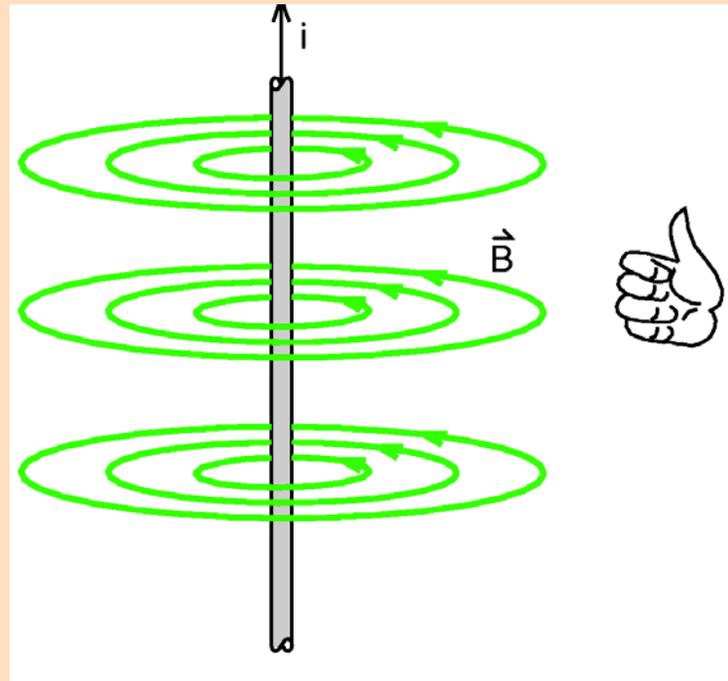
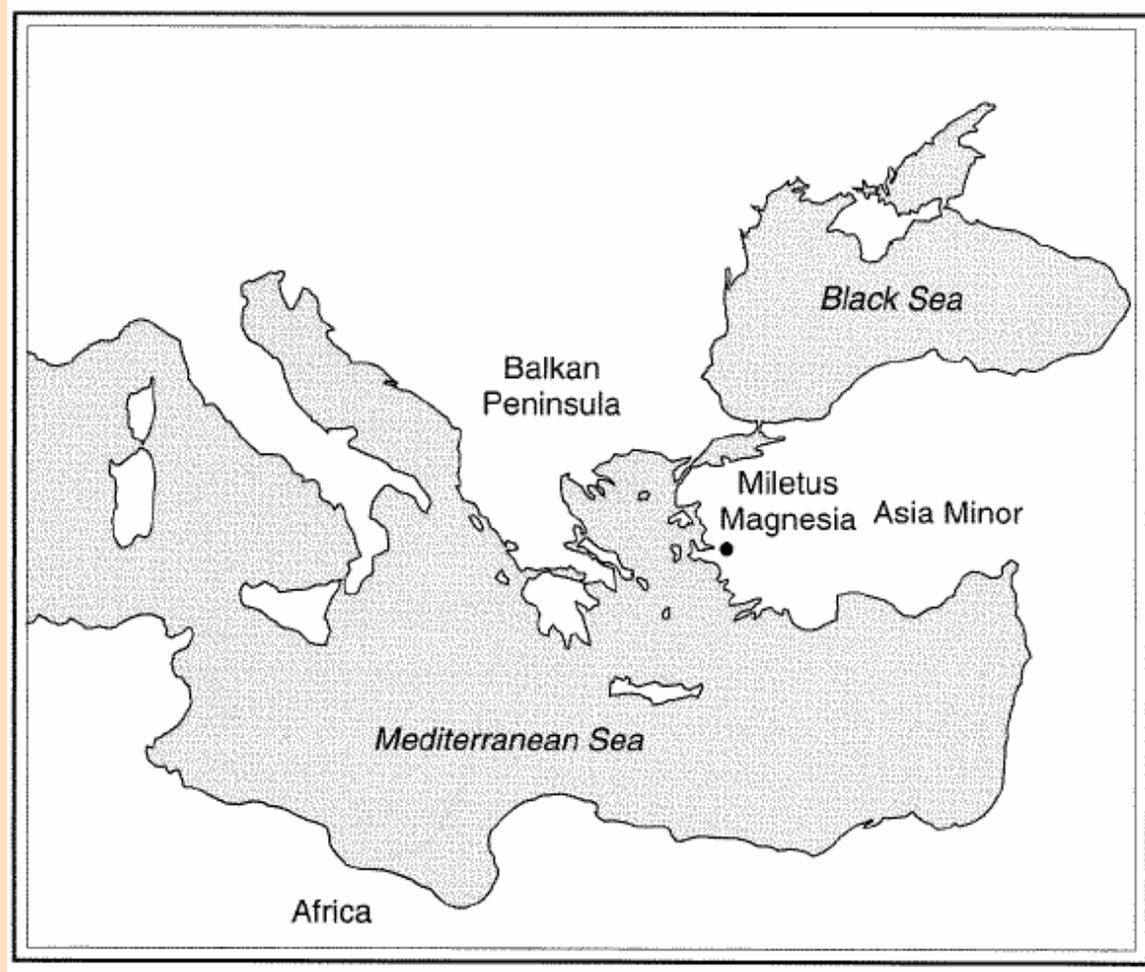


Capítulo 2:

Interacción Magnética



Un poco de historia



Magnetita

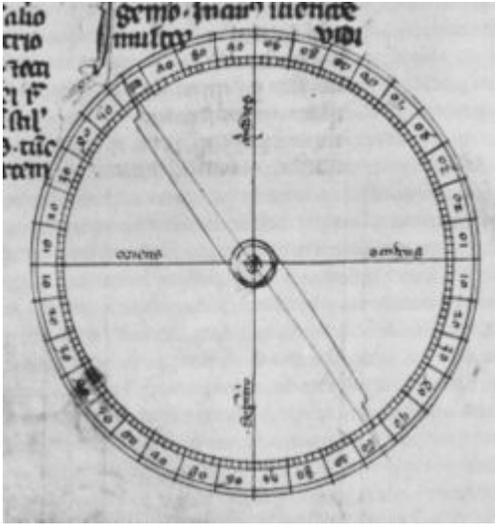
La **magnetita** es un mineral de hierro constituido por óxido Fe_3O_4 que debe su nombre a la ciudad de Magnesia.

Sus propiedades magnéticas las reconocieron los chinos en el Siglo XI a. c.

