Adaptive Output Tracking of a Single Link Robot Arm Using NONACODE

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1 The System

As an application example we consider a single link robot arm consisting of a rigid link coupled to the shaft of an armature controlled DC motor drive. A schematic diagram of the system is shown in Figure 1, while the model variables and parameters are listed in Table 1.

$\theta(t)$	:	angle of the link
$\omega(t)$	:	angular velocity of the link
J_m	:	rotational inertia of the motor
J_l	:	rotational inertia of the link
J	$=$	$J_m + J_l$
M	:	total mass of the link
l	:	distance from the joint axes to the link center of mass
b	:	dumping coefficient of the motor
L_a	:	armature inductance
R_a	:	armature resistance
K	:	electromechanical conversion coefficient
ϵ	:	back emf
i_a	:	armature current
$u(t)$	:	armature voltage

Table 1: Model Variables and Parameters

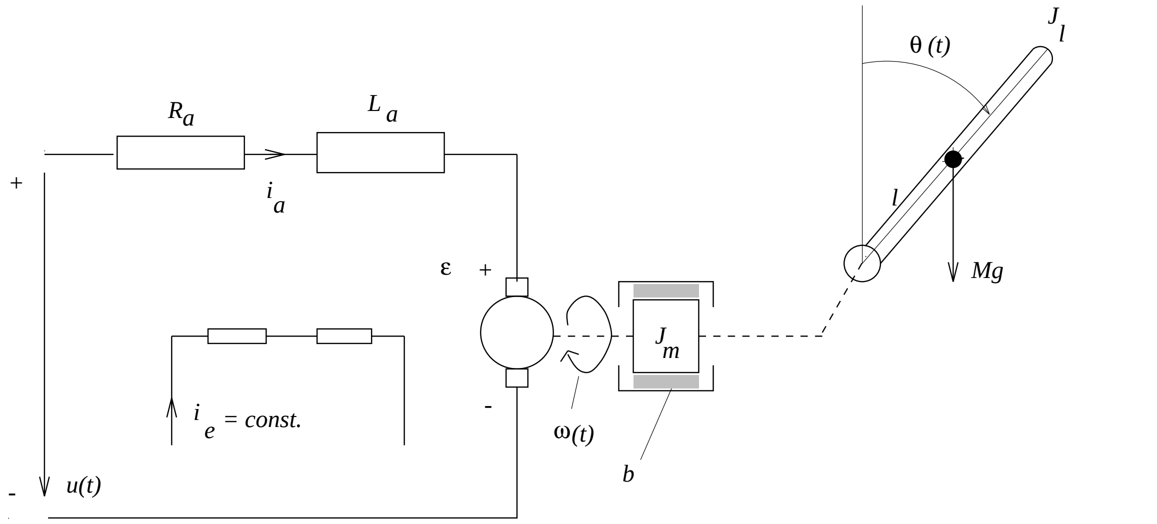

Figure 1: Single Link Robot Arm

Defining the state variables as

$$\begin{aligned} x_1(t) &:= \theta(t) \\ x_2(t) &:= \omega(t) \\ x_3(t) &:= i_a(t) \end{aligned} \tag{1}$$

the state equations of the system can be written as:

$$\dot{x}_1(t) = x_2 \tag{2}$$

$$\dot{x}_2(t) = -\frac{b}{J}x_2 + \frac{K}{J}x_3 + \frac{Mgl}{J}sin(x_1) \tag{3}$$

$$\dot{x}_3(t) = -\frac{K}{L_a}x_2 - \frac{R_a}{L_a}x_3 + \frac{1}{L_a}u(t) \tag{4}$$

or equivalently as

$$\dot{x}(t) = f(x) + g(x)u(t)$$

where we have defined

$$x(t) = [x_1(t), x_2(t), x_3(t)]^T \tag{5}$$

$$f(x) = \begin{bmatrix} x_2 \\ -\frac{b}{J}x_2 + \frac{K}{J}x_3 + \frac{Mgl}{J}sin(x_1) \\ -\frac{K}{L_a}x_2 - \frac{R_a}{L_a}x_3 \end{bmatrix} \tag{6}$$

