EXTENDED MPC FOR CLOSED-LOOP RE-IDENTIFICATION BASED ON PROBABILISTIC INVARIANT SETS

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Resumen: Recently, a Model Predictive Control (MPC) scheme suitable for closed-loop re-identification was proposed which solves, in a non-conservative form, the potential conflict between the persistent excitation of the system and the stabilization. The scheme uses the concept of probabilistic invariance to define the target set, exploiting in that way the knowledge of the probabilistic distribution of the excitation signal to design a non-competitive two-objective MPC formulation. In this work, we prove some theoretical properties of the scheme that have fundamental practical consequences, including the finite–time convergence to the target set and a lower probability bound about the period of time the state remains in that set for the identification procedure. We also include new simulation results comparing the performance of the proposed approach with those of a previous deterministic formulation.

Palabras Claves: Model predictive control, closed-loop identification, probabilistic invariant set, finite-time convergence.

1. INTRODUCTION

Model predictive control (MPC) is a popular control technique that based on a simplified model of the system under control, solves an on-line optimization problem to determine the current control action. As the system conditions change, the model requires an update (re-identification) that usually must be performed in a closed loop fashion in order. One problem of the closed-loop re-identification is that the control objective is opposite to those of exciting the system for identification: while the controller is devoted to maintain the system at a given equilibrium most of time, the excitation procedure agitates the system around it, with the objective of producing output-input data with enough dynamic information.

In the MPC framework, several strategies were developed to perform closed-loop re-identification. An early strategy, consisting in the addition of an excitation constraint, was presented in (Genceli and Nikolaou, 1996). In (Zacekova *et al.*, 2013) a two-stage controller approach is presented. Recently, a study of several MPC re-identification methods is made in (Potts *et al.*, 2014), emphasizing the so-called MPC Relevant Identification

(MRI). In (Patwardhan and Gopaluni, 2014), the generation of a persistent excitation (PE) signal by means of the maximization (instead of the minimization) of the MPC cost function is proposed. This way, the variance (variability) of the signal is maximized while the process variables fulfill the constraints.

The main theoretical drawback of all these schemes is that the formal feasibility and attractivity/stability properties are lost. In (González *et al.*, 2014), a MPC scheme suitable for re-identification is proposed, which ensures recursive feasibility and stability of a proposed invariant set, performing a safe closed-loop re-identification once the system reaches the set. However, the computation of the target invariant set is made according to the maximum value that the excitation signals, without exploiting the knowledge of their probabilistic distribution. This results in large target regions that conservatively contains the excited system evolution.

Recently, based on the concept of *Probabilistic Invariant Sets* (PIS) introduced in (Kofman *et al.*, 2012), a significant reduction of the conservativeness of the strategy was obtained in (Anderson *et al.*, 2016). The idea in this approach is to replace the target (robust) invariant sets by their probabilistic counterparts. So, once the excitation procedure starts, the state trajectories remain in the set with high probability (close to 1 for most practical problems), and if a state leaves the set, the control is resumed and the excitation aborted.

In this work, we prove some fundamental theoretical properties of the aforementioned strategy. In particular, we show that the target set is reached in finite time. Additionally, we prove that, with certain probability, the state remains inside the target set during a period of time that is long enough to ensure the correct re-identification of the model. We also extend the robustness analysis of (Anderson *et al.*, 2016) to ensure that the different properties are accomplished by a more general family of models. Finally, we included simulation results that compare the performance of this approach with a previous one based on deterministic sets.

2. PROBLEM STATEMENT AND BASIC DEFINITIONS

Consider a discrete time system described by a linear time-invariant model

$$x(k+1) = Ax(k) + Bu(k), \quad x(0) = x_0 \qquad (1)$$

where $x(k) \in \mathcal{X} \subset \mathbf{R}^n$ is the system state at the k–th sample time, x_0 is the initial state, and $u(k) \in \mathcal{U} \subset \mathbf{R}^m$ is the current control input. All along this work it is assumed that matrix $A \in \mathbf{R}^{n\times n}$ has all its eigenvalues strictly inside the unit circle, the pair (A, B) is controllable, the set $\mathcal{X}$ is convex and closed, the set $\mathcal{U}$ is convex and compact and both contain the origin in their interior.