El naturalista Plinio el Viejo habló de su existencia al sugerir que el nombre se deriva de Magnes, un pastor cuyos zapatos con clavos de hierro se fijaron en piedras que contenían dicho mineral.



Pier de Mauricourt (1269)

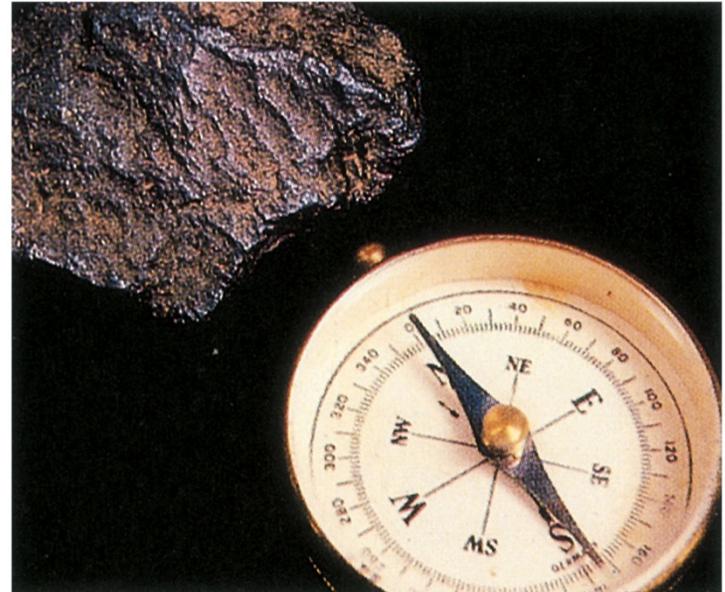


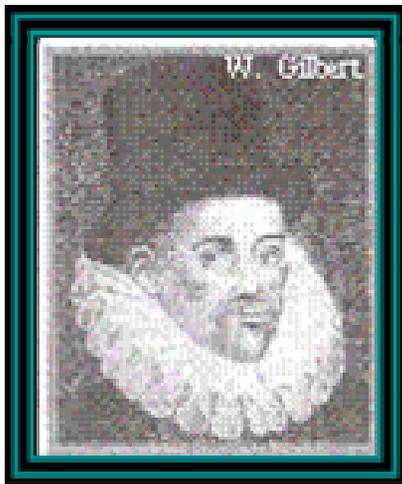
Su trabajo se destaca por la primera discusión detallada de una brújula.

Estudia una piedra imán de forma esférica mediante una aguja que se orienta por la acción del campo magnetico.

Aguja rotatoria de una brújula en una copia de la “Epistola de magnetete” de P. Mauricourt (1269)

Descubre los polos magnéticos

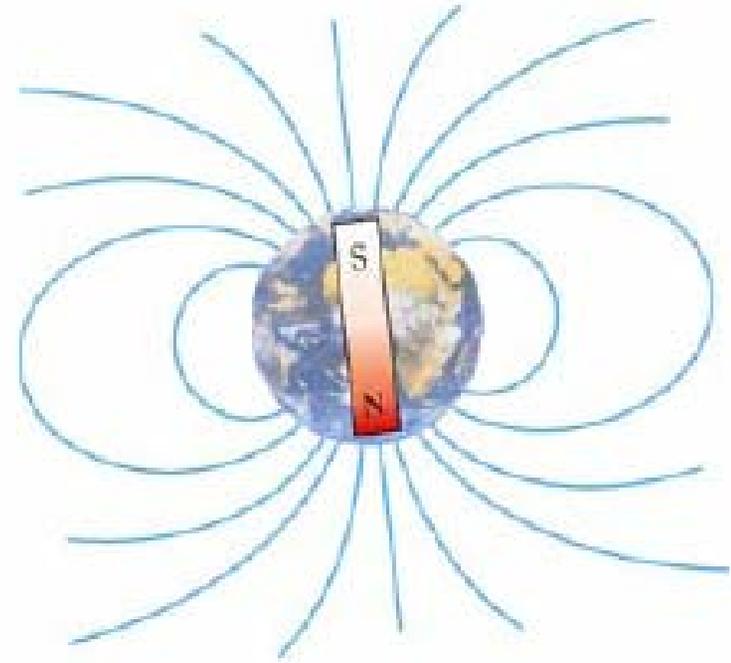




Willian Gilbert (1544-1603)

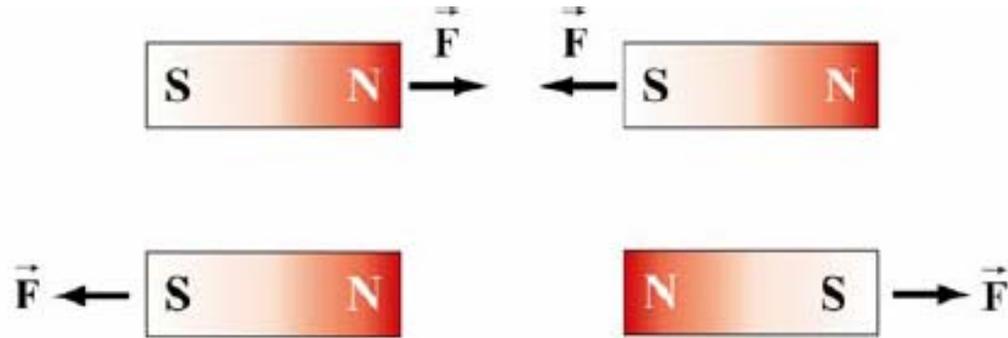
Descubrió la imantación por influencia, y observó que la imantación del hierro se pierde cuando se calienta al rojo.

Estudió la inclinación de una aguja magnética concluyendo que la Tierra se comporta como un gran imán.



