El flujo de Ricci en una clase de solvariedades.

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El *flujo de Ricci* es una muy conocida ecuación de evolución para una curva de métricas en una variedad Riemanniana. En el caso de los grupos de Lie, es equivalente de una manera natural y especícifa al *flujo de corchetes,* que es una ecuación diferencial ordinaria para una curva de álgebras de Lie. El objetivo de esta comunicación es analizar el flujo de Ricci de las solvariedades cuya álgebra de Lie posee un ideal abeliano de codimensión 1, usando el flujo de corchetes. Hemos probado que el intervalo de tiempo hacia adelante para el flujo es $[0, \infty)$, el ω-límite es un punto, i.e. no hay 'caos', y que el flujo de Ricci converge con la convergencia punteada a una variedad, la cual es localmente isométrica a una variedad plana. Para evitar que algunas soluciones converjan a la métrica plana, hemos estudiado una normalización del flujo de corchetes. Damos una función monótona decreciente que nos va a permitir probar que l ímites de subsucesiones son solitones algebraicos, y determinamos cuáles de estas soluciones convergen a una métrica plana. Finalmente, usaremos estos resultados para probar que si un grupo de Lie en esta clase admite una métrica Riemanniana de curvatura seccional negativa, entonces la curvatura de cualquier solución del flujo de Ricci se convertirá en negativa antes de la primera singularidad.

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Geometría de los grupos de Lie con estructura compleja abeliana

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En este trabajo describimos la estructura de los grupos de Lie que admiten una estructura compleja abeliana en términos de álgebras asociativas conmutativas. Dada una métrica invariante a izquierda y hermitiana con respecto a una estructura compleja abeliana, demostramos que dicha métrica es Kähler si y sólo si el grupo de Lie es un producto de varias copias del plano hiperbólico real por un factor euclídeo. Si la métrica hermitiana, en lugar de ser Kähler tiene primera conexión canónica plana, mostramos que el grupo de Lie es abeliano.

Polinomios de Secciones Normales planas en Hipersuperficies Isoparam?tricas

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Sea p un punto en una variedad Riemanniana M, compacta, conexa, esférica de dimensión n. Considerando un vector unitario X, en el espacio tangente $T_p(M)$, se define el subespacio afín de R^{n+k} por $Sec\,(p, X) = p + Span\{X, T_p^{\perp}\,(M)\}$. Si U es una vecindad de p en M, entonces la intersección $U \cap Sec\,(p, X)$ puede ser considerada una curva regular C^{∞}, $\gamma\,(s)$, parametrizada por longitud de arco, tal que, $\gamma\,(0) = p$, $\gamma^{'}\,(0) = X$, denominada *Sección Normal de M en p en la dirección X*. Se dice que la sección normal γ de M en p en la dirección X es *plana* en p si las derivadas $\gamma^{'}\,(0)$, $\gamma^{''}\,(0)$ y $\gamma^{'''}\,(0)$ son *linealmente dependientes*. Si M es una subvariedad esférica compacta en R^{n+k}, dado un punto p en M, se denota por $\widehat{X_p}\,[M]$ el conjunto algebraico definido por, $\widehat{X_p}\,[M] = \{X \in T_p\,(M) : \|X\| = 1, (\nabla_X \alpha)\,(X, X) = 0\}$.

Para estudiar las secciones normales en p, se consideran polinomios homogéneos de grado tres. En esta presentación se muestra la forma de calcular los polinomios que gobiernan el comportamiento de secciones normales planas en subvariedades isoparam?tricas de rango dos o equivalentemente Hipersuperficies Isoparamétricas.

