it to the averaged model simulation [13]. However, these approaches are based on state space representations that require a previous mathematical treatment for each topology. Although this is very useful for analysis and design purposes, it is not as useful when the goal is limited to build a circuit model and simulate it.

For these reasons, most power electronics simulation practitioners use different models to analyze the transient and the long term behavior of the circuits. In the first case, detailed switched models are used while in the second case, averaged models are preferred. This, besides duplicating the effort regarding the model construction, usually implies running several simulations in order to capture the different transients that may occur during a long term evolution.

Motivated by these problems, in this article we propose a mixed modeling methodology that combines switched and averaged models, using the first ones during transient evolutions and the second ones during slowly varying conditions. The commutation between models is automatically done by an algorithm that detects the type of evolution. In addition, while the precise switched model is used, the algorithm adjusts the parameters of the averaged model so that it is correct irrespective of the operating point and conduction mode. In this way, the resulting models provide detailed and precise information during transients and they save calculations –preserving accuracy– in steady state situations.

The manuscript also describes the implementation of the methodology in a SMPS library that was built using the object oriented Modelica language [14]. This library allows users to connect mixed mode SMPS models with arbitrary circuit components (and with other domain models such as thermal and mechanical components). In addition, the performance of the proposed methodology is analyzed in several simulation examples.

The article is organized as follows: Section II introduces the basic principles of DC–DC SMPS and the modeling tools used along the work. Then, Section III presents the proposed mixed modeling strategy and Section IV discusses its implementation in the Modelica language. Finally, Section V shows simulation results and Section VI concludes the article proposing also some future lines of research.

## II. Background

This section introduces the basic circuit topologies of DC–DC switched mode power supplies, the main strategies to represent them using averaged models, and a brief introduction to the Modelica modeling language.

### A. DC–DC Switched Mode Power Supplies

DC–DC Switched Mode Power Supplies [15], [16] are electronic devices that convert a DC input voltage level into a different DC output voltage level using components switching at high frequency. The most used and common topologies of SMPS are derived from the following three non insulated circuits:

- Buck converter: The output voltage is lower than the input voltage ($V_{\text{out}} < V_{\text{in}}$). A simplified schematic of the Buck converter is shown in Fig.1.
- Boost converter: The output voltage is higher than the input voltage. ($V_{\text{out}} > V_{\text{in}}$).
- Buck - Boost converter: the output voltage can be higher or lower than the input voltage.

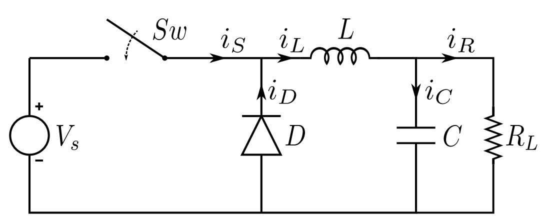


Fig. 1. Buck converter circuit

These circuits are called *second order* converters as they have two energy storage components (one inductor and one capacitor) [17]. There are also popular fourth order DC–DC converters[18], like the Cuk and SEPIC circuits, that contain two inductors and two capacitors.

In all cases, the output voltage is regulated by controlling the fraction of time that a switching component is in *on* state, i.e. the *duty cycle*. Usually, the duty cycle is defined by a PWM signal.

As it was already discussed in the introductory section, the simulation of these circuits imposes several difficulties to the numerical algorithms due to the presence of very frequent discontinuities. In consequence, long term simulations of these models yield a very high computational load, even using cheap fixed step numerical algorithms [5]. For these reasons, it is common to use simplified versions of the SMPS models obtained by *averaging* the different signals involved.

### B. Averaged SMPS Models

Averaged models [19], [20] approximate the behavior of power electronic switched models by describing the dynamics of the average values of currents and voltages over each period.

There are several strategies to obtain averaged models, that are usually classified as *state space averaging* [19], [21], [22] and *circuit averaging* [23], [24], [25], [26]. State space averaging techniques attempt to directly obtain pure continuous models in the form of state equations that have a similar behavior to that of the original switched models. Circuit averaging, in turn, is based on the direct replacement of the switching components of the original circuit.

Taking into account that this work aims to combine switching and averaged models, the second approach was adopted as it is simpler from a modeling perspective.

In circuit averaging, the continuous components (resistors, inductors and capacitors) remain unchanged while discontinuous components (switches and diodes) are replaced by controlled sources as shown in Figure 2. This replacement requires the parametrization of the controlled sources gain $K(d)$ as a function of the duty cycle. The parametrization can be easily done when the circuit works in *continuous conduction mode* (CCM), i.e. when the current in the inductors never becomes null. In that case, a simple analysis shows that function $K(d)$ can be expressed as

$$K(d) = \frac{1-d}{d} \tag{1}$$

However, in the discontinuous conduction case, the parametrization process becomes more complex.

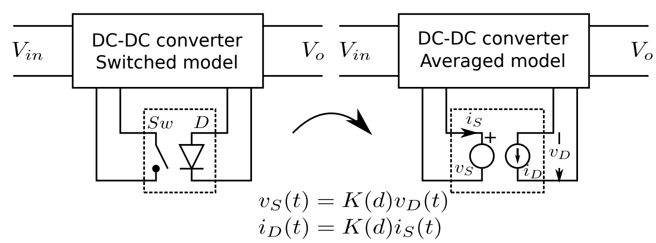


Fig. 2. Averaged model approximation of a DC-DC converter.