De Magnete, Magneticisque Corporibus, et de Magno Magnete Tellure (Sobre los imanes, los cuerpos magnéticos y el gran imán terrestre)

Experimentos posteriores revelaron:



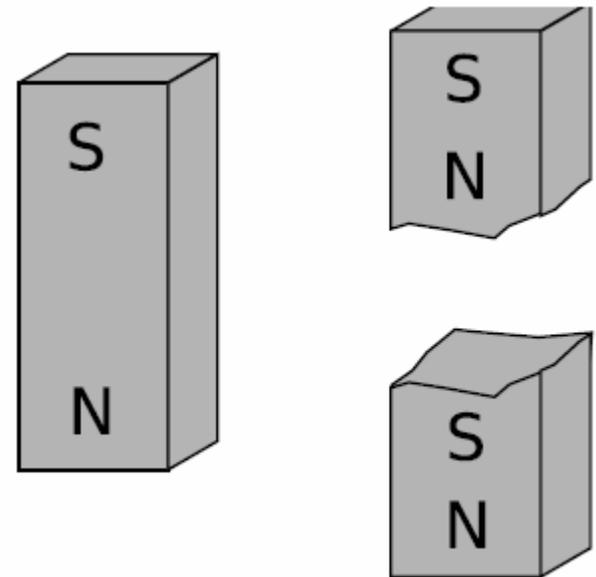
John Michell (1750)

Balanza de Torsion

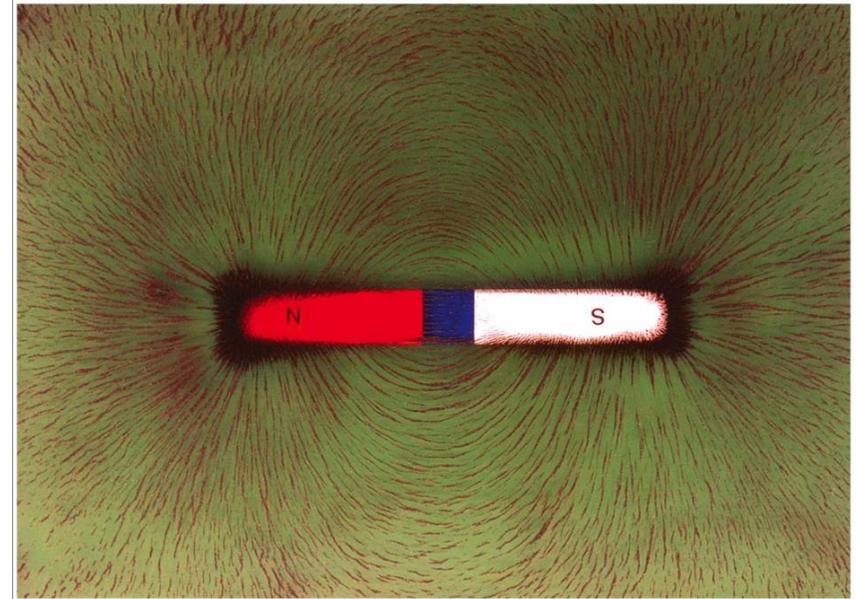
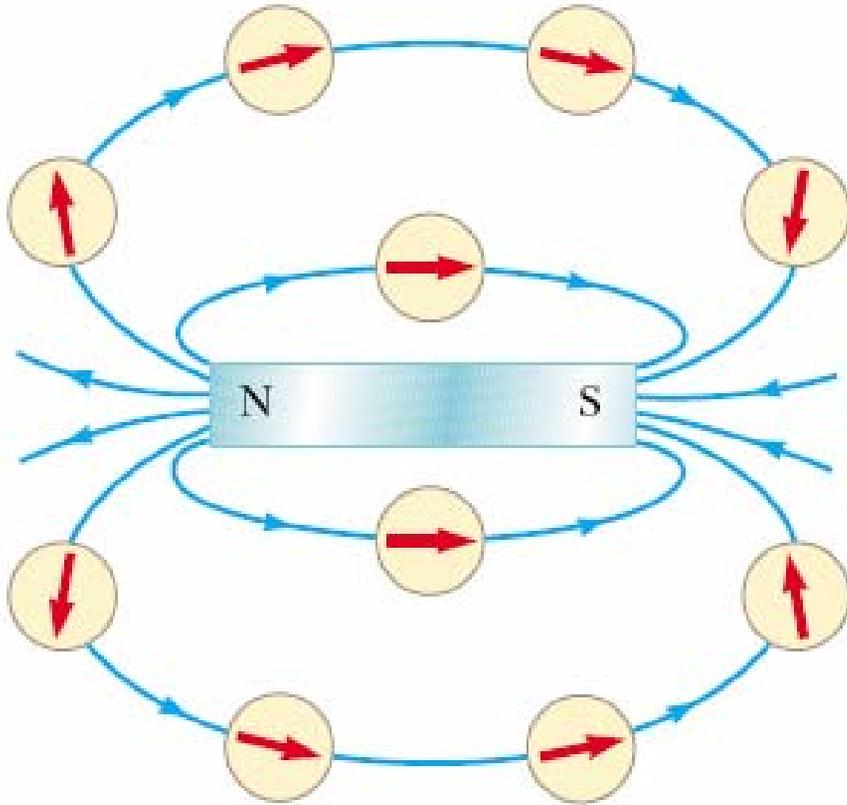
$$F \sim 1/r^2$$