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Índice de estabilidad de hypersurficies en esferas con curvatura media constante

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Es conocido que las hipersuperficies totalmente umbilicales en la esfera unidad (n+1)-dimensional, S^{n+1}, se caracterizan como las únicas con índice débil de estabilidad cero. Es decir, se tiene que toda hipersuperficie $M \subset S^{n+1}$ compacta, con curvatura media constante y diferente de una esfera Euclidiana debe tener índice débil de estabilidad mayor o igual que 1. En esta charla demostraremos que el índice débil de toda hipersuperficie $M \subset S^{n+1}$ compacta, con curvatura media constante y diferente de una esfera Euclidiana no puede tomar los valores $1, 2, 3, \ldots, n$

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Hodge Theory and Differential Systems

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The periods of a family of smooth projective varieties give integral submanifolds of the differential system defined by Griffithsínfinitesimal period relations. In this introductory expository talk I will review the basic properties of these systems together with old and recent results about them.

Topology of compact solvmanifolds

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Joint work with A. Fino, M. Macrì, G. Ovando, M. Subils

A compact solvmanifold is a quotient of a connected and simply connected solvable Lie group by a lattice. We consider the question of existence of lattices on solvable Lie groups and the problem of the computation of the de Rham cohomology on compact solvmanifolds. We will explain known results on the latter problem in special cases (completely solvable Lie groups, solvmanifolds for which the Mostow condition holds) as well as a general computation method ([1], following results by Guan and Witte). As applications we study the Nakamura manifold, the almost abelian Lie groups [2], the hyperelliptic surface and three families of lattices on the oscillator group [3]. We show that some of these solvmanifolds admit not invariant symplectic structures and we study their formality and Lefschetz properties.

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Geometric inequalities for black holes

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A geometric inequality in General Relativity relates quantities that have both a physical interpretation and a geometrical definition. It is well known that the parameters that characterize the Kerr-Newman black hole satisfy several important geometric inequalities. Remarkably enough, some of these inequalities also hold for dynamical black holes. This kind of inequalities, which are valid in the dynamical and strong field regime, play an important role in the characterization of the gravitational collapse. They are closed related with the cosmic censorship conjecture. The talk is based in the review article [1].

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A necessary condition for the existence of symplectic structures on nilmanifolds

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Our main interest is the existence of symplectic structures on nilmanifolds. An important consequence of Nomizu's Theorem is that any symplectic form on a nilmanifold $\Gamma\backslash G$ is cohomologous to a left invariant form on G. Therefore, the study of existence of symplectic structures on $\Gamma\backslash G$ reduces to the existence of a non degenerate closed 2-form on $\mathfrak{g}$.

It is well known that when a compact manifold M admits a symplectic form then $H^2_{dR}(M) \neq 0$. This obstruction does not apply for nilmanifolds since $\dim H^2(\Gamma\backslash G) \geq 2$ ([2]). Thus new obstructions need to be found for these compact quotients.

For any nilpotent Lie algebra $\mathfrak{g}$ it is possible to construct a spectral sequence $\{E_r^{p,q}\}$ that converges to the cohomology of $\mathfrak{g}$ with trivial coefficients. This implies in particular that

$$H^2(\mathfrak{g}) \cong \bigoplus_{p+q=2} E_\infty^{p,q}(\mathfrak{g}).$$

Under these definitions we prove the following obstruction to the existence of symplectic structures on nilmanifolds.

Theorem. Let G be a simply connected nilpotent Lie group with Lie algebra $\mathfrak{g}$ and Γ a co-compact discrete subgroup. If $E_\infty^{0,2}(\mathfrak{g}) = 0$ then $\Gamma\backslash G$ does not admit symplectic structures.

By making use of this result we prove that the real simply connected nilpotent Lie groups arising from the nilradicals of Borel subalgebras of the clasical simple Lie algebras over $\mathbb{C}$ cannot be endowed with symplectic structures.

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Reducción Óptima de variedades Kähler

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La reducción óptima es un método de reducción simpléctica que fue introducido por Ortega y Ratiu [2]. Una de sus características es que funciona para el caso en que no es posible aplicar la técnica de reducción simpléctica clásica de Marsden-Weinstein. En este trabajo describiremos el método de reducción óptima y mostraremos como extenderlo al caso de variedades Kähler, inspirándonos en la construcción clásica del cociente Kähler [1].