$$g(x) = \begin{bmatrix} 0 \\ 0 \\ \frac{1}{L_a} \end{bmatrix} \tag{7}$$

2 Adaptive Output Tracking Problem

The control objective is that the electromagnetic torque tracks a reference signal, assuming that the parameters are unknown. That is, we want to solve the **Adaptive Output Tracking Problem**.
We then choose $x_3(t)$ as the output, so that the output equation can be written as

$$y(t) = h(x) = x_3(t) = [0\,0\,1]\ x(t) \tag{8}$$

Now, since

$$L_g h(x) := \frac{\partial h}{\partial x}\ g(x) = [0\,0\,1] \begin{bmatrix} 0 \\ 0 \\ 1/L_a \end{bmatrix} = \frac{1}{L_a} \neq 0$$

then the system has **relative degree** $r = 1$.
Let us now define new parameters as follows

$$\begin{array}{ccc} k_{11} = \frac{b}{J} & k_{12} = \frac{K}{J} & k_{13} = \frac{Mgl}{J} \\ k_{14} = \frac{R_a}{L_a} & k_{15} = \frac{K}{L_a} & k_{21} = \frac{1}{L_a} \end{array}$$

Then, the system can be written as

$$\dot{x}(t) = \sum_{j=0}^{5} k_{1j} f_j(x) + k_{21} g_1(x) u(t) \tag{9}$$

where

$$\begin{array}{rcl} f_0(x) & = & [x_2\ 0\ 0]^T \\ f_1(x) & = & [0\ -x_2\ 0]^T \\ f_2(x) & = & [0\ x_3\ 0]^T \\ f_3(x) & = & [0\ sin(x_1)\ 0]^T \\ f_4(x) & = & [0\ 0\ -x_3]^T \\ f_5(x) & = & [0\ 0\ -x_2]^T \\ g_1(x) & = & [0\ 0\ 1]^T \end{array}$$

Furthermore, we assume that the parameters $k_{1j}, (j = 1, \cdots, 5)$ and k_{21} are **unknown**, while $k_{10} = 1$ is known.
Since equation (9) is of the form

$$\dot{x}(t) = \sum_{j=1}^{n_1} k_{1j}\ f_j(x) + \sum_{i=1}^{n_2} k_{2i}\ g_i(x)\ u(t) \tag{10}$$

then, the control law for adaptive output tracking is given by [2]

$$u = \frac{1}{\widehat{L_g h}}\ \left(-\widehat{L_f h} + \dot{y}_R + \alpha(y_R - y)\right) \tag{11}$$

where y_R is the reference signal, and where $\widehat{L_g h}$ and $\widehat{L_f h}$ stand for the estimates of $L_g h$ and $L_f h$ respectively

$$\begin{aligned} \widehat{L_g h} &= \sum_{i=1}^{n_2} \widehat{k_{2i}}(t)\ L_{g_i} h && (12) \\ \widehat{L_f h} &= \sum_{j=1}^{n_1} \widehat{k_{1j}}(t)\ L_{f_j} h && (13) \end{aligned}$$

Computing (11) for this particular example we obtain the following control law for adaptive output tracking

$$u = \frac{\widehat{k_{14}}\, x_3 + \widehat{k_{15}}\, x_2 + \dot{y}_R + \alpha(y_R - x_3)}{\widehat{k_{21}}} \tag{14}$$