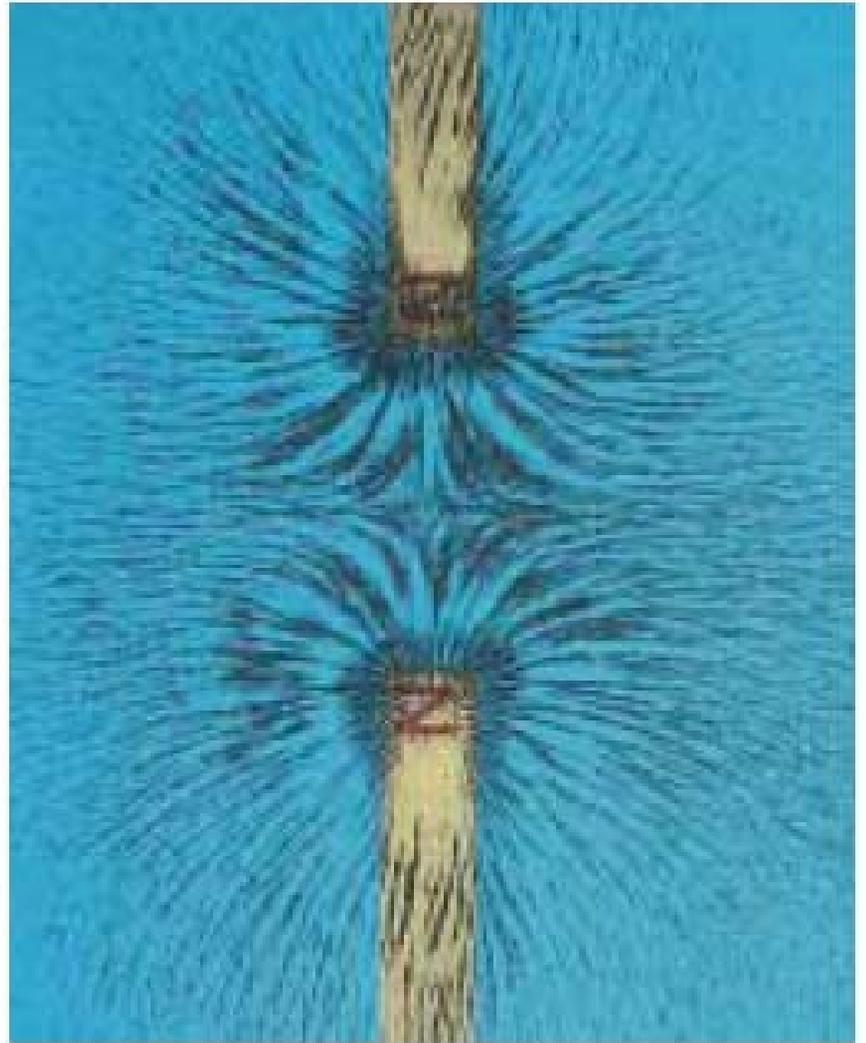
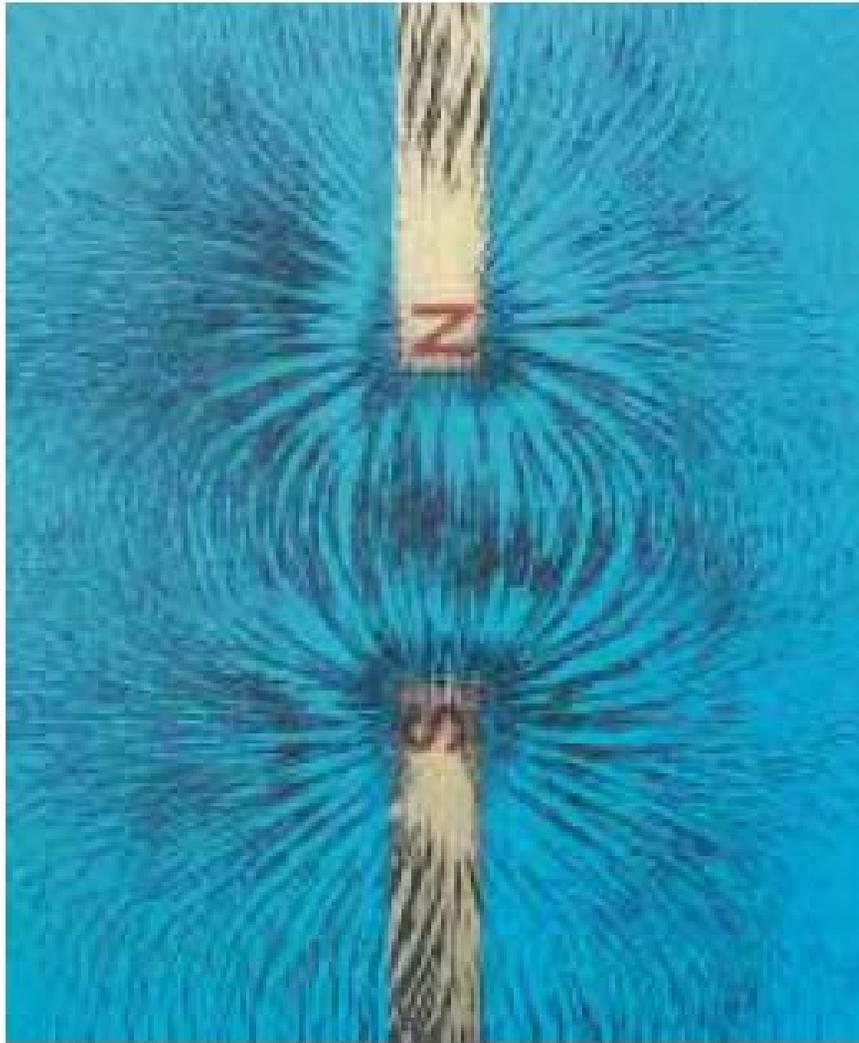
Diferencia importante con las cargas eléctricas:

Los polos magnéticos no se pueden aislar

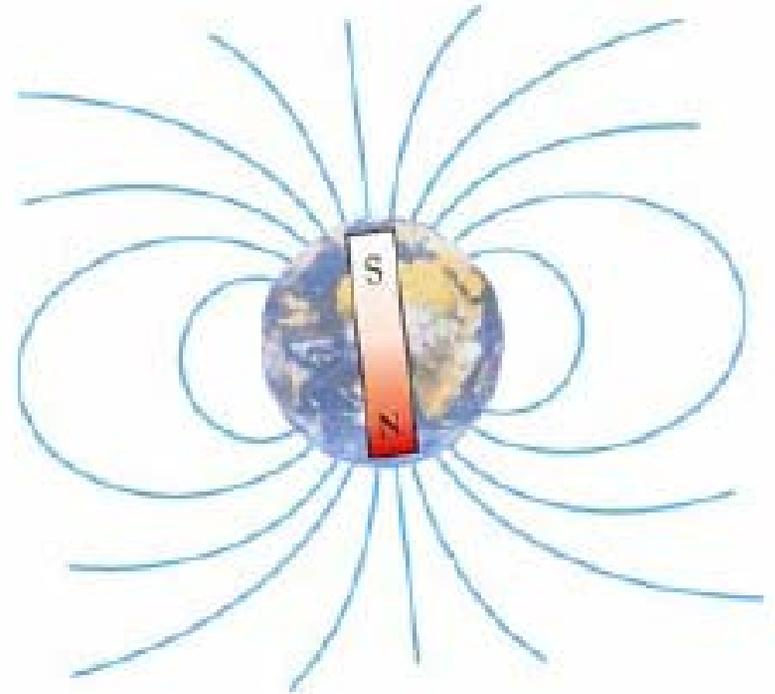
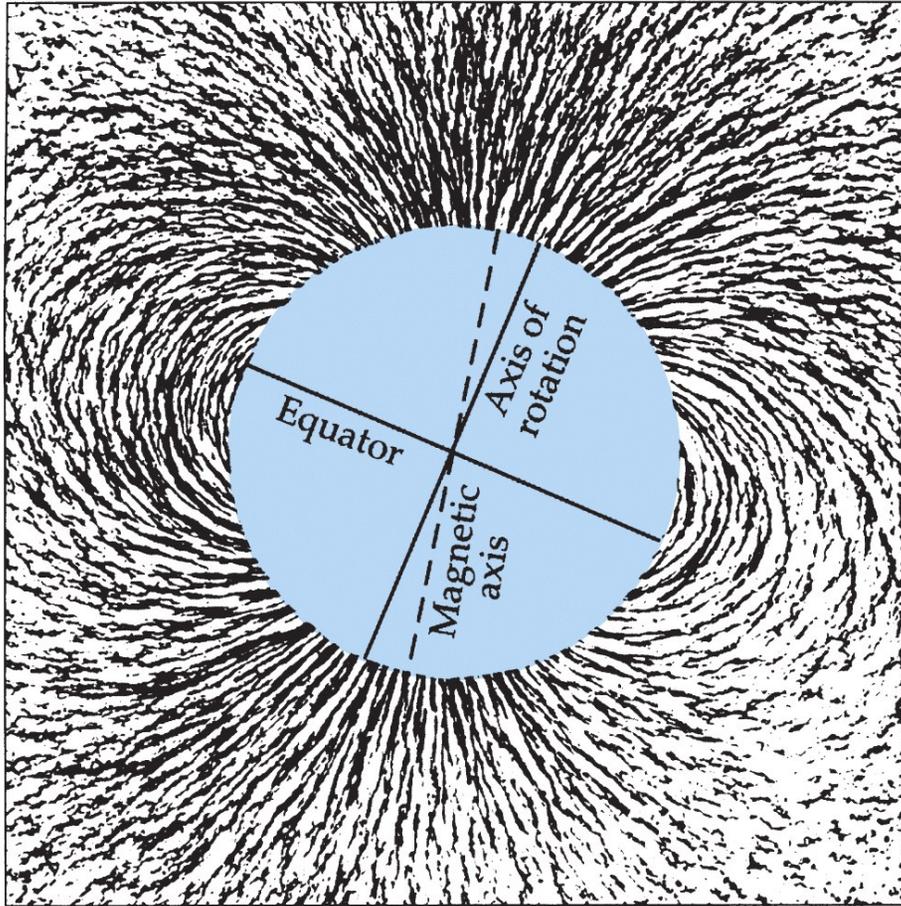


