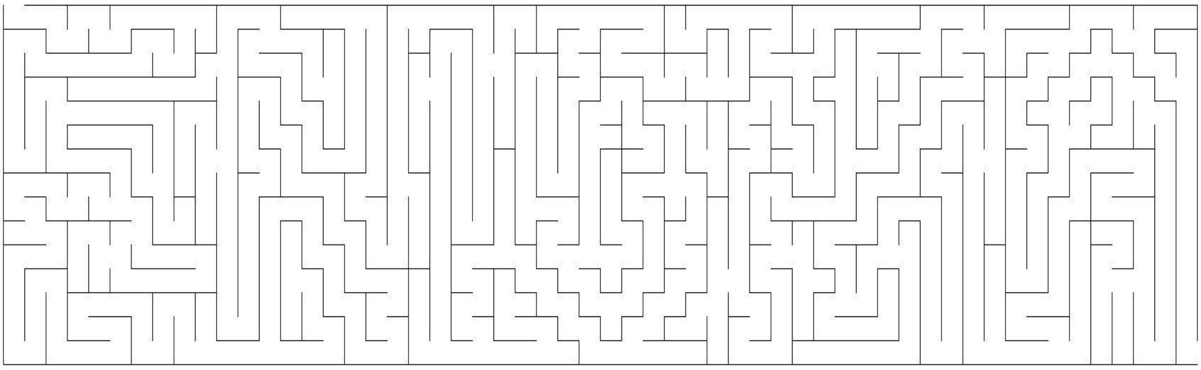
Enigma:
The Newsletter of PuzzleSIG



243

THE ONLY 3-DIGIT NUMBER THAT IS A FIFTH POWER

'Mathematical problems, or puzzles, are important to real mathematics, just as fables, stories, and anecdotes are important to the young in understanding real life'.

Terry Tao

September 2019

Electronic Version of this Newsletter
Email enigma.mensa@yahoo.co.uk and I'll send a pdf copy

About Enigma
Enigma is the newsletter of Puzzle SIG.
Puzzle SIG is the international special interest group for anyone interested in puzzles. The scope covers word puzzles and crossword puzzles, logic puzzles, Japanese puzzles, mathematical brain teasers, lateral thinking problems, quizzes and picture quizzes, discussion of physical / mechanical puzzles, computer / internet based puzzles and puzzle games, and puzzle books and publications. Contributions to the Enigma newsletter are always gratefully received, and whilst experimentation and innovation of puzzle types are more than welcome, traditional types of puzzles are equally appreciated.

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How to Join
You can join Puzzle SIG by contacting me with your name and membership number, and whether you want a postal or a email subscription, or via the members' area on the Mensa website, by emailing sigs@mensa.org.uk or completing a SIG membership application form and send it to British Mensa, St John's House, St John's Square, Wolverhampton WV2 4AH.

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Welcome to Enigma 243

Hello and welcome to another issue of Enigma.

You may notice I have changed the logo on the front page: it is now an extra bonus puzzle for you to complete; I hope you like it!

I've also changed the quotation from the George Polya one to one by Terry Tao, who is regarded as one of the top mathematicians working today. Of course I should point out that the scope of this group extends beyond just maths puzzles and we welcome word puzzles, logic puzzles and much more. See the 'About Enigma' section on the previous page. If you have a puzzle or an article for this newsletter I'd love to hear from you.

Many thanks to Chris Finn, Peter and Jenny Nichols and Christa Ramonat for their contributions to this issue. The majority of the rest of the puzzles are taken from my website www.elliottline.com, so I apologise if you've seen them before.

Please try all of the puzzles, and if you have any feedback, let me know and I'll pass it on to the puzzle's creator.

As ever, if you get stuck and need a hint, drop me a line and I'll be happy to help.
Please keep your puzzles, answers, comments, queries, suggestions, etc coming in.

Happy puzzling
Elliott.