The parameter update law based on Lyapunov criterion is given by [2]

$$\dot{\phi} = -\gamma\ e\ W \tag{15}$$

where $\phi := k - \widehat{k}$ is the parameter estimation error, $e := y - y_R$ is the output error, and γ is a positive coefficient that fix the estimation error dynamics, and where:

$$W = \left[W_1^T W_2^T\right]^T \tag{16}$$

with:

$$W_1 = \begin{bmatrix} L_{f_1} h \\ \vdots \\ L_{f_{n1}} h \end{bmatrix} \tag{17}$$

and

$$W_2 = \begin{bmatrix} L_{g_1} h \\ \vdots \\ L_{g_{n2}} h \end{bmatrix} . \frac{-\widehat{L_f h} + \dot{y}_R + \alpha(y_R - y)}{\widehat{L_g h}} \tag{18}$$

Computing (15) for this particular example we obtain

$$\begin{aligned} \dot{\widehat{k_{10}}} &= 0 \\ \dot{\widehat{k_{11}}} &= 0 \\ \dot{\widehat{k_{12}}} &= 0 \\ \dot{\widehat{k_{13}}} &= 0 \\ \dot{\widehat{k_{14}}} &= -\gamma(x_3 - y_R)x_3 \\ \dot{\widehat{k_{15}}} &= -\gamma(x_3 - y_R)x_2 \\ \dot{\widehat{k_{21}}} &= \frac{\gamma(x_3 - y_R)\left[\widehat{k_{14}}\, x_3 + \widehat{k_{15}}\, x_2 + \dot{y}_R + \alpha(y_R - x_3)\right]}{\widehat{k_{21}}} \end{aligned} \tag{19}$$

It becomes clear that only three parameters need to be estimated. Namely, we only need to compute $\widehat{k_{14}}, \widehat{k_{15}}$ and $\widehat{k_{21}}$.

3 Adaptive Output Tracking using NONACODE

- The system model can be defined in **Mathematica** as follows

$$
\begin{array}{rrcl}
\text{In}[1] := & x & := & \{x1, x2, x3\} \\
\text{In}[2] := & f0 & := & \{x2, 0, 0\} \\
\text{In}[3] := & f1 & := & \{0, -x2, 0\} \\
\text{In}[4] := & f2 & := & \{0, x3, 0\} \\
\text{In}[5] := & f3 & := & \{0, \sin[x1], 0\} \\
\text{In}[6] := & f4 & := & \{0, 0, -x3\} \\
\text{In}[7] := & f5 & := & \{0, 0, -x2\} \\
\text{In}[8] := & g1 & := & \{0, 0, 1\} \\
\text{In}[9] := & f & := & k10\ f0 + k11\ f1 + k12\ f2 + k13\ f3 + k14\ f4 + k15\ f5 \\
\text{In}[10] := & g & := & k21\ g1 \\
\text{In}[11] := & h & := & x3
\end{array}
$$

- The relative degree can be computed using the function **RELDEG** in **NONACODE** [1], as follows

$$
\begin{array}{rrcl}
\text{In}[11] := & r & = & RELDEG\ [f, g, h, x] \\
\text{Out}[11] := & & & 1
\end{array}
$$

- The control law for Adaptive Output tracking can be computed using the function **LYADAPTRACK** as follows

$$
\begin{array}{rrcl}
\text{In}[12] := & k1 & := & \{k10, k11, k12, k13, k14, k15\} \\
\text{In}[13] := & k2 & := & \{k21\} \\
\text{In}[14] := & k & := & \{k1, k2\} \\
\text{In}[15] := & k1es & := & \{k10es, k11es, k12es, k13es, k14es, k15es\} \\
\text{In}[16] := & k2es & := & \{k21es\} \\
\text{In}[17] := & kes & := & \{k1es, k2es\} \\
\text{In}[18] := & F & := & \{f0, f1, f2, f3, f4, f5\} \\
\text{In}[19] := & G & := & \{g1\}
\end{array}
$$

$$
\begin{array}{rl}
\text{In}[20] := & LYADAPTRACK\ [F, G, k1, k2, k1es, k2es, h, x, yR(t), \text{alpha}, \text{gamma}] \\
\text{Out}[20] := & \\
& kesdot = \{0, 0, 0, 0, -\text{gamma}\ (x3 - yR)\ x3, -\text{gamma}\ (x3 - yR)\ x2, \\
& \text{gamma}\ (x3 - yR)\ (k14esx3 + k15esx2 + yR' + \text{alpha}\ (yR - x3))\ /\ k21es\} \\
& u = (k14es\ x3 + k15es\ x2 + yR' + \text{alpha}\ (yR - x3))\ /\ k21es
\end{array}
$$