The goal in this work is to develop a MPC strategy that accounts for the closed-loop re-identification of such a system.

2.1 Invariant sets and control

Next, some definitions and properties that will be used later to derive the main results of the work, are recalled.

Definition 1. **(γ-Control Invariant Set, γ-CIS)** Given $\gamma \in [0, 1]$, a set $\Omega \subseteq \mathcal{X}$ is γ-control invariant for system (1) associated with set $\mathcal{U}$, if $x(k) \in \Omega$ implies that $x(k+1) \in \gamma\Omega$ for some $u(k) \in \mathcal{U}$.

Definition 2. **(Controllable Set)** Given the set $\Omega \subset \mathcal{X}$, the one step controllable set to Ω, $\mathcal{Q}(\Omega)$, associated to the input set $\mathcal{U}$, is the set of all $x \in \mathcal{X}$ for which there exists $u \in \mathcal{U}$ such that $Ax + Bu \in \Omega$.

Definition 3. **(N–Steps Controllable Set)** Given the set $\Omega \subset \mathcal{X}$, the N steps controllable set to Ω, $\mathcal{Q}_N(\Omega)$, associated to the input set $\mathcal{U}$, is defined by $\mathcal{Q}_N(\Omega) \triangleq \mathcal{Q}(\mathcal{Q}_{N-1}(\Omega))$ with $\mathcal{Q}_1(\Omega) \triangleq \mathcal{Q}(\Omega)$.

Definition 4. **(Strict Interior Set)** Given a set $\Omega_1 \subset \mathbb{R}^n$, a set $\Omega_2 \subset \Omega_1$ is a strict interior set of Ω_1, if there exists $\varepsilon > 0$ such that for any $x \in \Omega_2$ the ball $\mathcal{B}_\varepsilon(x) = \{y : \|y - x\| < \varepsilon\} \subseteq \Omega_1$.

Next, a property relating the γ-CIS and its corresponding controllable set is presented:

Lemma 1. Let $\Omega \subset \mathbb{R}^n$ be a closed and convex γ-CIS, with $\gamma < 1$, for system (1). Then, Ω is a strict interior set of $\mathcal{Q}(\Omega)$.

Proof: Let the state $x \in \Omega$, and consider

$$\varepsilon = \inf\{\|y - z\| : y \in \partial\Omega,\ z \in \gamma\Omega\}, \qquad (2)$$

Taking into account that Ω is convex and $\gamma < 1$, it results $\varepsilon > 0$. Given that Ω is a γ-CIS for system (1), there exists $u \in \mathcal{U}$ such that $x^+ = Ax + Bu \in \gamma\Omega$. Let $\bar{x}$ be any point such that $\|x - \bar{x}\| < \varepsilon$ and define $\bar{x}^+ \triangleq A\bar{x} + Bu$. Then,

$$\left\|\bar{x}^+ - x^+\right\| = \|A(\bar{x} - x)\| \leq \|A\| \, \|x - \bar{x}\| < \varepsilon,$$

where the last inequality follows from the fact that A is Hurwitz (i.e, $\|A\| < 1$). Taking into account that Ω is closed, and recalling the definition of ε in Eq.(2), the fact that $\|\bar{x}^+ - x^+\| < \varepsilon$ implies that

$\bar{x}^+ \in \Omega$, and then, by definition of controllable set, $\bar{x} \in \mathcal{Q}(\Omega)$.

Then, the ball $\mathcal{B}_\varepsilon(x) = \{\bar{x} : \|\bar{x} - x\| < \varepsilon\} \subseteq \mathcal{Q}(\Omega)$, for all $x \in \Omega$, which means that Ω is a strict interior set of $\mathcal{Q}(\Omega)$. ■

3. PROBABILISTIC INVARIANT SETS

The concept of probabilistic invariant set associated to the excitation requirements necessary to perform suitable identifications is central in this work. So, we first define the excitation signal, according to the identification requirements.