Another approach for circuit averaging is that of the Switched Inductor Model (SIM). There are methods for CCM [27], DCM [28] and unified methods that takes into account continuous and discontinuous conduction modes [12], [26]. In this approach, the averaged model is obtained replacing a non linear subcircuit called *switched inductor* (Fig.3a.) by an equivalent continuous circuit (Fig.3b.). The switched inductor consists of an inductor that has the particularity that one of its ports switches between two terminals $B$ and $C$ at a frequency $f_s = 1/T$. This port is connected to the terminal $B$ at the duty ratio $D_{ON}$ and to $C$ during the $D_{OFF}$ duty ratio where

$$\begin{array}{rcll} D_{OFF} & = & 1 - D_{ON} & \text{for } CCM \\ D_{OFF} & < & 1 - D_{ON} & \text{for } DCM \end{array} \tag{2}$$

Assuming that the switching period much smaller than the time constants of the SMPS, the parameters of the switched inductor averaged model can be computed as in [12]:

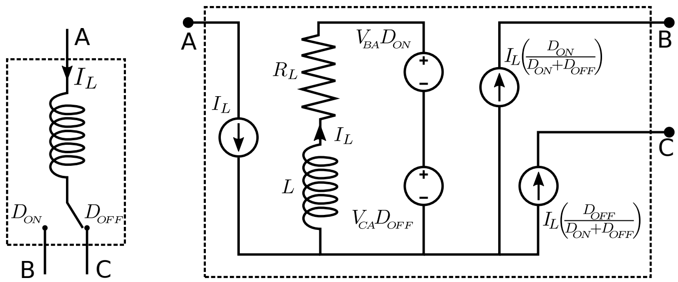


Fig. 3. Switched Inductor: a) Real configuration b) Unified switched inductor averaged model

$$\begin{array}{rcl} I_A & = & I_L \\ I_B & = & \frac{D_{ON}}{D_{ON}+D_{OFF}} I_L \\ I_B & = & \frac{D_{OFF}}{D_{ON}+D_{OFF}} I_L \end{array} \tag{3}$$

$$D_{OFF} = \begin{cases} 1 - D_{ON} & \text{when} \quad D^*_{OFF} \geq 1 - D_{ON} \\ D^*_{OFF} & \text{otherwise} \end{cases} \tag{4}$$

$$D^*_{OFF} = \frac{2L}{T} \frac{|I_L|}{|V_{CA}|(D_{ON} + D^*_{OFF})} \tag{5}$$

Unlike switched models, averaged SMPS models can be simulated very fast and they are the usual choice when the power supplies are part of larger systems that require long term simulations. However, they hide some details (like the voltage and current ripple) and they may require some mathematical effort to include non ideal and parasitic components.

### C. The Modelica Language

Switched and averaged models of SMPS can be modeled and simulated using several tools that range from specific circuit simulation packages like PSPICE [29] to general purpose simulation software like Matlab/Simulink [30]. Among them, Modelica is a unified modeling language that combines the graphical modeling facilities of SPICE with the general purpose features of the other tools [14].

Modelica models are usually grouped in libraries, like the *Modelica Standard Library* (MSL), an open repository of components of different domains (electrical, mechanical, hydraulic, thermal, etc.). It is worth mentioning that this library contains an implementation of SPICE for modeling and simulation of electrical circuits [31]. The resulting models can be built and simulated using different software tools. OpenModelica [14] is the most complete open source package, while Dymola [32] and Wolfram System Modeler are the most used commercial tool.

Besides the already mentioned implementation of SPICE, Modelica was widely used for modeling and simulation of switched mode power supplies [33], [34], [35].

### D. Related Work

To the best of the authors knowledge, the idea of mixed mode modeling combining switched and averaged models of SMPS was never proposed before. However, there is a similar idea for the simulation of AC networks [36], where the authors propose to use a time domain model during transients and a phasor-based model during steady state. This methodology is also implemented using the Modelica language.

## III. Mixed Modeling Methodology

This section introduces the proposed mixed modeling methodology, presenting first the basic idea and the general scheme, and then describing the different sub-models and the commutation strategy.

### A. Basic Idea and General Scheme

As it was mentioned in the introductory section, the proposed methodology is based on the alternate usage of switched and averaged models. More precisely, the model works in precise switched mode until a slowly varying condition is detected and the model commutes to the averaged mode. The model then works in this mode until some change is detected and it goes back to the precise switched mode.

The general scheme is shown in Figure 4. The *Mixed Model* contains the circuit with the continuous components (capacitors, inductors and resistors) connected through a *Commutator* to either the *Switched Two–Port* (formed by the switch and the diode) or the *Averaged Two–Port* (formed by the controlled sources). The commutator is driven by a *Mode Selector*, that uses information of the continuous components to detect the transient or slowly varying condition in order to select the mode of operation. The complete mixed model is fed by an input voltage $V_{in}(t)$ and produces an output voltage $V_{out}(t)$. It also receives the duty cycle signal $d(t)$.

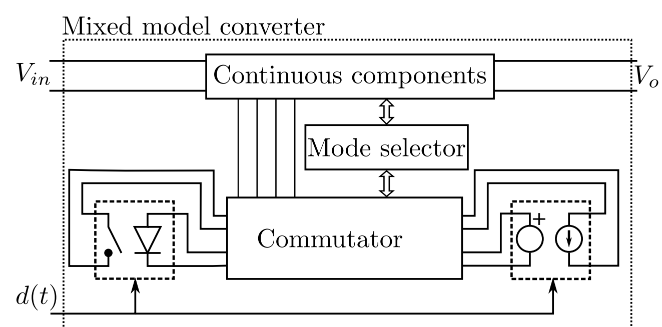


Fig. 4. Mixed model schema of a DC/DC converter.

We describe next the different components of the model.

### B. Continuous Components

This part of the mixed model contains the complete circuit without the switches and diodes. Among the components, the second order converters (Buck, Boost, and Buck–Boost) contain one inductor and one capacitor, while the fourth order converter (Cuk and SEPIC, for instance) contain two inductors and two capacitors. The current by the inductors and the voltage of the capacitors are taken as state variables and they are monitored by the Mode Selector to decide the operation mode.