Campo Magnético producido por un imán de barra

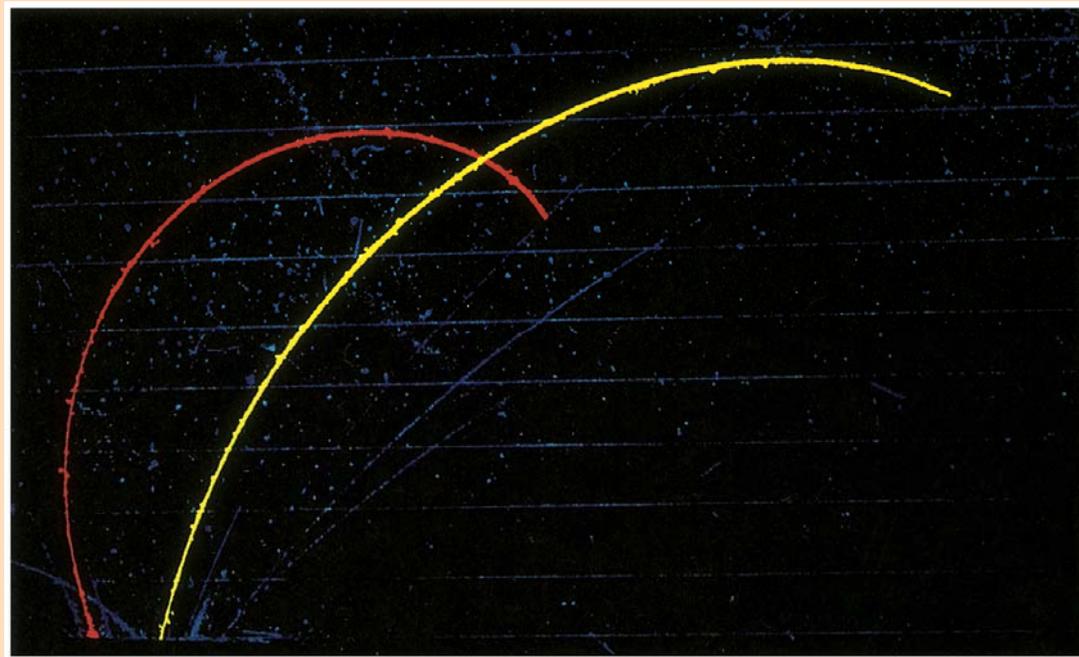




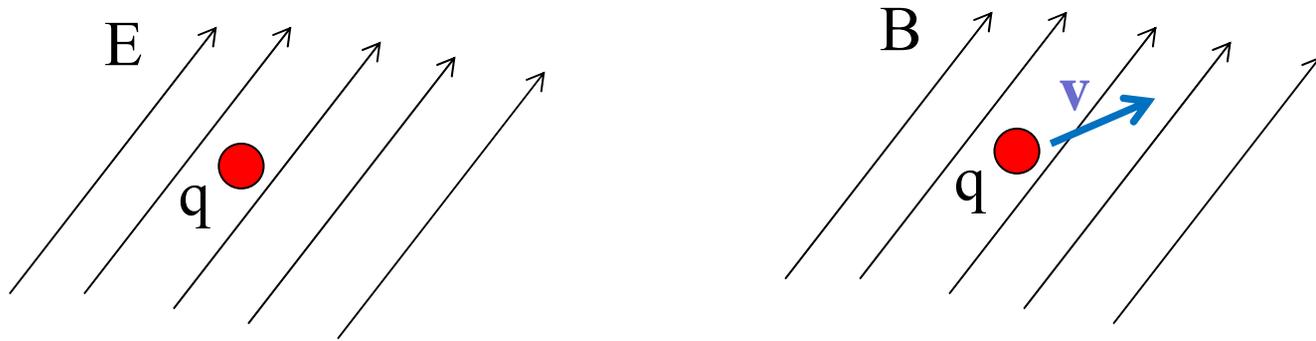
Campo magnético terrestre



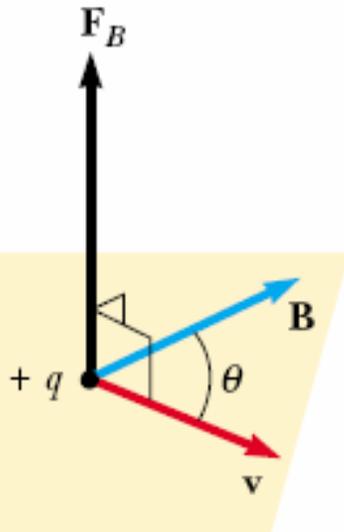
Fuerza ejercida por un campo magnético sobre una carga