242.01 - COMPETITION: Palindrome - Elliott Line

Base 3: 22222
Base 7: 464
Base 21: (11)(11)
Base 120: (2)(2)
Base 241: (1)(1)

WELL DONE TO:
Johann Muller
Michael Kennedy
Andrew Yeo
Jakub Waniek
Julie Harkin
Paul Clark
Chris Finn
Christa Ramonat
Ivor Cornish
Stuart Nelson
Abhilash Unnikrishnan
Matt Francis
Agnijo Banerjee
Maire Dowling
Ben Betts

243.01 - COMPETITION: Unique Circle - Elliott Line

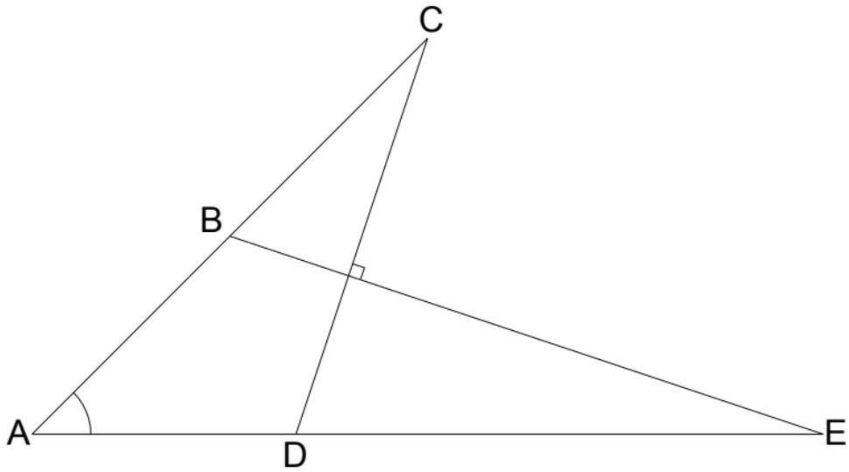
I have a circle, whose diameter is an exact integer. I calculate the radius, circumference and the area, and round these to the nearest integer (precisely 0.5 gets rounded upwards, as usual).
I discover that none of the four integer values contain the digits 2, 4 or 3 (243 being the number of this issue of Enigma).

What was the diameter of the circle?

This is a competition, but not for prizes, only bragging rights. Every correct answer I receive will get an honourable mention in the next issue of Enigma. Send your answer to me at enigma.mensa@yahoo.co.uk .

243.02 – Angle – Elliott Line

B is the midpoint of AC
D is one third of the way along AE
CD is perpendicular to BE
AE is (√2) times the length of AC



What is the angle at A?

243.03 – Coin vs Dice – Elliott Line

Which of the following scenarios should on average* take the most throws:
Repeatedly tossing a coin until you have seen at least 6 heads and at least 6 tails, or
Repeatedly throwing a dice until you have seen each of the numbers 1 to 6 at least once each?
*(simple arithmetic mean)

243.04 – Collate – Elliott Line

Given a string with three each of three different letters ordered thus:

A A A B B B C C C

It is possible through a series of three moves to change the order into:

A B C A B C A B C

A ‘move’ consists of taking a section of the string and reversing the order of the items within it. The brackets show the section to be reversed in the subsequent move:

A(AABB)BCCC -> ABBA(ABC)CC -> AB(BACBAC)C -> ABCABCABC

Using the exact same idea, how many moves will it take to change between the following?

A A A A A B B B B B C C C C C D D D D D E E E E E

A B C D E A B C D E A B C D E A B C D E A B C D E

243.05 – Crypto Sum – Christa Ramonat

Solve the following Crypto Sum. Each letter stands for a number, the only restrictions are that each letter represents a different digit and that E and A cannot be zero.