### C. Swithed Two–Port Sub–Model

The Switched Two–Port sub–model contains one switch and one diode and it is connected to the Commutator. This sub–model receives the duty cycle signal $d(t)$, used to generate a PWM signal with period $T$ that drives the switch. This PWM signal is only generated when the model works in switched mode.

### D. Averaged Two–Port Sub–Model

The Averaged Two–Port sub–model contains one controlled voltage source and one controlled current source, whose gains depend on the duty cycle signal $d(t)$. The controlled voltage and current sources obey the equations

$$v_S(t) = K(d) \cdot v_D(t) \tag{6}$$
$$i_D(t) = K(d) \cdot i_S(t) \tag{7}$$

Here, $v_S(t)$ and $v_D(t)$ are the voltages at the voltage and current sources, respectively. Analogously, $i_S(t)$ and $i_D(t)$ are the currents at the voltage and current sources.

The expression for function $K(d)$ for any conduction mode (continuous or discontinuous) can be derived as follows. Let $\langle i_D \rangle$ be the mean value of the diode current during the "off" state of the switch (with duration $(1-d)\cdot T$) and let $\langle i_S \rangle$ be the mean value of the switch current during its "on" state (with duration $d \cdot T$). Then, the mean values of these signals in a complete period (with duration $T$) can be computed as

$$\bar{i}_D = \langle i_D \rangle \cdot (1-d) \tag{8}$$
$$\bar{i}_S = \langle i_S \rangle \cdot d \tag{9}$$

Taking into account that the averaged two–port sub–model does not take power, the following equation must be verified

$$P_D = \bar{i}_D \bar{v}_D = \bar{i}_S \bar{v}_S = P_S, \tag{10}$$

and then the controlled source gain $K(d)$ verifies

$$K(d) = \frac{\bar{v}_S}{\bar{v}_D} = \frac{\bar{i}_D}{\bar{i}_S} = \frac{\langle i_D \rangle}{\langle i_S \rangle}\frac{1-d}{d} = k_1 \cdot \frac{1-d}{d} \tag{11}$$

where $k_1 = \frac{\langle i_D \rangle}{\langle i_S \rangle}$ is the parameter computed by the Mode Selector (described later on this section). Notice that when $k_1 = 1$, Eq.(11) coincides with Eq.(1), which is the correct expression when the converter works in continuous conduction mode.

### E. Mode Selector

This component is the core of the strategy. It implements an algorithm that distinguishes the transient from the slowly varying condition, and drives the commutator accordingly.

The selector initially assumes that the system is in transient state and activates the switched mode. Working in this mode, at the end of each period of the PWM signal with duration $T$, the algorithm computes the mean value of each state variable $\bar{x}_i(t_k)$ and the corresponding ripple amplitude $r_i(t_k)$. It also computes the mean switch voltage of the switched two–port component $\bar{v}_S(t_k)$ and the mean diode voltage $\bar{v}_D(t_k)$

Then, the algorithm checks if the mean value variation in $\bar{x}_i$ since the last period is less than a fraction of its ripple, i.e.,

$$|\bar{x}_i(t_k) - \bar{x}_i(t_{k-1})| \leq \alpha \cdot r_i(t_k) \tag{12}$$

where $\alpha$ is a user defined parameter.

In addition, the algorithm calculates the correct parameter $k_1$ of the controlled sources of the averaged model, that can be derived from Eqs.(6) and (11) as:

$$k_1 = \frac{\bar{v}_S(t_k)}{\bar{v}_D(t_k)} \cdot \frac{d(t_k)}{1-d(t_k)} \tag{13}$$

and checks the condition

$$|k_1 - k_1^{prev}| \leq \kappa \tag{14}$$

where $k_1^{prev}$ was the value of $k_1$ calculated for the previous PWM period and $\kappa$ is another user defined parameter.

Provided that the inequality (12) is verified for all the state variables, and that inequality (14) is also accomplished, the algorithm decides that it is in a slowly varying condition so the averaged model can be used. Then, the last value computed for $k_1$ is used to parametrize the controlled sources and the commutation to the averaged mode is performed. During the commutation, the state variables of the continuous components are reset to their last average values, i.e., $x_i(t_a^+) = \bar{x}_i(t_k)$.

Once the model works in averaged mode, the selector algorithm only verifies that the states and the duty cycle do not experience significant changes with respect to their mean values evaluated at the last commutation time when the model was working in switched mode. This is, while the condition $|x_i(t) - \bar{x}_i(t_k)| \leq \sigma \cdot r_i(k)$ is met for all the states, and additionally $|d(t) - d(t_k)| \leq \Delta$ the model remains in averaged mode. Here, $\sigma$ is a parameter that defines the maximum variation that can occur in a state variable before turning back to the switched mode. Similarly, $\Delta$ is the maximum variation of the duty cycle in averaged mode.

Whenever a state or the duty cycle change beyond the prescribed limits, the mode selector decides that the switched mode must be used. Notice that $\bar{x}_i(t_k)$ and $d(t_k)$ are the mean state values and $DC$ computed in the last commutation time when the switched sub-model was running. In this way, even an arbitrarily slow varying unidirectional drift in the parameter will eventually result in a difference larger than the detection threshold.

In order to avoid a difference in the PWM phase, the commutation is performed when the time elapsed since $t_k$ reaches an integer number of periods. During the commutation to the switched mode, the selector also resets the state variables according to $x_i(t_s^+) = x_i(t_k) + x_i(t_s) - \bar{x}_i(t_k)$. That way, the value of $x_i$ at the beginning of the period is that it had at the beginning of the last period in switching mode (i.e., $x_i(t_k)$) plus the variation of the mean value during the averaged mode evolution.