Definition 5. **(Bounded White Noise)** Given a compact non empty set $V \subset \mathbf{R}^m$, we say that a stationary process $v : \mathbf{N} \to V$ is a bounded white noise if it satisfies $E[v(k)] = 0$ and $cov[v(k)] > 0$ for all $k \in \mathbf{N}$, and, additionally $v(k)$ is uncorrelated with $v(j)$, for $k \neq j$.

Notice that the fact that $v(k)$ is bounded white noise implies that it is also a persistent excitation of any order (Ljung, 1999).

Probabilistic Invariant Sets (Kofman *et al.*, 2012) ensure that the state trajectories remain inside them for all future time, with certain probability. However, in the context of the MPC scheme that will be proposed, it will suffice with ensuring that the trajectories remain in the set at the following step. For this reason, the concept of One Step Probabilistic Invariant Sets (OSPIS) is introduced next.

Definition 6. **(γ–OSPIS)** Let $p \in (0, 1]$ and $\gamma \in (0, 1]$. A set $S \subseteq \mathcal{X}$ is a γ–One Step Probabilistic Invariant Set with probability p of system (1) with $u(k)$ being a bounded white noise on $V \subset \mathcal{U}$, if and only if $\Pr[x(k + 1) \in \gamma S \mid x(k) \in S] \geq p$.

When $\gamma = 1$ a γ–OSPIS is simply an OSPIS. Furthermore, when $p = 1$ a γ-OSPIS is a γ-ISI set, as the one defined in González *et al.* (2014).

The following property relates the probability p of remaining in an OSPIS after one step with that of remaining longer inside that set. This property will play an important role to ensure the feasibility of the re-identification procedure.

Lemma 2. Let $p \in (0, 1]$. Let S be an OSPIS with probability p for System (1) with $u(k)$ being a bounded white noise on $V \subset \mathcal{U}$. Then, provided that $x(k) \in S$, it results that $Pr[x(k + 1) \in S \wedge x(k + 2) \in S \wedge \cdots \wedge x(k + q) \in S] \geq p^q$.

Proof: The fact that $u(k)$ is bounded white noise implies that $x(k)$ has a Markov property, i.e., given $x(k)$, the value of $x(k + 1)$ does not depend on past values of the state prior to time k. That way, the OSPIS property that $Pr[x(k + 2) \in S|x(k+1) \in S] > p$ is accomplished independently on the fact that $x(k) \in S$. Thus, $Pr[x(k + 2) \in S^t|x(k + 1) \in S \wedge x(k) \in S] > p$.

Then, subject to $x(k) \in S$, it results that

$$\begin{aligned} & Pr[x(k+2) \in S \wedge x(k+1) \in S] \\ &= Pr[x(k+2) \in S|x(k+1) \in S] \cdot Pr[x(k+1) \in S] \\ &\geq p^2 \end{aligned}$$

and the proof concludes by the recursive use of this reasoning. ■

4. MAIN RESULT

In this Section, the MPC formulation that uses OSPIS as target sets is presented.

4.1 Proposed Scheme

The basic idea consists in using a control law that drives the trajectories to a target set. Once the state enters the target set, the scheme should introduce a bounded input white noise signal that allows to perform the re–identification procedure. For that goal, the cost function is defined as follows:

Let $u_{pe}(k) \in \mathcal{U}^t$ be a bounded white noise signal for which $\mathcal{S}^t$ is an OSPIS with probability p for System (1). Then, being k the current sample time, the cost function is given by:

$$\begin{aligned} & V_N(x, \mathcal{S}^t, u_{pe}(k); \mathbf{u}) \\ &= (1 - \rho(x)) \sum_{j=0}^{N-1} [\alpha d_{\mathcal{S}^t}(x(j)) + \beta d_{\mathcal{U}^t}(u(j))] \\ &+ \rho(x)\|u(0) - u_{pe}(k)\|, \end{aligned}$$

where $\rho(x) = 1$ if $x \in \mathcal{S}^t$, and $\rho(x) = 0$ otherwise. Here, $d_{\mathcal{S}^t}(x(j))$ and $d_{\mathcal{U}^t}(u(j))$ represent the distance between the set and the point, α and β are positive real numbers and $N \in \mathbf{N}$ is the control horizon.