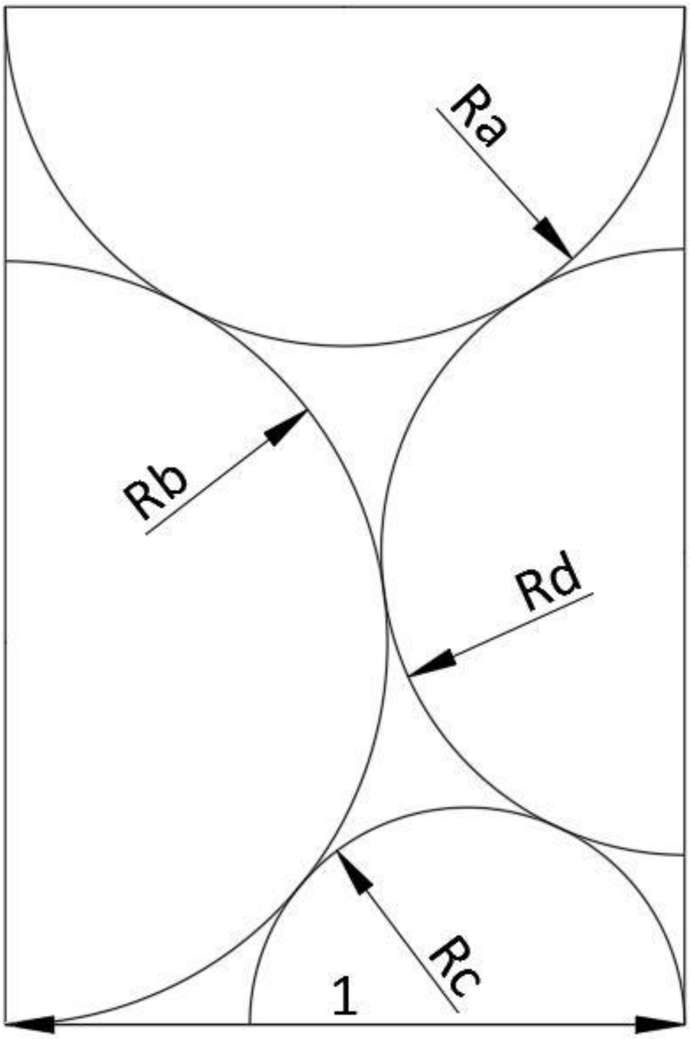
```
E N I G M A
E N I G M A +
-----------
A N S W E R
```

243.06 – Fast Flowing River – Elliott Line

Grace and Ruby live in a house next to a fast flowing river. One day they decide to travel a few miles down the river and then return home. Grace is on foot, jogging along the riverbank at a speed of 5mph. Ruby takes the boat, which can travel at 9mph relative to the current of the river. They both set off at the same time, turn around at the same spot and arrive home at the same time. How fast was the current of the river?

243.07 – Four Semicircles in a Rectangle – Elliott Line

Four semicircles are arranged around the edges of a rectangle. What are their respective radii? This is a great deal more difficult than 'Three Semicircles in a Square' (243.14). If it helps I can tell you that there is exactly one solution, and that each of the radii are rational numbers (so if you find it useful – and I'm not at all sure you will - you can scale the whole thing up by some factor, solve as a Diophantine set of equations, and scale back down again). I myself found this very difficult to solve, so if you discover a route to the solution that is not too difficult, I would be interested in knowing it.



243.08 – Hidden Phrase – Peter and Jenny Nichols

Find a five-word phrase hidden in the following passage. The letters of each word of the phrase appear consecutively (just as ‘word’ appears in ‘slow or dangerous’, for example).

“If economy is important to you, try making do.
Maybe discover bargains or forage in the hedgerows.
Search or seek out berries, herbs, etc.”

243.09 – Identical Twins – Elliott Line

Phyllis and Dilys are identical twins. They are each independently given the same 4-digit number.
Phyllis takes the number and converts it from decimal (base 10) to base 4, and writes down the 6-digit result.
Dilys simply writes the first and last digits of the number followed by the number in its entirety.
They are astonished to find that they have both written down the same 6-digit number!
What was the original number?

In other words, which number ABCD, when converted from decimal to base 4 becomes ADABCD?

243.10 – Infinite Sum – Elliott Line

The convergent sum of the following infinite sequence, in its simplest terms, is a fraction with a square number on the top and a factorial number below.

$1/7 + 1/16 + 1/27 + 1/40 + 1/55 + 1/72 + \dots + 1/n(n+6) + \dots$

What is it?