Figures 5 and 6 illustrate the evolution of the current in the inductance of a Buck DC-DC converter, where the commutation conditions between both modes can be observed.

The behavior of the mode selector can be formalized by Algorithms 1–2.

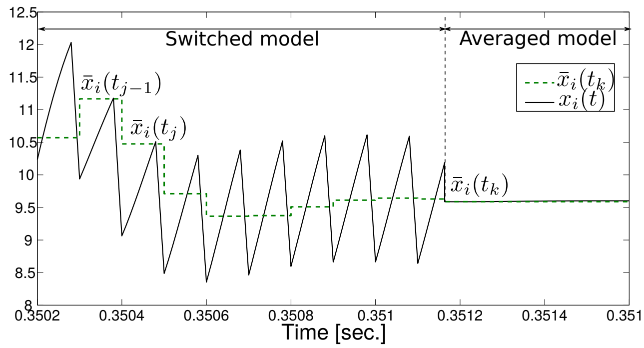


Fig. 5. Detail of the current in the Buck converter when the hybrid model switches from the switched to the averaged submodel

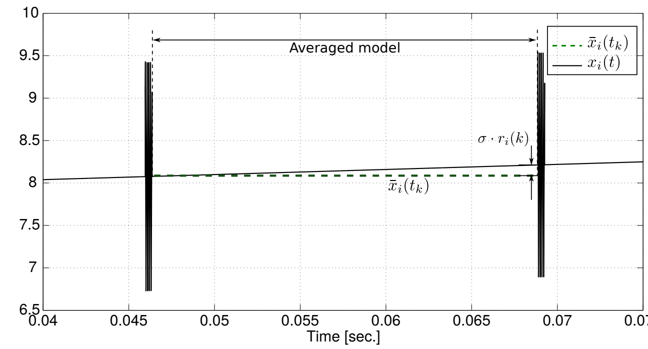


Fig. 6. Detail of the current in the Buck converter when the hybrid model switches from the averaged to the switched submodel

**Algorithm 1** Commutation from switched to averaged mode

**when** $mode == Switched$ **and** $endOfPeriod$ **then**
$t_k \leftarrow t$ // store the final time of the last switching period
$\bar{v}_S \leftarrow \frac{1}{T} \cdot \int_{t_k-T}^{t_k} v_S(\tau)\mathrm{d}\tau$ // Compute the mean switch voltage value.
$\bar{v}_D \leftarrow \frac{1}{T} \cdot \int_{t_k-T}^{t_k} v_D(\tau)\mathrm{d}\tau$ // Compute the mean diode voltage value.
$toAveragedEnabled \leftarrow True$ // Initial flag.
**for** $i \leftarrow \{1,..,nstates\}$
$\bar{x}_i^{prev} \leftarrow \bar{x}_i$ // Store the previous mean value of each state variable
$\bar{x}_i \leftarrow \frac{1}{T} \cdot \int_{t_k-T}^{t_k} x_i(\tau)\mathrm{d}\tau$ // Compute each state mean value.
$r_i \leftarrow \max x_i(\tau) - \min x_i(\tau)$ // last period ripple amplitude
**if** $|\bar{x}_i - \bar{x}_i^{prev}| > \alpha \cdot r_i$ **then**
$toAveragedEnabled \leftarrow False$ // cannot go to average mode
**end if**
$\hat{x}_i \leftarrow x_i$ // Store the actual state value
**end for**
$k_1^{prev} \leftarrow k_1$ // Store the previous parameter $k_1$
$k_1 \leftarrow \frac{\bar{v}_S}{\bar{v}_D} \cdot \frac{d}{1-d}$ // Compute the actual controlled source parameter $k_1$
**if** $|k_1 - k_1^{prev}| > \kappa$ **then**
$toAveragedEnabled \leftarrow False$ // cannot go to averaged mode
**end if**
**if** $toAveragedEnabled$ **then**
**for** $i \leftarrow \{1,..,nstates\}$
$x_i \leftarrow \bar{x}_i$ // Reset the state variables with their mean values
**end for**
$mode \leftarrow Averaged$
$toAveragedEnabled \leftarrow False$ // Reset to "false" the flag
**end if**
**end when**

**Algorithm 2** Commutation from averaged to switched mode

**when** $mode == Averaged$ **and** $(\exists i : |x_i - \bar{x}_i| \geq \sigma \cdot r_i(k)$ **or** $|d - d^{prev}| \geq \Delta)$ **then**
$toSwitchedEnabled \leftarrow True$
**end when**
**when** $toSwitchedEnabled$ **and** $t == t_k + T \cdot \text{floor}\left(\frac{t-t_k}{T} + 1\right)$ **then**
**for** $i \leftarrow \{1,..,nstates\}$
$x_i \leftarrow \hat{x}_i + x_i - \bar{x}_i$ // Reinitialize state variables to their last value in switched mode plus their variation in averaged mode.
**end for**
$mode \leftarrow Switched$
$toSwitchedEnabled \leftarrow False$
**end when**

## IV. Modelica implementation

Based on the proposed mixed modeling strategy, a library of SMPS was developed in the Modelica language. This library includes mixed models of five SMPS topologies: Buck, Boost, Buck-Boost, Cuk, and a bidirectional variant of the Boost converter. These models, built using components from the Electric Library of the MSL, can be used as regular Modelica electrical components to be part of more complex circuits and models.

The core component of the implementation is the mixed two–port commutation cell, containing a pair switch–diode for the switching mode, a pair of current and voltage sources for the averaged mode, and the algorithms for the automatic selection of the mode (Algorithms 1–2). This mixed commutation cell can be connected to standard circuit components (inductors, capacitors, resistors, additional diodes, etc.) in order to build mixed models of SMPS with different topologies. Figure 7, for instance, shows a Buck SMPS that was built making use of the mixed two–port commutation cell.