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D′Atri spaces of type k and related classes of geometries

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A Riemannian manifold M of $\dim n$ is called a DÁtri space of type k (or k-DÁtri space), $1 \leq k \leq n-1$, if the geodesic symmetries preserve the k-th elementary symmetric functions of the principal curvatures of small geodesic spheres. In this work, joint with Arias Marco T., we continue the study of the geometry of k-DÁtri spaces M began by [1], where it was shown that k-DÁtri spaces, $k \geq 1$, are related to properties of Jacobi operators R_v along geodesics, as the invariance of $\mathrm{tr}R_v$, $\mathrm{tr}R_v^2$ under the geodesic flow for any unit tangent vector v. Now, assuming that M is a DÁtri space, we prove in our main result that $\mathrm{tr}R_v^3$ is also invariant under the geodesic flow if $k \geq 3$. Other properties of Jacobi operators related to the Ledger conditions are obtained with applications to Iwasawa type spaces: in the class of DÁtri spaces, the symmetric of noncompact type are exactly the $\mathfrak{C}$-spaces and on the other hand they are characterized as the k-DÁtri spaces for some $k \geq 3$. In the last case, they are k-DÁtri for all $k = 1, ..., n-1$ as well. In particular, Damek-Ricci spaces that are k-DÁtri for some $k \geq 3$ are symmetric.

Finally, we describe the k-DÁtri spaces for all $k = 1, ..., n-1$ as the $\mathfrak{SC}$-spaces (geodesic symmetries preserve the principal curvatures of small geodesic spheres). As a consequence, in the 4-dim homogeneous spaces M, the properties of M being a DÁtri (1-DÁtri) space, 3-DÁtri space or k-DÁtri space for all $k = 1, 2, 3$ are equivalent.

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Growth estimates for orbits of self adjoint groups

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Let G be a closed, connected, noncompact subgroup of $GL(n, \mathbb{R})$ that is closed under the transpose operation. For v in $\mathbb{R}^n$ let G_v denote the subgroup of G that fixes v, and let $d_L(g, G_v)$ denote the left invariant distance in G between an element g and G_v. We obtain simple algebraic upper and lower bounds for the asymptotic growth rate of $log|g(v)|/d_L(g, G_v)$ as $d_L(g, G_v) \rightarrow \infty$. These bounds may be regarded as generalizations of the Lyapunov exponents for flows, where the additive group $\mathbb{R}$ is replaced by G. The bounds are sharp if G_v is discrete.

The canonical contact structure on the space of oriented null geodesics

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Let N be a pseudo-Riemannian manifold such that $\mathcal{L}^0(N)$, the space of all its oriented null geodesics, is a manifold. B. Khesin and S. Tabachnikov introduce a canonical contact structure on $\mathcal{L}^0(N)$ (generalizing the definition given by R. Low in the Lorentz case), and study it for the pseudo-Euclidean space. We continue in that direction for other spaces. Let S be the pseudosphere of signature (k, m). We show that $\mathcal{L}^0(S)$ is a manifold and find a contactomorphism with some standard contact manifold, namely, the unit tangent bundle of some pseudo-Riemannian manifold. We present an application to the null billiard operator. For N the pseudo-Riemannian product of two Riemannian manifolds, we give geometrical conditions on the factors for $\mathcal{L}^0(N)$ to be a manifold, and exhibit a contactomorphism with a concrete contact manifold.

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Isometry groups of Lie groups.

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Let G be a simply-connected Lie group with left-invariant metric. As the metric can be completely recovered from the inner product on Lie G, all of the geometry of G is encoded in this metric Lie algebra, at least in principle. In practice, however, it is usually very difficult to reconstruct even the isometry group from this metric Lie algebra.