For any initial state x in $\mathcal{X}_N \triangleq \mathcal{Q}_N(\mathcal{S}^t)$, the optimization problem $P_N(x, \mathcal{S}^t, u_{pe}(k), k)$, to be solved at each time instant k, is given by:

Problem $P_N(x, \mathcal{S}^t, u_{pe}(k), k)$

$$\begin{aligned} & \min_{\mathbf{u}} V_N(x, \mathcal{S}^t, u_{pe}(k); \mathbf{u}) \\ & s.t. \\ & \quad x(0) = x, \\ & \quad x(j+1) = Ax(j) + Bu(j), \quad j = 0, ..., N-1 \\ & \quad x(j) \in \mathcal{X}, \; u(j) \in \mathcal{U}, \quad j = 0, ..., N-1 \\ & \quad x(N) \in \mathcal{S}^t \end{aligned} \tag{3}$$

Notice that $\rho(x)$ is a discontinuous function necessary to cancel the persistent excitation in case the state leaves $\mathcal{S}^t$. This could occur due to the presence of an external disturbance or even, with a small probability, lesss than $(1-p)$, due to the persistent excitation itself.

The control law resulting from the application of the receding horizon policy is given by a function $\kappa_N(x, \mathcal{S}^t) = u^0(0; x)$, where $u^0(0; x)$ is the first element of the (optimal) solution sequence $\mathbf{u}^0(x)$. This way, the closed-loop system under the MPC law is described as $x(j) = \phi_{\kappa_N}(j; x, \mathcal{S}^t) = A^j x + \sum_{i=0}^{j-1} A^{j-i-1} B\kappa_N(x, \mathcal{S}^t)$.

In (Anderson *et al.*, 2016), the asymptotic convergence of the closed loop trajectories to the target set $\mathcal{S}^t$ of the scheme was proved. However, it is also important to ensure convergence in finite time, as the identification procedure has to be applied inside that set. The following results establish that property

Lemma 3. Let $\mathcal{S}^t \subseteq \mathcal{X}$ be an OSPIS with probability $p \in (0, 1]$ for system (1) with $u(k)$ being a bounded white noise signal on $\mathcal{U}^t$. Let $x(0) = x \in \mathcal{X}_1$, where $\mathcal{X}_1$ is the one-step controllable set to $\mathcal{S}^t$ (i.e., $\mathcal{X}_1 = \mathcal{Q}(\mathcal{S}^t)$). Then, the target set $\mathcal{S}^t$ is reached in one step for the closed-loop system $x(j) = \phi_{\kappa_N}(j; x, \mathcal{S}^t)$, with $j \in \mathbf{N}$.

Proof: We need to consider the case when $x(0) \notin \mathcal{S}^t$, then $\rho(x) = 0$. Notice that the fact that $x(0) = x \in \mathcal{X}_1$ implies that a control action $u(0) \in \mathcal{U}^t$ exists such that $x(1) \in \mathcal{S}^t$, and then successive control actions exist for which $x(j)$ remain in $\mathcal{S}^t$ (from the invariance of that set). Then, the use of that control sequence has null cost, while any control action that leaves $x(1)$ outside $\mathcal{S}^t$ has a positive cost. Thus, the MPC will drive the state to the target set in one step. ■ Now, from Lemmas 1 and 3, the following result regarding the convergence on finite time to the target set $\mathcal{S}^t$ is established.

Theorem 1. Let $\mathcal{S}^t \subseteq \mathcal{X}$ be a $\gamma-$OSPIS with probability $p \in (0, 1]$ and $\gamma < 1$ for system (1) with $u(k)$ being a bounded white noise signal on $\mathcal{U}^t$. Then, $\mathcal{S}^t$ is reached in finite time for the closed-loop system $x(j) = \phi_{\kappa_N}(j; x, \mathcal{S}^t)$, with $x(0) = x \in \mathcal{X}_N$ and $j \in \mathbf{N}$.