As it can be observed, the construction of the mixed model of the source is very simple as it consists in the connection of the cell with the remaining circuit components following the SMPS topology. The mixed SMPS model is then completed by declaring the correspondence between the state variables of the algorithms and those of the SMPS circuit components. While this is the procedure to build new topologies of SMPS sources, the library already contains SMPS models for the basic SMPS topologies, as it was mentioned above.

The basic commutation cell (and also the mixed SMPS models of the library) have some user defined parameters like the PWM frequency, the maximum variation of the states as a fraction of the ripple in averaged mode ($\sigma$), its minimum fraction in switched mode ($\alpha$), and the maximum variation of the duty cycle in averaged mode ($\Delta$). In addition, all the circuit components can be parametrized (including non ideal parameters of switches and diodes).

Besides the mixed models, the library contains pure switched and pure averaged SMPS models allowing the users to perform comparisons. The latter were built using the Unified Switched Inductor Model approach [12], so they work for continuous and discontinuous conduction modes.

## V. Examples and Results

In this section, we present simulation results in order to analyze the performance of the proposed mixed modeling approach under different situations.

### A. Experimental Setup

All the experiments were run under the following settings:

- A computer with an Intel (R) Core (TM) i3-2350M CPU @ 2.30GHz under a Linux operating system was used.

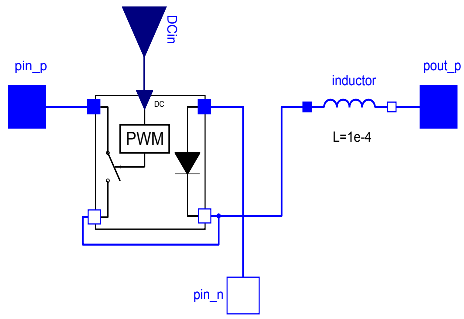


Fig. 7. Modelica mixed model of a Buck converter.

- The simulations were run on WolframSystemModeler v4.2.0 with DASSL solver (relative tolerance $tol = 10^{-6}$).
- In the mixed models, the parameters to commutate between both modes were set as $\sigma = 0.03$ and $\alpha = 0.6$.
- Results with pure switched and pure averaged models correspond to those of the mixed SMPS library.

### *B. Open Loop DC-DC Converters*

We first simulated the five topologies of the library in open loop configuration with the following additional settings:

- The PWM frequencies were: 10KHz (Buck and Buck-Boost), 50KHz (Cuk) and 100KHz (Boost and bidirectional Boost).
- In all cases the duty cycle $d(t)$ followed the profile depicted in Figure 8.
- Besides the duty cycle changes, a resistive load $R_l = 1\Omega$ was initially used, and at time $t = 0.65$ its value was changed to $R_l = 100\Omega$.
- The final simulation time was set $T_{stop} = 10$ seconds.

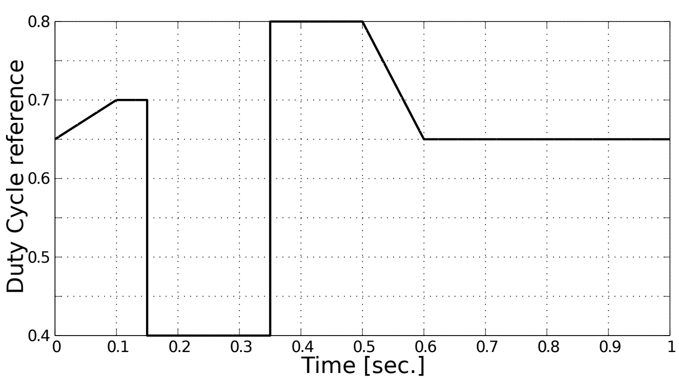


Fig. 8. Duty cycle used in all the SMPS topologies

The changes in the duty cycle and the load were included in order to show the behavior of the mixed model under different transient conditions.

The Buck converter output voltage and inductor current evolutions obtained with the mixed, the pure averaged and the pure switched model are compared in Figs.9 to 10.

The figures show a perfect match between the switched and the mixed model trajectories. The only difference between both evolutions is observed when the mixed model works in averaged mode. However, that difference is only due to the absence of the ripple information. A more detailed plot of the trajectories is shown inside Figure 10, where the perfect adjustment between both models can be observed.

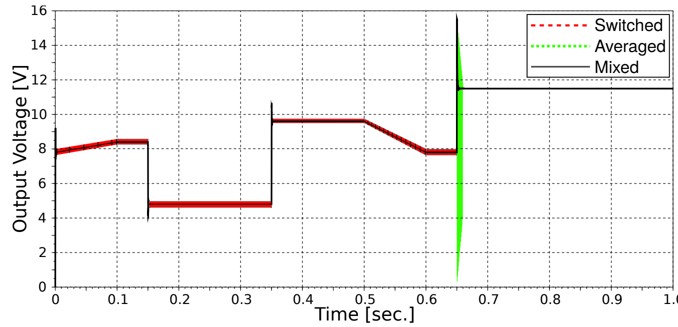


Fig. 9. Buck converter output voltage.

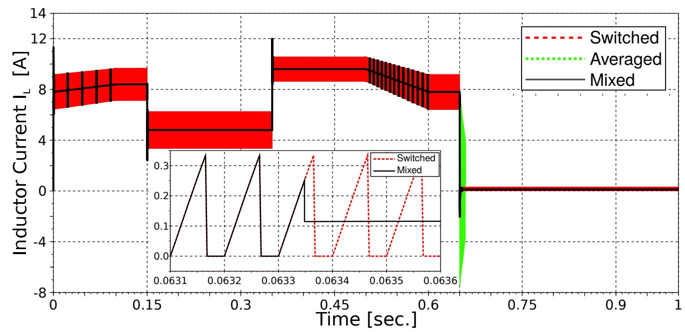


Fig. 10. Buck converter inductor current.