Remark: Compare the above expressions with the corresponding ones computed analytically in the previous section (equations (19) and (14) respectively).

4 Simulation results

For the simulation we assume the following nominal values of the parameters

$$\begin{array}{rclcrclcrcl} R_a & = & 1 & \quad & L_a & = & 0.002 & \quad & K & = & 10 \\ J & = & 4 & & b & = & 1 & & \ell & = & 1 \\ Mg & = & 20 & & & & & & & & \end{array}$$

that correspond to

$$\begin{array}{rclcrclcrcl} k_{11} & = & 0.25 & \quad & k_{12} & = & 2.5 & \quad & k_{13} & = & 5 \\ k_{14} & = & 500 & & k_{15} & = & 5000 & & k_{21} & = & 500 \end{array}$$

The reference output $y_R(t)$ is the output of the following linear system:

$$G_R(s) = \frac{1}{s^2 + 3s + 2}$$

when the input is:

$$u(t) = 20\ \mu(t)$$

where $\mu(t)$ stands for the unit amplitude step function.
The following initial conditions have been used in the simulation experiment

$$\begin{array}{rclcrclcrcl} x_1(0) & = & 0 & \quad & x_2(0) & = & 15 & \quad & x_3(0) & = & 15 \\ \widehat{k_{14}}(0) & = & 500 & & \widehat{k_{15}}(0) & = & 5000 & & \widehat{k_{21}}(0) & = & 550 \\ y_R(0) & = & 2 & & \dot{y}_R(0) & = & -1 & & & & \\ \gamma & = & 2 & & \alpha & = & 1 & & & & \end{array}$$

Figure 2 shows the reference signal $y_R(t)$ and the output signal $y(t)$. In figure 3, the control signal $u(t)$ is shown, while in figures 4, 5, and 6 the parameter estimates $\widehat{k_{21}}(t)$, $\widehat{k_{14}}(t)$, and $\widehat{k_{15}}(t)$ are shown, respectively.

References

[1] J. C. Gómez, *Using symbolic computation for the computed aided design of nonlinear (adaptive) control systems*, Tech. Rep. EE9454, Dept. Of Electrical and Computer Engineering, University of Newcastle, Callaghan, NSW 2308, September 1994.

[2] S. S. Sastry and A. Isidori, *Adaptive control of linearizable systems*, IEEE Transactions on Automatic Control, 34 (1989), pp. 1123–1131.