Proof: In (Anderson *et al.*, 2016) it was proved that $\mathcal{S}^t$ is also a $\gamma-$CIS. Then, by Lemma 1, $\mathcal{S}^t$ is in the interior of $\mathcal{X}_1 = \mathcal{Q}(\mathcal{S}^t)$. Since we now also by (Anderson *et al.*, 2016) that $x(j)$ tends to $\mathcal{S}^t$ as j goes to ∞, then, a finite time K there exists such that $x(K) \in \mathcal{X}_1$. Then, by Lemma 3, the state $x(K+1) = \phi_{\kappa_N}(1; x(k), \mathcal{S}^t)$ will be in $\mathcal{S}^t$, which concludes the proof. ■

From the re-identification point of view, it is important to ensure that the trajectory is kept inside the target set $\mathcal{S}^t$ long enough to apply the identification procedure. Let us suppose that $T_{id} \in \mathbf{N}$ is the length of the data necessary to perform a suitable identification of the system of Eq. (1). Taking into account Lemma 2, whenever the trajectory enters set $\mathcal{S}^t$, it will remain inside it during T_{id} units of time with a probability greater than $p^{T_{id}}$. That way, if p is chosen to be sufficiently large, it can be ensured that the re-identification procedure can be frequently performed.

5. ROBUSTNESS ANALYSIS

The MPC scheme proposed above uses a target set $\mathcal{S}^t$ computed as an OSPIS for the system under certain input noise signal. The problem is that $\mathcal{S}^t$ depends on the model parameters (A and B), and we cannot assume that they are accurately known (and that is the reason why the re-identification procedure is needed).

In (Anderson *et al.*, 2016), it was shown that when the set $\mathcal{S}^t$ was computed as a γ–OSPIS, then it results an OSPIS for a family of models around the nominal one, and the properties derived about stability and convergence are still valid. However, the mentioned family of models corresponded to an affine parametrization by a scalar parameter around the nominal model. Here, we extend that result to a wider family under a generic parametrization.

Let $\mathcal{W} \subseteq \mathbf{R}^p$ be a proper C-set, and consider that matrices A and B in the nominal system of Eq.(1) are parametrized by $w \in \mathcal{W}$.

$$x(k+1) = A(w)x(k) + B(w)u(k), \qquad (4)$$

where $A(w)$ and $B(w)$ are Lipschitz functions on $\mathcal{W}$ satisfying $A(0) = A$, and $B(0) = B$. Then, the following theorem can be established

Theorem 2. Let $\mathcal{S}^t \subset \mathcal{X}$ be a γ-OSPIS with probability $p \in (0, 1]$ and $\gamma \in [0, 1)$ for system (1), with $u(k)$ being a bounded white noise signal on $\mathcal{U}^t$. Then, there exists a proper C-set $\mathcal{W}^r \subseteq \mathcal{W}$ such that for any $w \in \mathcal{W}^r$ the set $\mathcal{S}^t$ is an OSPIS with probability p for system (4) under the same bounded white noise signal.

Proof: Let $x(k) \in \mathcal{S}^t$. Compute $x(k+1) = Ax(k) + Bu(k)$, and $\bar{x}(k+1) = A(w)x(k) + B(w)u(k)$. Then, substracting both future values of the state we obtain

$$\bar{x}(k+1) - x(k+1) = [A(w) - A]x(k) + [B(w) - B]u(k)$$

applying norms and triangular inequality, it results that

$$\begin{aligned}\|\bar{x}(k+1)-x(k+1)\| &= \|[A(w)-A]x(k)\\ &\quad+[B(w)-B]u(k)\|\\ &\leq \|A(w)-A\|\cdot\|x(k)\|\\ &\quad+\|B(w)-B\|\cdot\|u(k)\|\\ &\leq L_A\cdot\|w\|\cdot\|x(k)\|\\ &\quad+L_B\cdot\|w\|\cdot\|u(k)\|\end{aligned}$$

where L_A and L_B are the Lipschitz constants of $A(w)$ and $B(w)$ in $\mathcal{W}$. Then,

$$\|\bar{x}(k+1)-x(k+1)\| \leq (L_A\cdot r_x+L_B\cdot r_u)\cdot\|w\| = \alpha\cdot\|w\| \tag{5}$$

where $r_x = \max_{x\in\mathcal{S}^t}\|x\|$ and $r_u = \max_{u\in\mathcal{U}^t}\|u\|$.