In all cases, the pure averaged model also provides accurate results, except during the transient caused by the sudden increment in the resistance load at time $t = 0.65s$. In the switched model, this transient provokes an oscillation between the capacitor and inductor that eventually results in a negative current through the inductor, an ill behavior is not properly captured by the pure averaged model.

For reasons of space, the trajectories obtained with the remaining four SMPS topologies were not included. However, they show very similar features to those observed in the Buck converter.

Regarding the computational costs, Table I compares the CPU time and the number of discontinuities of the switched and the mixed models. The Buck converter was also simulated using PSIM, arriving to identical results to those of the switched model in Modelica. Unlike Modelica, PSIM uses a fixed step solver that was setup to the maximum admitted step size.

The comparison of PSIM and Modelica for the same switched model reveals that PSIM is about twice as fast as Modelica. A simple analysis allows to conclude that this difference is preserved in the remaining examples, so we only report Modelica results.

As expected, the number of discontinuities of the switched models is similar to the number of PWM commutations (adding the number of changes in the diode conduction state when the source is in discontinuous conduction mode).

| Model | No. of Disc. | CPU Time (s) |
|---|---|---|
| Buck (Mixed) | 2518 | 0.23 |
| Buck (Switched) | 293491 | 62.9 |
| Buck (PSIM) | N/A | 9 |
| Boost (Mixed) | 49030 | 1.53 |
| Boost (Switched) | 2001675 | 463.7 |
| Boost (PSIM) | N/A | 72 |
| Bi. Boost (Mixed) | 42922 | 5.47 |
| Bi. Boost (Switched) | 2001251 | 443.1 |
| Bi. Boost (PSIM) | N/A | 76 |
| Cuk (Mixed) | 14978 | 8.6 |
| Cuk (Switched) | 586963 | 141.2 |
| Cuk (PSIM) | N/A | 127 |
| Buck–Boost (Mixed) | 2785 | 1.36 |
| Buck–Boost (Switched) | 296488 | 61.6 |
| Buck–Boost (PSIM) | N/A | 47 |

TABLE I

COMPUTATIONAL COSTS OF THE DIFFERENT SMPS TOPOLOGIES USING SWITCHED AND MIXED MODELS.

However, in the mixed model, discontinuities only occur when it works in switching mode, so their number is significantly smaller. The difference in the number of discontinuities is effectively translated into a reduction in CPU time taken by the simulations.

It is worth mentioning that in the examples analyzed we introduced several transient situations (changes in the load and in the duty cycle) in a short interval of time in order to show the behavior of the proposed method under different transients. There are many cases where the systems remain in steady state for longer periods or where the variables evolve very slowly. Thus, in those cases, the mixed model would work in averaged mode most of the time exhibiting even more computational advantages than those observed here.

In order to support this observation, we simulated several times the buck converter introducing a different number of transients in each simulation. In every case, we measured the fraction of time that the mixed model uses the switched mode $T_{SW}/T_{stop}$ (i.e., the fraction of time that the system is in transient state). We also measured the CPU time taken by the mixed model simulation and the CPU time taken by the switched model simulation.

The results are shown in Figure 11, where the CPU time of the mixed model divided by the CPU time of the switched model is plotted against the fraction of time that the mixed model uses switched mode. The plot shows that when the switched mode is used more than $15.5\%$ of the time, then the switched simulation is faster than the mixed mode simulation. Otherwise, when the system remains in a slowly varying situation most of the time, then the mixed model is several times faster.

## C. Closed loop battery charger

In this example, a buck converter with non-ideal components is used for charging a Lead-Acid battery with the scheme shown in Figure 12.

The buck converter includes a non linear inductor, a parasitic resistor in series with the inductor, and the forward voltage drop in the diode. The construction of this mixed model circuit is essentially identical to that of the ideal Buck converter

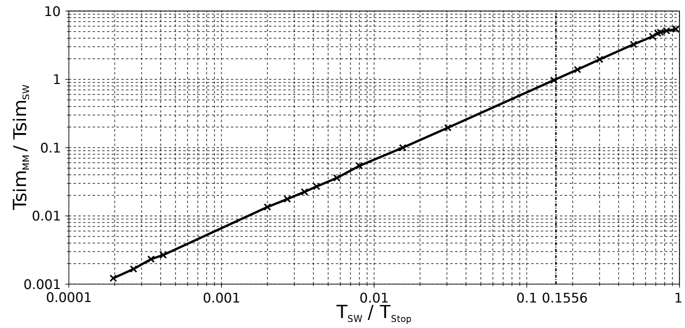


Fig. 11. CPU time of mixed mode and switched models vs. fraction of time in transient operation.

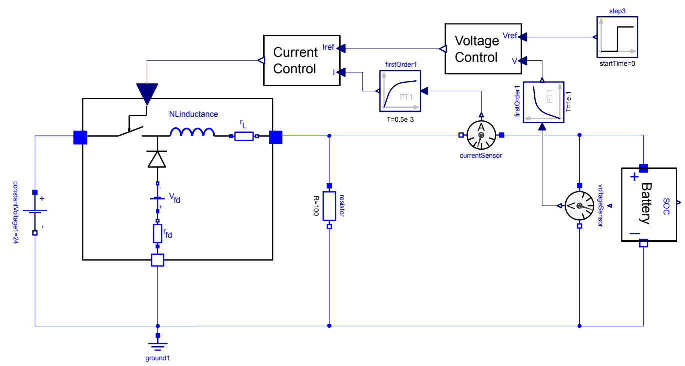


Fig. 12. Battery charger Modelica representation.

of Fig.7, except for the use of the non linear inductor and the additional components that are just connected around the commutation cell.