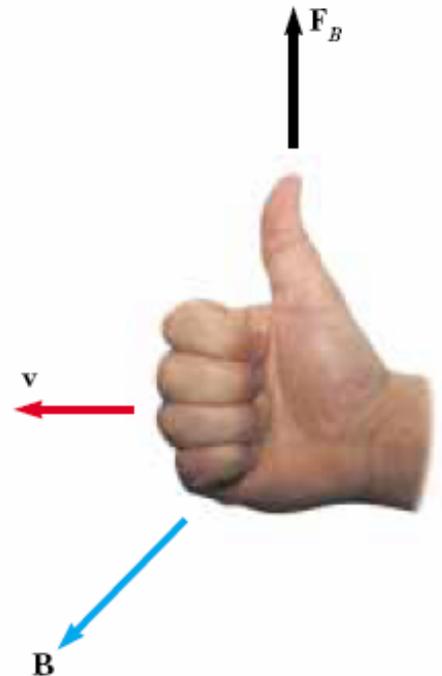
Definiremos el campo magnético (\mathbf{B}) en un punto del espacio en términos de la fuerza ejercida sobre un objeto de prueba apropiado

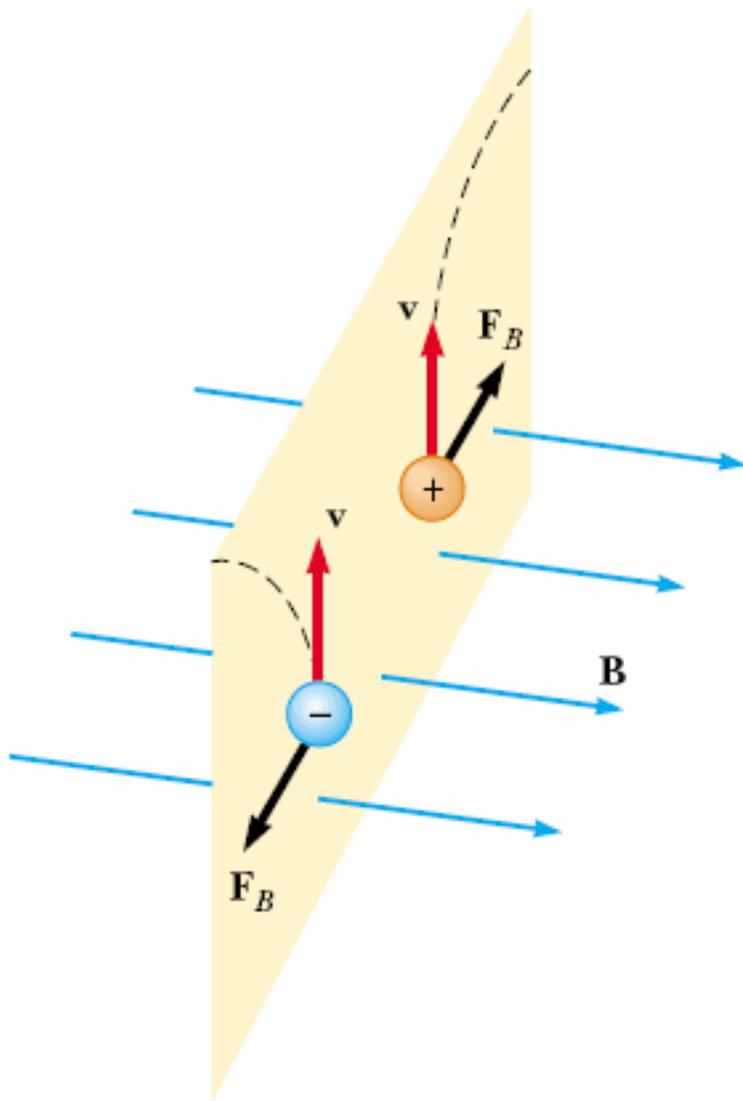


A partir de mediciones experimentales:



$$\mathbf{F}_B = q\mathbf{v} \times \mathbf{B}$$





$$\mathbf{F}_B = q\mathbf{v} \times \mathbf{B}$$

$$F_B = |q|vB \sin \theta$$

$$[B] = [F] / [q] [v] = \text{N} / \text{C} (\text{m/s}) = \text{T} \text{ (Tesla)}$$

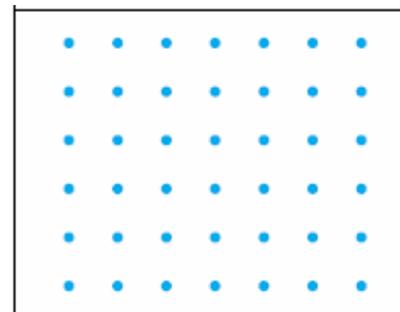
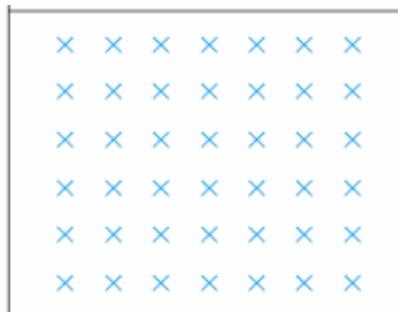
$$1\text{T} = 10^4 \text{ G (Gauss)}$$

Some Approximate Magnetic Field Magnitudes

Source of Field	Field Magnitude (T)
Strong superconducting laboratory magnet	30
Strong conventional laboratory magnet	2
Medical MRI unit	1.5
Bar magnet	10^{-2}
Surface of the Sun	10^{-2}
Surface of the Earth	0.5×10^{-4}
Inside human brain (due to nerve impulses)	10^{-13}

Representación:

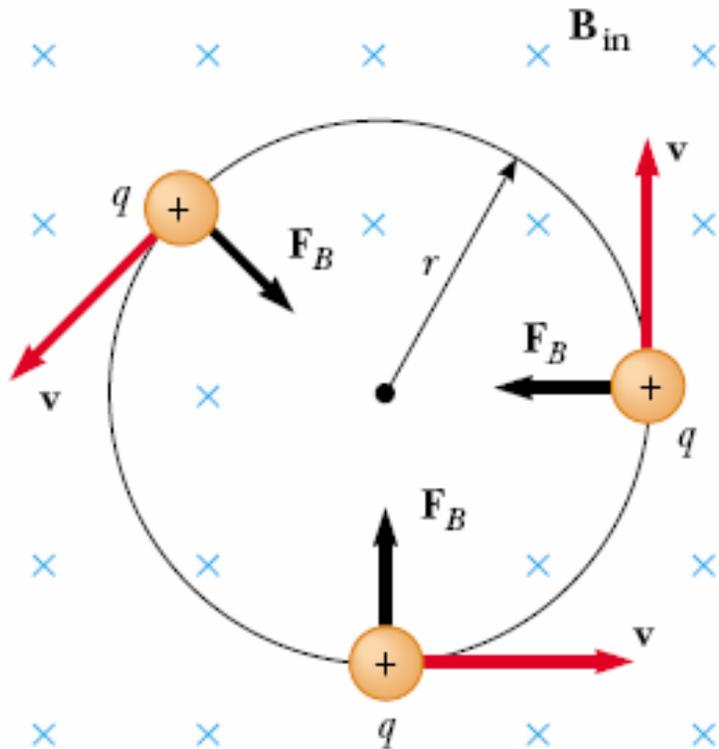
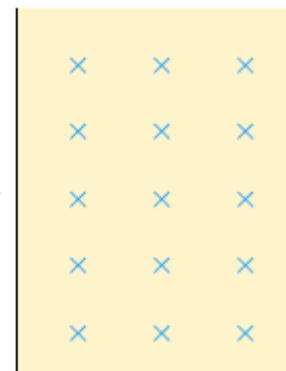
B entra
pantalla



B sale
pantalla

Ejemplo:

B homogéneo
y perpendicular a v



Movimiento circular

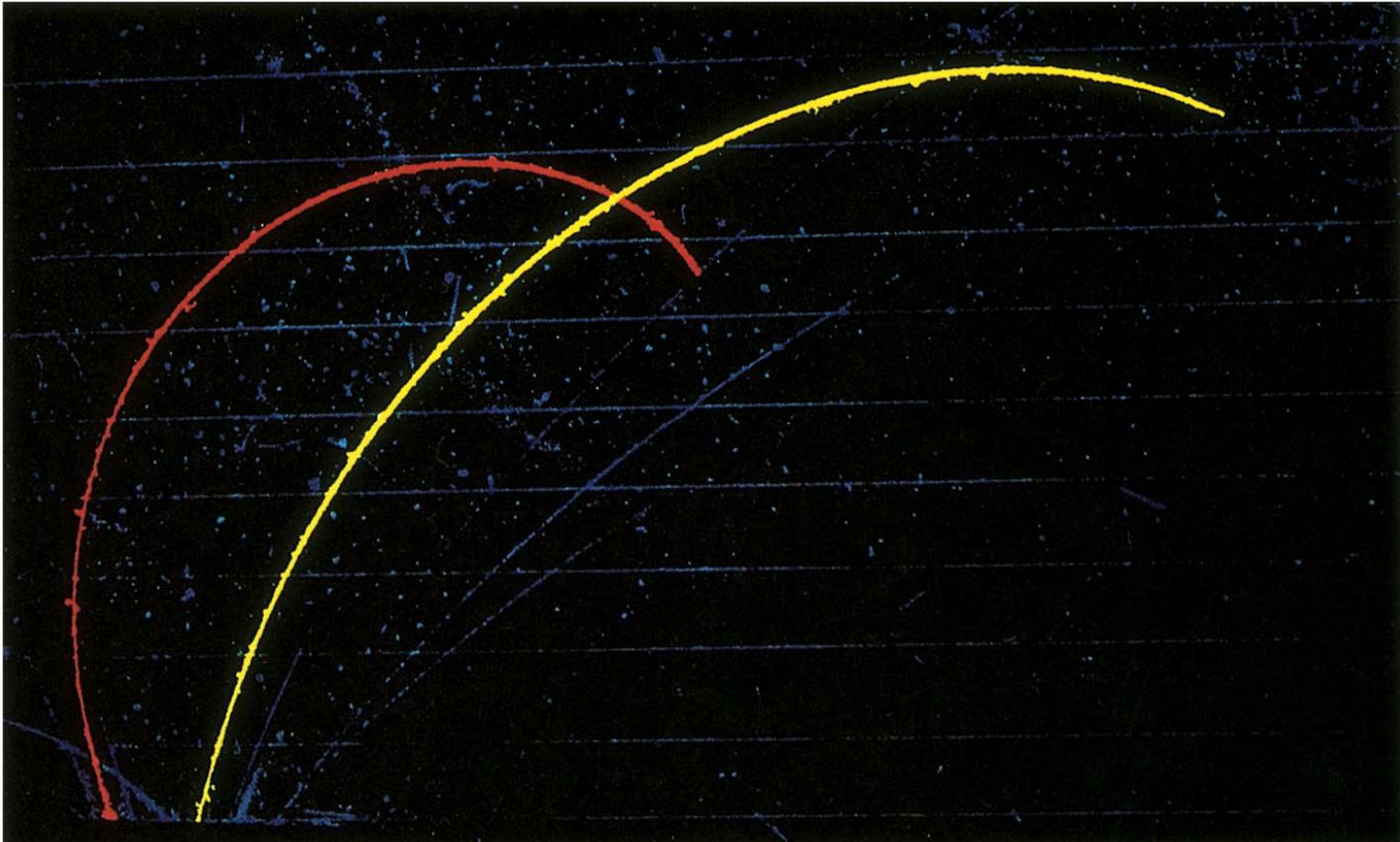
$$\sum F = ma_c$$

$$F_B = qvB = \frac{mv^2}{r}$$

$$r = \frac{mv}{qB}$$

$$\omega = \frac{v}{r} = \frac{qB}{m}$$

$$T = \frac{2\pi r}{v} = \frac{2\pi}{\omega} = \frac{2\pi m}{qB}$$



Trayectoria de un protón (rojo) y una partícula alfa (amarillo) en una cámara de niebla.