In this talk, we will address the question of how much one can say about the isometry group of G using information coming from the metric Lie algebra $Lie(G)$. Particular attention will be given to semi-simple and solvable groups.

Cohomología dinámica y rigidez topológica/geométrica

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La siguiente ecuación diferencial es frecuentemente llamada *ecuación cohomológica:*

$$Xu = \phi. \tag{1}$$

Aquí X es un campo vectorial diferenciable sobre una variedad M, $\phi\colon M \to \mathbb{R}$, una función suave dada y $u\colon M \to \mathbb{R}$, la incógnita de la ecuación.

A pesar de su (aparente) simplicidad, esta ecuación aparece muy a menudo en el estudio de Sistemas Dinámicos y, por lo general, resulta un problema bastante complicado el determinar si (1) admite o no soluciones regulares.

Esto ha generado un estudio bastante intensivo de este tipo de ecuaciones y, recientemente, se ha mostrado que la resolubilidad de éstas guarda una estrecha relación con la topología/geometría de la variedad soporte M.

En esta charla haremos una breve reseña de estos resultados [1, 2, 3], (y conjeturas [4, 5, 2]) y discutiremos la construcción de nuevos "contraejemplos" en grupos de Lie compactos y nilvariedades, recientemente obtenidos en un trabajo conjunto con A. Avila y B. Fayad.

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Solitones de Ricci homogéneos

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Los *solitones de Ricci* son una clase especial de métricas Riemannianas en una variedad, definidas por el hecho de que su geometría no cambia a lo largo del flujo de Ricci. El objetivo principal de esta charla es estudiar dichas métricas en el caso de variedades homogéneas. Para esto, introduciremos las nociones de solitón de Ricci *semi-algebraico* y *algebraico* (que son cierta clase de solitones de Ricci homogéneos con propiedades algebraicas especiales). Presentaremos un teorema sobre la estructura algebraica que debe tener un solitón de Ricci homogéneo, el cual por un lado generaliza resultados previamente conocidos sobre la estructura de solvsolitones y solvariedades Einstein, y además puede ser usado para probar que los solitones de Ricci con grupo de isometrias unimodular son todos del tipo algebraicos. Basándonos en este resultado explicaremos además un vínculo muy particular entre las métricas de Einstein homogéneas y los! solitones de Ricci homogéneos.

Este es un trabajo realizado en colaboración con Jorge Lauret.

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The Ricci flow and its solitons for homogeneous manifolds and the Alekseevskii conjecture

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We consider an ODE for a one-parameter family of Lie algebras which is equivalent, in a natural and specific sense, to the Ricci flow starting at any homogeneous Riemannian manifold. Such a flow is however much more friendly in some particular cases, and is useful to better visualize the possible (nonflat) pointed limits of Ricci flow solutions, under diverse rescalings, as well as to determine the type of the possible singularities. Ancient solutions arise naturally from the qualitative analysis of the evolution equation. Convergence issues will be considered.

A panoramic view on the current state of the following generalization of the long standing Alekseevskii's Conjecture on Einstein homogeneous manifolds of negative scalar curvature proposed in Besse's book will also be given:

Any connected (nontrivial) homogeneous Ricci soliton is diffeomorphic to a euclidean space (i.e. it is isometric to a simply connected solvmanifold, essentially).

The index of symmetry

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In this talk, based on joint work with Silvio Reggiani, we would like to draw the attention to some concept that we call *index of symmetry* $i_{\mathfrak{s}}(M)$ of a Riemannian manifold M^n, $0 \leq i_{\mathfrak{s}}(M) \leq n$. One has that M is symmetric if and only if $i_{\mathfrak{s}}(M) = n$

We are, of course, interested on non-symmetric spaces with positive index of symmetry. In this case one can prove that $i_{\mathfrak{s}}(M) \leq n - 2$ (in other words the *co-index of symmetry* is at least 2). These examples are known homogenous spaces but endowed with a very particular Riemannian metric.