Footnote: the partial sum converges extremely slowly, such that if you add the first million terms you will only get the first 5 decimal places. However there exists a very nice trick to allow you to work out exactly the number the infinite sum converges upon (without having to do an infinite number of calculations!).

243.11 - Reducing the Irreducible – Elliott Line

The fraction 1174/5063 cannot be reduced further in the usual way, however it can be expressed exactly using only six digits instead of eight.
How?

243.12 – Return Journey – Elliott Line

The towns of Abbottsville, Beresford and Christchurch all lie on one straight road. I embark on a journey from Abbottsville, through Beresford, to Christchurch, and back the same way. Each day I cover 1 mile more than the day before. It takes me 10 days to travel from Abbottsville to Beresford, 11 days to travel from Beresford to Christchurch and 12 days to travel from Christchurch (through Beresford) to Abbottsville.
What are the distances between the towns?

243.13 – Scrabble Maths – Chris Finn

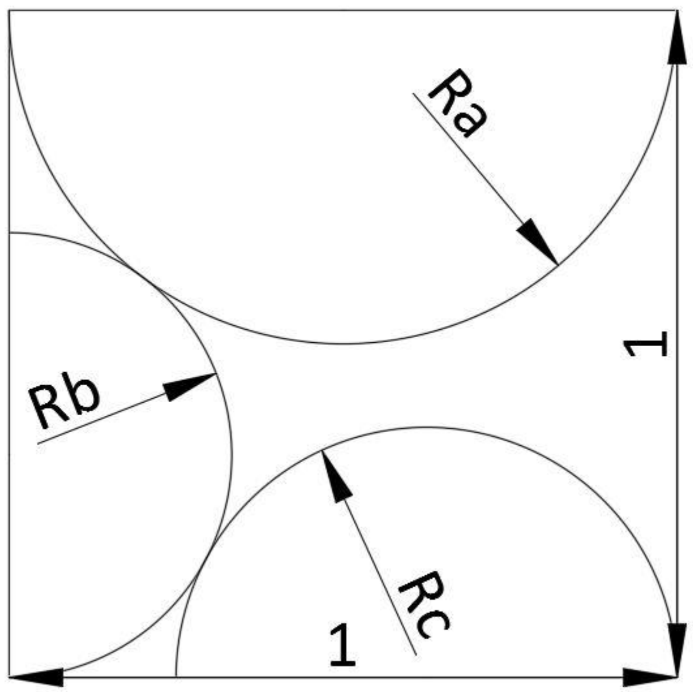
The English Scrabble tiles have the following values -

1 AEIOU LNRST
2 DG
3 BCMP
4 FHVWY
5 K
8 JX
10 QZ

Which positive integer has the same value as its English Scrabble tiles, when it is spelt out in full?

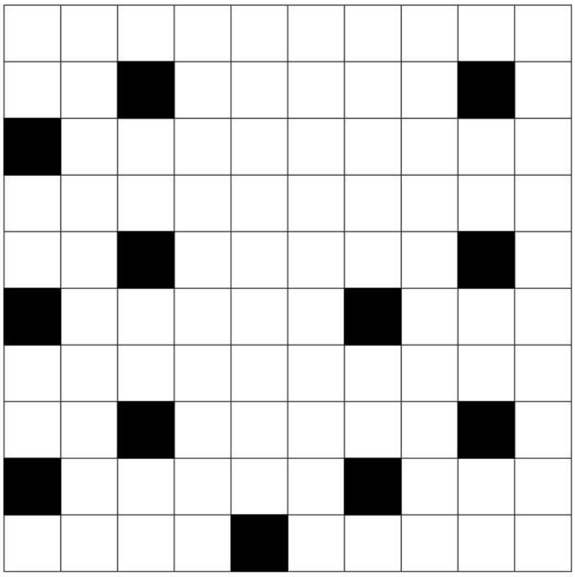
243.14 – Three Semicircles in a Square – Elliott Line

Three semicircles are arranged around the edges of a unit square. What are their respective radii?