Let $d = \inf_{x\notin\mathcal{S}^t} d_{\gamma\mathcal{S}^t}(x)$, i.e., the minimum distance from the border of $\mathcal{S}^t$ to set $\gamma\mathcal{S}^t$. Then, consider the set

$$\mathcal{W}^r = \{w\in\mathcal{W} : \|w\| \leq \frac{d}{\alpha}\}$$

Thus, $w\in\mathcal{W}^r$ implies that $\alpha\|w\|\leq d$, and, from Eq.(5), we have

$$w\in\mathcal{W}^r \Rightarrow \|\bar{x}(k+1)-x(k+1)\| \leq d$$

Taking into account that d is the minimum distance from the border of $\mathcal{S}^t$ to the set $\gamma\mathcal{S}^t$, the later condition establishes that $x(k+1)\in\gamma\mathcal{S}^t \Rightarrow \bar{x}(k+1)\mathcal{S}^t$. Then,

$$Pr[\bar{x}(k+1)\in\mathcal{S}^t] \geq Pr[x(k+1)\in\gamma\mathcal{S}^t] \geq p$$

what proves that $\mathcal{S}^t$ is an OSPIS for the system of Eq.(4). ■

6. EXAMPLE

The idea now is to test the proposed MPC scheme in an uncertainty scenario. We consider the second order stable system introduced in (González *et al.*, 2014), given by Eq.(4), with $w\in\mathcal{W} = [-0.22, 0.22] \subseteq \mathbf{R}$ and

$$A(w) = \begin{bmatrix}0.42 & -0.28\\ 0.02 & 0.6\end{bmatrix} + w\begin{bmatrix}-0.6 & 0.4\\ -0.6 & -0.85\end{bmatrix},$$
$$B(w) = \begin{bmatrix}0.3\\ -0.4\end{bmatrix} + w\begin{bmatrix}-0.2\\ -0.4\end{bmatrix},$$

The unknown *real model* is given by $A(w_r)$ and $B(w_r)$ with $w_r = -0.2$. The constraints of the system are given by $\mathcal{X} = \{x\in\mathbf{R}^2 : \|x\|_\infty \leq 17\}$ and $\mathcal{U} = \{u\in\mathbf{R} : \|u\|_\infty \leq 1\}$.

The EIS set has been selected to be $\mathcal{U}^t = \{u\in\mathbf{R} : \|u\|_\infty \leq 0.8\}$. The Bounded White Noise signal $u_{pe}(k)$ is assumed to have a truncated normal distribution, and lies within $\mathcal{U}^t$, with mean $\mu = 0$ and standard deviation $\sigma = 0.4$.

6.1 Simulating the Re-identification Control and Exciting modes

Two indexes are defined to evaluate the benefits of having a reduced target set for identification, $\mathcal{S}^t$, from the control point of view. The first one, denoted as I_1, is defined by the cumulative distance from the states to the target equilibrium set $\mathcal{X}_{ss}^t$, which represents the objective target for the **Control Operation Mode** (i.e., when no re-identification is needed):

$$I_1 = \sum_{i=1}^{T_{sim}} d_{\mathcal{X}_{ss}^t}(x(i)) + d_{\mathcal{U}_{ss}^t}(u(i)),$$

where T_{sim} is the simulation time, $\mathcal{T}_{ar}$ is the objective set of the MPC cost function (i.e., an invariant set in the Re-identification Operation Mode; an equilibrium set in the Control Operation Mode) and $x = x(0)$ is the initial state. In fact, this index is directly given by the MPC cost proposed for the Control Operation Mode in (González *et al.*, 2014).