The nonlinear characteristic of the inductor is modeled as in [37], using a (saturated) fourth order equation:

$$L(i) = \begin{cases} L_0 + L_1 i + L_2 i^2 + L_3 i^3 + L_4 i^4 & \text{if } i < i_{\max} \\ L_{\max} & \text{otherwise} \end{cases}$$

where $L_{\max} = L_0 + L_1 i_{\max} + L_2 i_{\max}^2 + L_3 i_{\max}^3 + L_4 i_{\max}^4$ and with coefficients $L_0 = 3.5 \cdot 10^{-3}$, $L_1 = -1.2 \cdot 10^{-4}$, $L_2 = -1.3 \cdot 10^{-4}$, $L_3 = 3 \cdot 10^{-5}$, $L_5 = -1.9 \cdot 10^{-6}$, and $i_{\max} = 100$A.

The battery model is that of [38], consisting in a controlled voltage source with a serial resistance $R$. The open circuit source voltage $E(t)$ is calculated by a non linear equation dependent on the actual charge of the battery ($\int i_b \mathrm{d}t$):

$$E(t) = E_0 - K \frac{Q}{Q - \int i_b dt} + A e^{-B \int i_b \mathrm{d}t} \tag{15}$$

with the parameters listed in Table II

| Symbol | Parameter | Value |
|---|---|---|
| $E_0$ | Constant Voltage | 12.6463 |
| $K$ | Polarization Voltage | 0.33 |
| $Q$ | Nominal battery capacity | 20 |
| $A$ | Amplitude of the exponential zone | 0.66 |
| $B$ | Inverse of the exponential zone time constant | 2884.61 |
| $R$ | Serial resistance | 0.25 |

TABLE II

BATTERY PARAMETERS.

The battery charger implements two saturated Proportional Integral (PI) controllers defined by:

$$
\begin{aligned}
e(t) &= u_{ref}(t) - u(t) \\
y(t) &= \begin{cases} Y_{up} & \text{if } K + K_p e(t) + K_i \int e(t)\, d \geq Y_{up} \\ Y_{low} & \text{if } K + K_p e(t) + K_i \int e(t)\, d \leq Y_{low} \\ K + K_p * e(t) + K_i * \int e(t)\, dt & \text{otherwise} \end{cases}
\end{aligned}
$$

where $Y_{up}$ is the upper saturation level and $Y_{low}$ is lower simulation level. The PI parameters for the current controller are $K = 0.5$, $K_p = 0.05$, $K_i = 0.01$, $Y_{up} = 0.99$ and $Y_{low} = 0.01$, while the PI parameters for the voltage controller are $K = 0$, $K_p = 9.5$, $K_i = 0.9$, $Y_{up} = 3$ and $Y_{low} = -3$.

The simulation reveals that reaching the full charge of the battery takes almost 4 hours. The battery voltage transient is shown in Figure 13, exhibiting a perfect match between the mixed and the switched models (since the mixed model works in switched mode during the transient) while there is a noticeable difference with the averaged model, which cannot accurately take into account the presence of nonlinear and parasitic components.

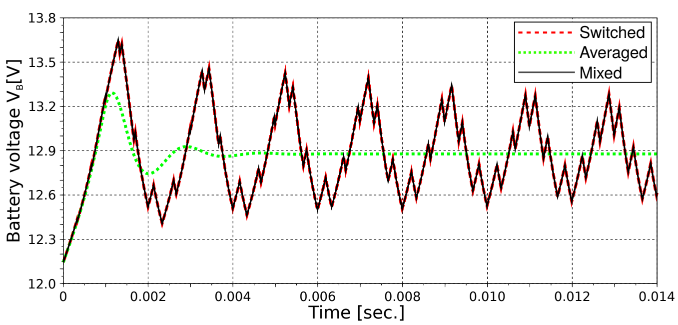


Fig. 13. Battery voltage trajectory (startup).

Table III compares the simulation time of the switched and averaged models. This time, the difference is huge as the switched model needs more than 18 hours to complete the simulation against the 4 seconds required by the mixed model.

| Model | No. of Disc. | CPU Time (seg) |
|---|---|---|
| Mixed | 12443 | 4.05 |
| Switched | 174084201 | 65971.4 |

TABLE III
SIMULATION COMPUTATIONAL COSTS OF THE BATTERY CHARGER USING SWITCHED AND MIXED MODELS.

### D. Regulated Buck Converter

Figure 14 shows a regulated Buck converter model taken from [39] with a controller implemented using circuit components, and containing a current limiter device. The model parameters are listed in Table IV and the experiment, reproducing the results of the cited reference, incudes an input voltage change from $20V$ to $40V$ at $t = 100ms$.

The model was simulated until a final time $t_f = 120$ms using the switched, the averaged, and the mixed models of the SMPS library. Figure 15 shows the output voltage simulated waveform, where a perfect match between the switched and

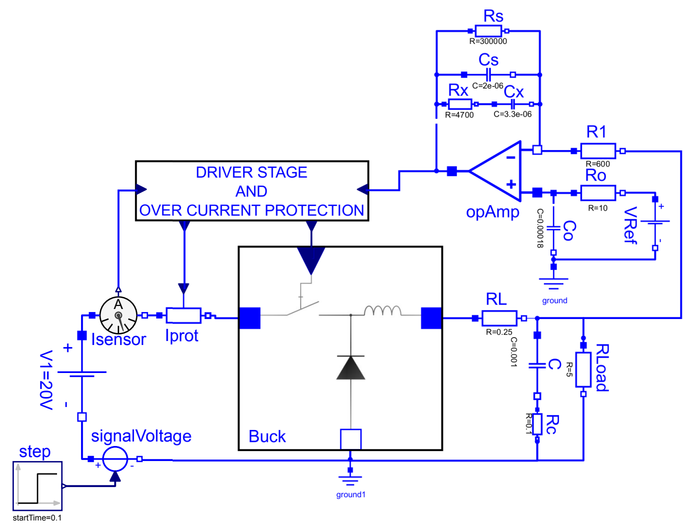


Fig. 14. Modelica model of a regulated Buck.