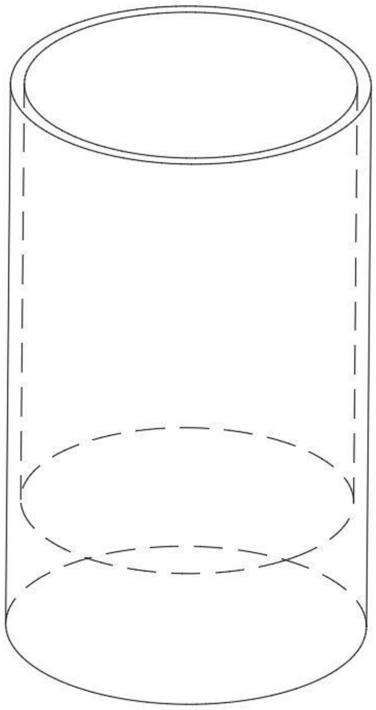
243.15 – Travelling Salesman – Elliott Line

A travelling salesman needs to visit each of the black squares in turn and return to where he started. He can choose to start wherever he wishes. He traces a zero-thickness path, can travel at any angle (this gridlines are only a reference) and doesn't need to go to the middle of each black square, it is sufficient for his route to touch a corner or an edge. If each small square measures 1 x 1, what is the length of the shortest route?



243.16 – Tumbler – Elliott Line

A glass tumbler has an outside diameter of 78mm, and inside diameter of 72mm and a solid base that is 30mm deep.
Its centre of gravity is exactly 30mm from the table, ie, at the height of the bottom of the inside of the glass.
What is the overall height of the glass tumbler?

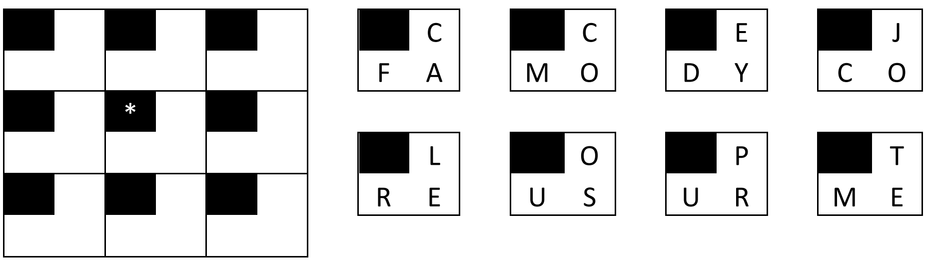


243.17 – Word Morph – Christa Ramonat

Morph the word ENIGMA into the word WORTHY, at each step changing one letter and rearranging.

243.18 – Wordwall – Elliott Line

Reassemble this word wall using the bricks provided. Unfortunately the brick that goes in the position marked with an asterisk is missing, and must be reconstructed.



~~~~**~~~~ SOLUTIONS ~~~~****~~~~**

243.02 – Angle – Elliott Line
45 degrees
ACE forms a right angled isosceles triangle

243.03 – Coin vs Dice – Elliott Line
Coins take on average 14.70703125 throws, dice on average takes 14.7 throws.
(18825/1280 compared to 18816/1280). Coin very slightly longer!

243.04 – Collate – Elliott Line
10 moves, for example:

```
AAAAABBBBBCCCCCDDDDDEEEEE
   AABBBB
   BBBBAA
AAABBBBAABCCCCCDDDDDEEEEE
      BAABCCCC
      CCCCBAAB
AAABBBCCCCBAABCDDDDDEEEEE
       CCCBAABCDDDD
       DDDDCBAABCCC
AAABBBCDDDDCBAABCCCDEEEEE
        DDDCBAABCCCDDD
        EEDCCCBAABCDDD
AAABBBCDEEDCCCBAABCDDDEEE
  ABBBCDE       ABCDDDE
  EDCBBBA       EDDDCBA
AAEDCBBBAEDCCCBAEDDDCBAEE
 AEDCB BAEDC CBAED DCBAE
 BCDEA CDEAB DEABC EABCD
ABCDEABCDEABCDEABCDEABCDE
```

(The last-but-one change shows 2 independent moves and last change shows 4 independent moves).
Since the original string had 5x4=20 adjacent identical pairs and the final string has none, and each move can eliminate at most two of those adjacent identical pairs, 10 moves is definitely the minimum.
Generally, with n copies of n letters, this minimum is n(n-1)/2.
So when n=2 (AABB), 1 move is the minimum, when n=3 (AAABBBCCC), 3 moves is the minimum (as shown in the example).
For the case of n=4 (AAAABBBBCCCCDDDD), that theoretical lower bound is 6 moves, but another more subtle consideration comes into play, meaning that 7 moves is in fact the minimum achievable.
If you're interested in what that more subtle consideration is, read on: If we confine ourselves to only splitting identical pairs at either end of our chosen segment (which we have to if n(n-1)/2 moves is achievable), the number of, say, AB/BA pairs in the overall string will begin as 1 and will either stay the same, or increase by

two (if the segment splits a pair of As at one end and a pair of Bs at the other end). An AB pair might change to a BA pair if it's in the middle of the segment, but since we are considering the total of AB and BA pairs, this doesn't affect us. So the number of AB/BA pairs will always be odd. That's fine when n is odd (as in the example n=3 and the puzzle n=5), however when n is even, the number of AB pairs in the target string is even. So our initial assumption that we are able to confine ourselves to only splitting identical pairs is not in fact the case, and so when n=4, 6 moves are not sufficient. A special case is when n=2 which, despite being even, begins with an odd number of AB/BA pairs, and so the minimum 1 move is achievable.
In actual fact the minimum number of moves it will take is n(n-1)/2 when n is odd and (n^2-2)/2 when n is even.

243.05 – Crypto Sum – Christa Ramonat
There are two possible answers: 203564 + 203564 = 407128, and 201764 + 201764 = 403528

243.06 – Fast Flowing River – Elliott Line
The current is 6mph, meaning Ruby's boat travels at a speed of 15mph downstream and 3mph upstream. Regardless of the distance (as long as she travels the same distance downstream and upstream), her average speed will be 5mph, matching Grace on foot.
Interestingly, assuming the boat is faster than the pedestrian, there is always a current speed which means the average speed of the boat will match the pedestrian. For instance, if the boat was 100mph and the pedestrian was 1mph, the current would have to be almost 99.5mph!

243.07 – Four Semicircles in a Rectangle – Elliott Line
Ra = 1/2
Rb = 9/16
Rc = 8/25
Rd = 25/56

243.08 – Hidden Phrase – Peter and Jenny Nichols
My kingdom for a horse

~~~~**~~~~ SOLUTIONS ~~~~****~~~~**

243.09 – Identical Twins – Elliott Line

3320 (both end up with 303320).
This can obviously be solved using a spreadsheet, but actually with a mixture of maths and logic can also be solved on paper:
If we call the decimal number ABCD, then the base 4 number will be ADABCD. Since the value of these two numbers is equal:

$$1000A + 100B + 10C + D = 1024A + 256D + 64A + 16B + 4C + D$$

Where A must be either 1, 2 or 3, and the other must be 0, 1, 2 or 3. This becomes:

$$88A - 84B - 6C + 256D = 0$$

$$44A - 42B - 3C + 128D = 0$$

$$2A + 42(A-B) - 3C + 128D = 0$$

In order of the whole thing to be 0, logically A-B = 0 (ie, A=B) and D = 0. This leaves:

$$2A = 3C$$

By inspection, the only solution of this is A = 3 and C = 2.