We are able to classify the spaces with small co-index of symmetry, after proving a bound on the dimension n of the space, for a fixed positive co-index of symmetry k (for irreducible spaces, since the product by a symmetric space does not change the co-index of symmetry, but increases the dimension). Namely, we prove that there is a transitive group of isometries of dimension at most $k(k+1)/2$.

The concept of index of symmetry came out from the study of compact naturally reductive spaces such that the isotropy has non-trivial fixed vectors (and so the full isometry group is bigger than the presentation group). For such spaces it is not hard to prove that the index of symmetry is al least the dimension of the fixed vectors of the isotropy representation.

Recently, with Reggiani and Tamaru, we proved the equality, if the space is irreducible, non-locally symmetric and it is presented by means of the transvection group. Then, for a normal homogeneous space $M = G/H$, the index of symmetry coincides with the dimension of the fixed vectors of H (since H must coincide with the transvections). Or equivalently, the distribution of symmetry coincides with the distribution of fixed vectors of the isotropies. This gives a geometric meaning of this last distribution (observe that the full isometry group of M is bigger than G if this distribution is non-trivial).

Secciones normales de subvariedades isoparamétricas.

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Las subvariedades isoparamétricas de los espacios Euclídeos son las subvariedades con *"curvaturas principales constantes"* y *"fibrado normal plano"*. Se dice que una subvariedad $M^n \subset \mathbb{R}^{n+k}$ tiene curvaturas principales constantes si para todo campo normal paralelo $\xi(t)$ definido a lo largo de toda curva diferenciable a trozos en M^n , los autovalores del operador forma $A_{\xi(t)}$ son constantes. Las secciones normales de una subvariedad $M^n \subset \mathbb{R}^{n+k}$ en un punto $p \in M^n$ son las curvas "cortadas" de M^n por los subespacios afines de $\mathbb{R}^{n+k}$ de la forma

$$p + Span\left\{Y, T_p(M^n)^{\perp}\right\} \qquad Y \in T_p(M^n) \quad ||Y|| = 1$$

De estas curvas interesa su comportamiento en el punto p y este comportamiento permite "separar" los vectores tangentes unitarios en p y sus clases en los espacios proyectivos $\mathbb{R}P(T_p(M^n))$, en conjuntos algebraicos reales. La naturaleza y propiedades de estos conjuntos algebraicos son propiedades de la subvariedad M^n en consideración y esto motiva su estudio.

Se presentarán diversos resultados y ejemplos con énfasis en las subvariedades isoparamétricas homogéneas de "rango" o codimensión $k = 2$.

El problema de equivalencia para distribuciones y la prolongación de Tanaka

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La geometría de las distribuciones no integrables despierta gran interés por su aplicación a la teoría de control geométrico y a la teoría de Hodge entre otras. Para describirla debemos recurrir a G-estructuras de tipo infinito lo que hace muy difícil estudiarla del punto de vista clásico [2]. Por eso N. Tanaka en [3] desarrolló un esquema de prolongación más adecuado para estructuras asociadas a distribuciones, que coincide con el clásico cuando la distribución es todo el fibrado tangente. En esta charla daremos una breve reseña sobre este esquema de prolongación, enunciaremos el teorema de Tanaka y algunas de sus consecuencias, y mostraremos una aplicación a las distribuciones FAT o de policontacto con estructuras conformes compatibles.

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On local normal forms of completely integrable systems

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The purpose of this talk is to present a local normal form of completely integrable systems. Using Poisson geometry tools, we show that a C^1 differential system on $\mathbb{R}^n$ which admits a set of $n-1$ independent C^2 conservation laws defined on an open subset $\Omega \subseteq \mathbb{R}^n$, is essentially C^1 equivalent on an open and dense subset of Ω, with the linear differential system $u_1' = u_1, u_2' = u_2, \ldots, u_n' = u_n$. The main results will be illustrated in the case of some concrete dynamical systems.

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