| Power Stage | Control Circuit | PWM modulator |
|---|---|---|
| $V_1 = 20V$ | $R_1 = 0.6k\Omega$ | $\alpha = 0.2V/\mu s$ |
| $C = 1mF$ | $R_S = 300k\Omega$ | $DC_{max} = 0.85$ |
| $L = 200\mu H$ | $R_x = 4.7k\Omega$ | $I_{max} = 4A$ |
| $R_L = 0.25\Omega$ | $C_S = 2\mu F$ | $F_s = 20kHz$ |
| $R_C = 0.1$ | $C_x = 3.3\mu F$ | |
| $R_{ON} = 0.05$ | $R_o C_o = 1.8mF$ | |
| $R_{LOAD} = 5\Omega$ | | |

TABLE IV
REGULATED BUCK MODEL PARAMETERS.

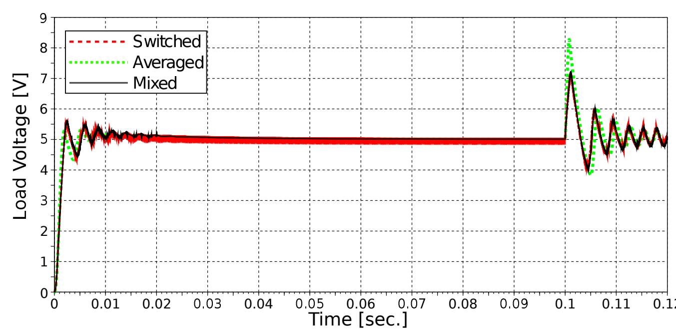


Fig. 15. Simulated load voltage.

mixed models can be observed, while the averaged model contains a noticeable error during the transient evolutions.

The reason for the averaged model mismatch can be easily understood by looking at the switch current evolution in Figure 16. There, in the switched model, the current limiter action cuts the peak ripple over its maximum allowed value $I_{\max} = 4$A. This behavior is not captured by the averaged model since the mean current never reaches this limit. The mixed model, however, also cuts the ripple peaks when it works in switched mode, and it uses these correct trajectories to identify the averaged model parameters. In consequence, the mixed model provides correct switched and averaged results.

Table V reports the CPU time and the number of discontinuities involved in each experiment. As expected, the mixed model is again faster than the switched model. However, the

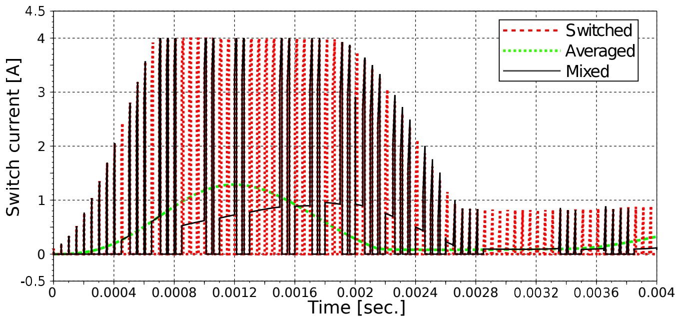


Fig. 16. Switch Current Startup in the Regulated Buck Model

difference is not as noticeable as in the previous examples since in this experiment the model remains most of the time in transient situation.

| Model | No. of Disc. | CPU Time (s) |
|---|---|---|
| Mixed | 2319 | 6.53 |
| Switched | 5077 | 15.42 |

TABLE V
COMPARISON OF THE CLOSED-LOOP BUCK SIMULATION PERFORMANCE

## VI. CONCLUSIONS

A novel modeling approach that combines switched and averaged models for fast and precise simulation of DC–DC SMPS was introduced. This mixed methodology was used to build a SMPS library in the Modelica language that was then tested in the simulation of different circuit topologies in open and closed loop configurations.

The simulation results show that the mixed modeling approach can significantly reduce the CPU times without sacrificing accuracy or hiding important phenomenas. The CPU time reduction is caused by the absence of discontinuities when the strategy selects the averaged mode. The accuracy of the results is ensured by the use of the detailed switched model in transient situations and by the validity of the averaged model, that is automatically re–parametrized after any significant change in the operating conditions.

Although the mode selector algorithm has some complicated details, the entire methodology is simple and can be easily extended to different circuit topologies. More precisely, adding a new topology to the Mixed SMPS Modelica Library only requires to build the switched circuit replacing the pair formed by the switch and diode by the mixed model commutation cell. Moreover, the addition of nonlinear and parasitic components is straightforward since the methodology does not rely on any assumption on the remaining circuit.

Regarding future work, we are currently extending the results to analyze their usage in large systems involving several SMPS. In particular, we want to apply the strategy to hybrid power generation systems, where the use of pure switched models yields non-affordable computational costs [40]. Also, we are studying the advantages of simulating mixed SMPS models using Quantized State Systems algorithms [1], as they are considerably faster in the simulation of switched models of SMPS [8].

In its actual state, the proposed technique is only useful to simulate PWM regulated DC-DC converters. However, we think that the idea might be also applied to simulate more general switched circuits such as current-mode controlled converters, switched mode rectifiers, DC-AC inverters, etc.

All the models simulated in Section V are part of the Mixed SMPS Modelica Library, that can be downloaded from http://www.fceia.unr.edu.ar/~kofman/files/MixModDcDc.mo.

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