The second index, denoted as I_2, simply gives the quantity of states in open-loop:

$$\begin{aligned}I_2 = T_{sim} &- \\ &\#\{x(i) : x(i) = \phi(i; x, \mathcal{T}_{ar}),\quad i = 1,\cdots,T_{sim}\}\end{aligned}$$

The idea behind these indexes is that the less time the system is in open-loop for the identification procedure (provided that a proper excitation is performed), the better for safety control purposes. In this context, the ideal scenario is given when the Control Operation Mode is implemented, in which case, the system is never in open-loop, and so $I_1 = I_1^{cm}$ (minimum value) and $I_2 = I_2^{cm} = 0$. In order to standardize the indexes we define:

$$I_1^{std} = \frac{I_1 - I_1^{cm}}{I_1}; \qquad I_2^{std} = \frac{I_2}{T_{sim}}$$

which are values between 0 and 1, being the smallest values that represent the best scenario, in both cases.

The simulation scenario, devoted to show how the proposed MPC works in the Re-identification Operation Mode, consists in a sequence of disturbances that enters the system while the excitation procedure is being performed. The simulation starts at an initial state inside the target set $\mathcal{S}^t$, which corresponds to the Re-identification Exciting Mode. Then, three disturbances takes the system states outside $\mathcal{S}^t$, which makes that the controller automatically switch to the Re-identification Control Mode, to steers the state back to $\mathcal{S}^t$, and once it occurs, to switch again to the Re-identification Exciting Mode, to resume the exciting procedure.

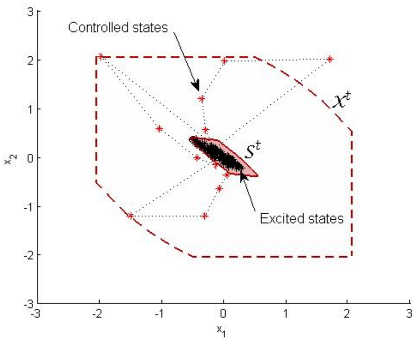

Fig. 1. The Re-identification Operation Mode simulation with the OSPIS $\mathcal{S}^t$ as target set of the MPC cost function.

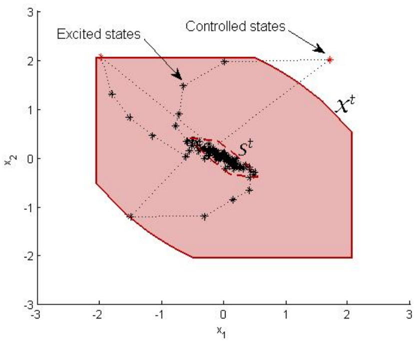

Fig. 2. The Re-identification Operation Mode simulation with the ISI $\mathcal{X}^t$ as target set of the MPC cost function.

Table 1 show the indexes for both criterion of re-identification process using the minimal invariant set (ISI): (González *et al.*, 2014) and the probabilistic invariant set (OSPIS), respectively, also in contrast to the ideal case (Control Operation Mode) when no re-identification process is done. As it can be seen, the proposed strategy shows a significant improvement in both indexes, which is due to the use of a smaller target set.

Target Set	Eq set $\mathcal{X}^t_{ss}$	OSPIS $\mathcal{S}^t$	ISI $\mathcal{X}^t$
Index 1	0	0.38	0.45
Index 2	0	0.88	0.98

Tabla 1. Both indexes for the **control operation mode** with the equilibrium objective set $\mathcal{X}^t_{ss}$: First column. The same indexes for the **re-identification operation mode** with the objective sets $\mathcal{S}^t$ and $\mathcal{X}^t$: Second and third column, respectively.

7. CONCLUSION

In this work, we proved some fundamental theoretical properties of a novel MPC scheme that allows closed-loop re-identification making use of probabilistic invatiant sets. We showed that, using the proposed scheme, the state reaches the target set in finite time, and then the state remains in that target set during the time necessary to perform the re-identification procedure with a given probability. We also extended the robustness analysis to ensure that the scheme can properly work in a wider family of models around the nominal one.

Additionally, we included new simulation results that show the advantages of the methodology with respect to a previous approach that used deterministic target sets.

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