Therefore ABCD = 3320.

~~~~**~~~~ SOLUTIONS ~~~~****~~~~**

243.10 – Infinite Sum – Elliott Line
49/120

If you evaluate the first few partial sums you get 1/7, 23/112, 733/3024, 4043/15120... No obviously helpful pattern. And even if you use a spreadsheet to calculate the partial sum up to n in the thousands, it isn't immediately obvious what rational number it is converging on, or why.

However, the key observation, which initially seems like it is making the sum more rather than less complicated, is to notice that:
1/7 = 1/6 – 1/42,
1/16 = 1/12 – 1/48,
1/27 = 1/18 – 1/54,
etc.
Or, in general:
1/n(n+6) = 1/6n – 1/6(n+6).
(The reader can easily verify this general case is true.)

This is particularly useful as when you apply this to the infinite sum, almost all terms appear as a positive in one expansion and a negative in another expansion (six terms later on), and are cancelled out, leaving only a finite sum of the first six positive terms:
1/6 + 1/12 + 1/18 + 1/24 + 1/30 + 1/36
This then gives the answer of 49/120.

243.11 - Reducing the Irreducible – Elliott Line
9/61 + 7/83

243.12 – Return Journey – Elliott Line
A to B 165 miles
B to C 297 miles
C to A 462 miles
I travelled 12 miles on the first day and on the last day I travelled 44 miles

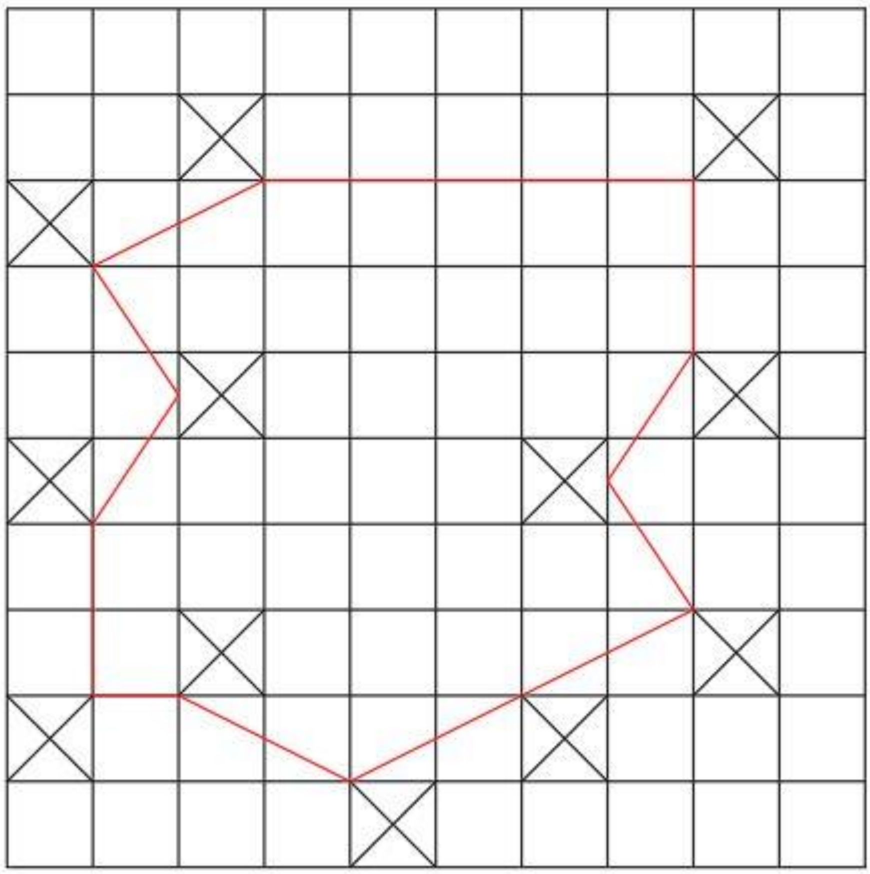
243.13 – Scrabble Maths – Chris Finn
T+W+E+L+V+E = 1+4+1+1+4+1 = 12

243.14 – Three Semicircles in a Square – Elliott Line
Ra = 1/2
Rb = 1/3
Rc = 3/8

~~~~****~~~~ SOLUTIONS ~~~~****~~~~

243.15 – Travelling Salesman – Elliott Line

26.1554 ($10 + 4\sqrt{5} + 2\sqrt{13}$)



243.16 – Tumbler – Elliott Line

108mm

243.17 – Word Morph – Christa Ramonat

Here is one possible solution:

ENIGMA : MIRAGE : TRIAGE : GOITER : RIGHTO : GROWTH : WORTHY

243.18 – Wordwall – Elliott Line

	C		C		O
F	A	M	O	U	S
	J	*	Y		P
C	O	L	O	U	R
	L		T		E
R	E	M	E	D	Y
~~~~