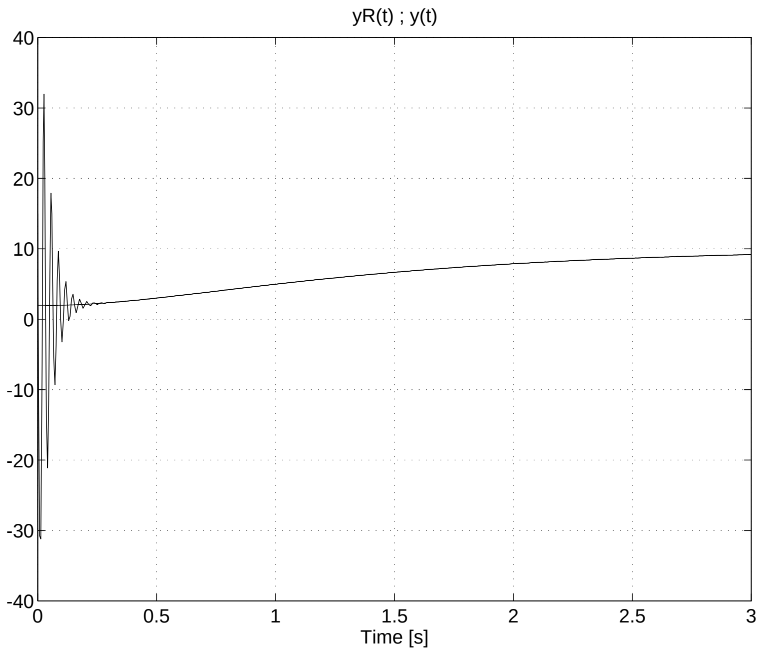

Figure 2: Reference signal $y_R(t)$ and Output signal $y(t)$

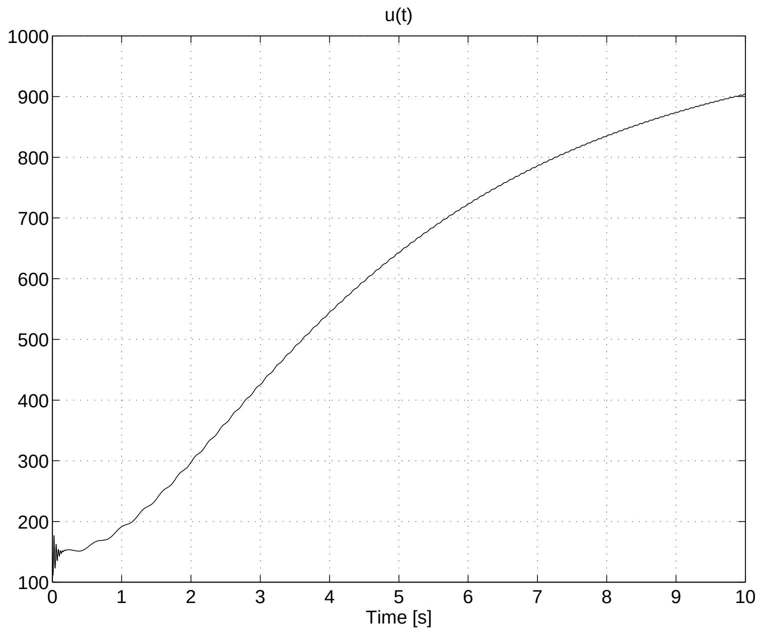

Figure 3: Control signal $u(t)$

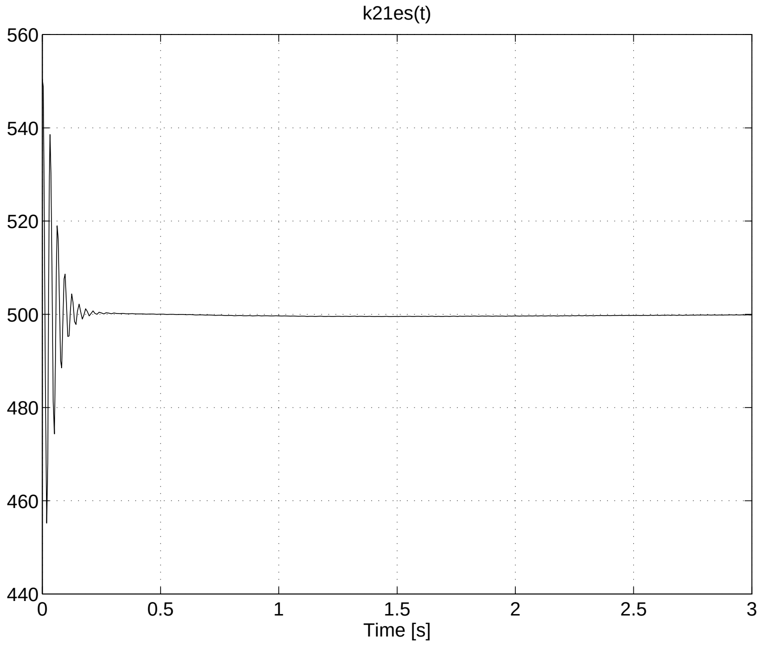

Figure 4: Parameter Estimate $\widehat{k_{21}}(t)$

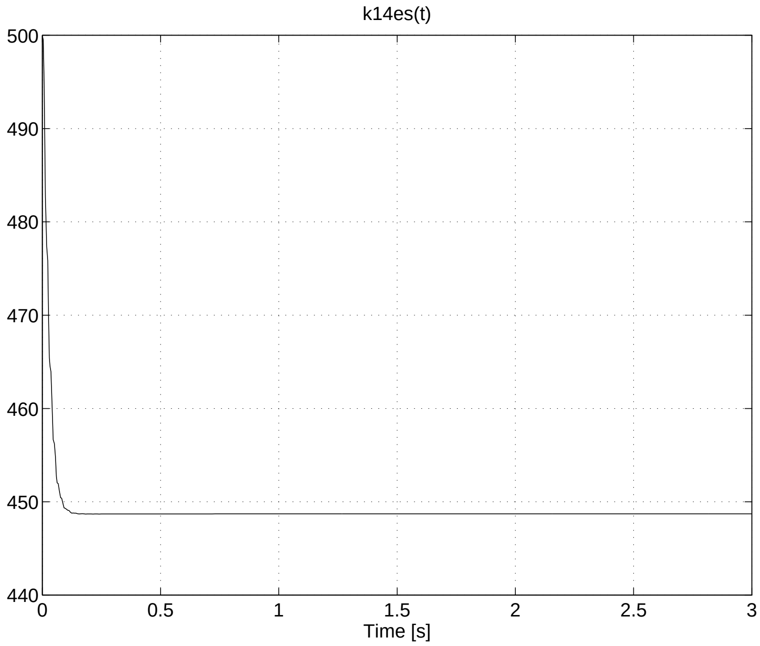

Figure 5: Parameter Estimate $\widehat{k_{14}}(t)$

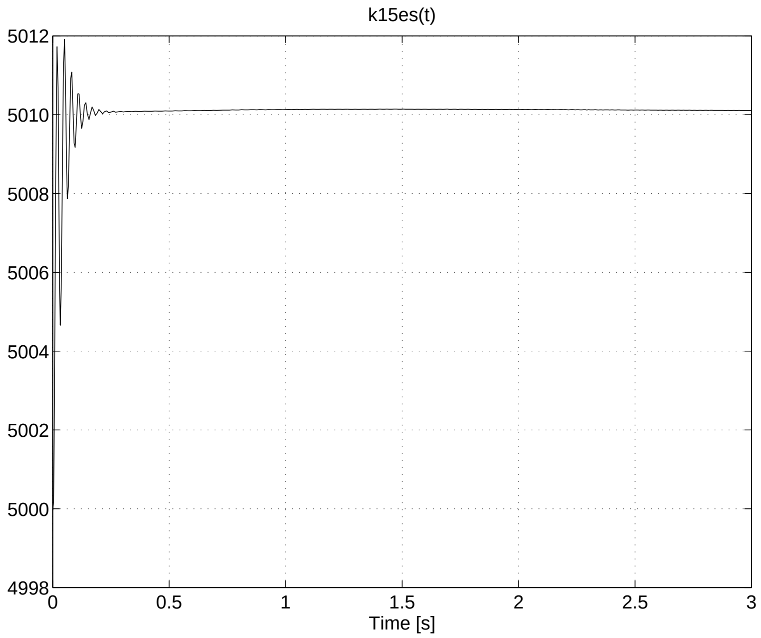

Figure 6: Parameter Estimate $\widehat{k_{15}}(t)$