Si la velocidad no es perpendicular a B:

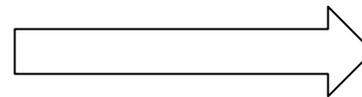
$$\vec{v} = \vec{v}_{\parallel} + \vec{v}_{\perp}$$

$$\vec{F}_{\parallel} = q(\vec{v}_{\parallel} \wedge \vec{B}) = 0 \rightarrow v_{\parallel} = cte$$

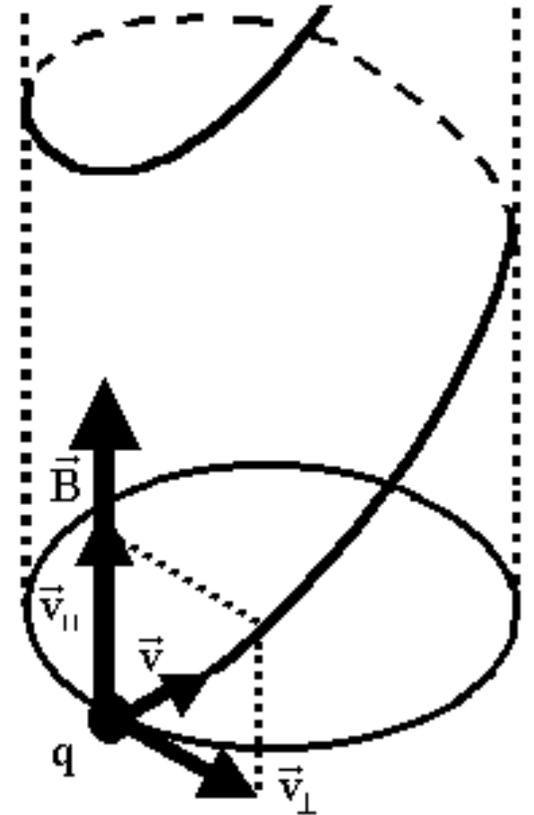
$$\rightarrow \vec{F}_{\perp} = q(\vec{v}_{\perp} \wedge \vec{B}) \rightarrow$$

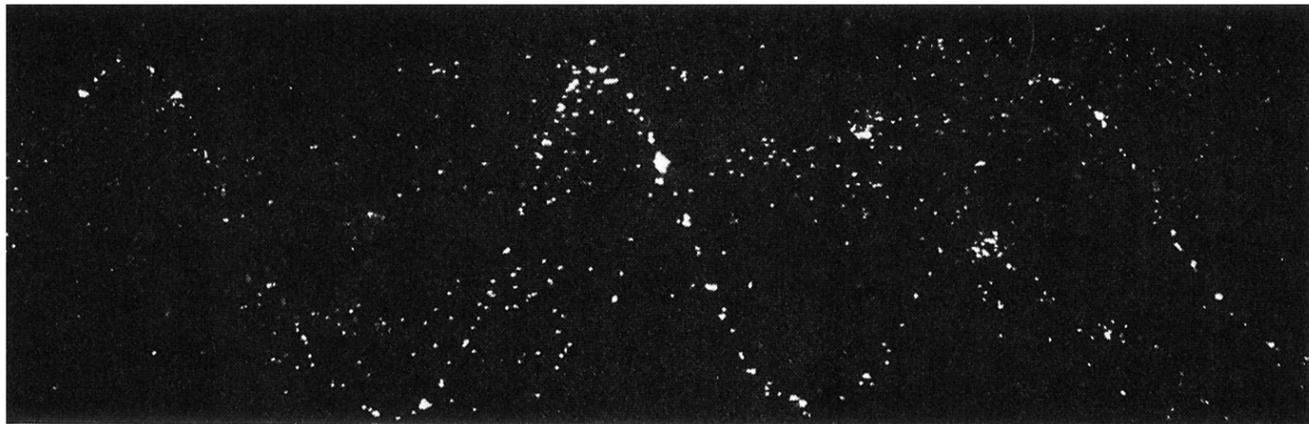
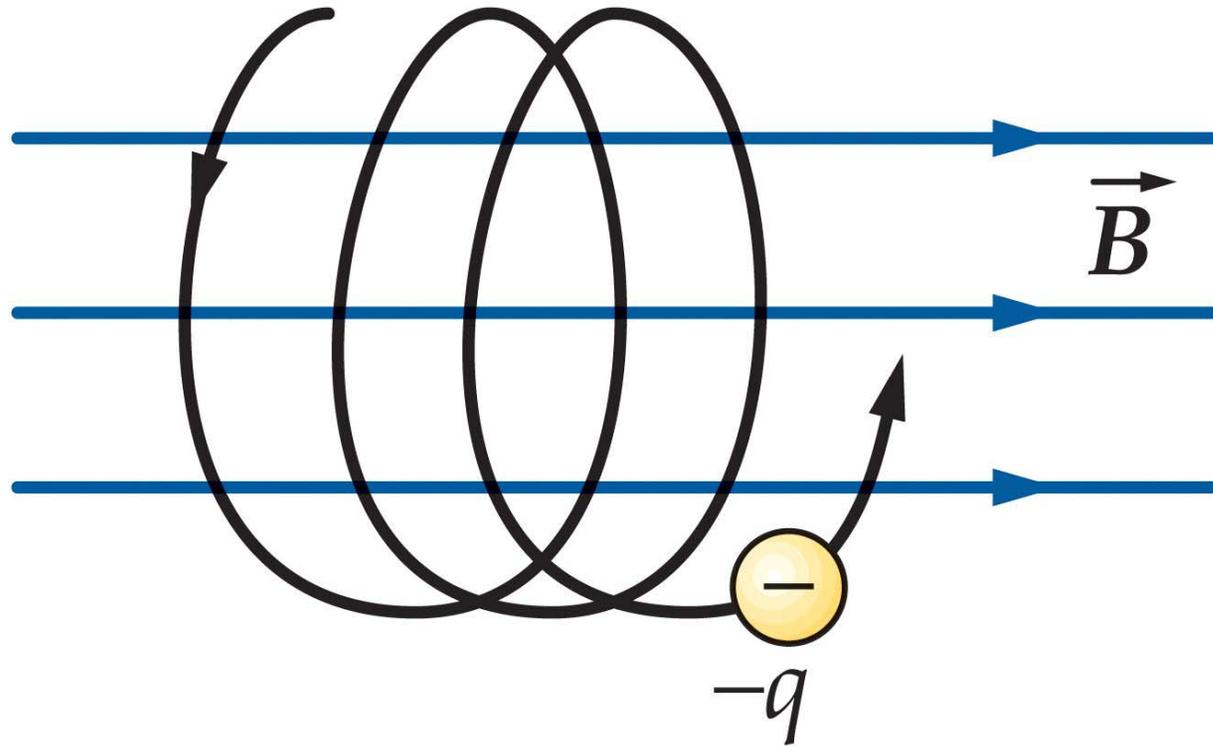
$$\rightarrow F_{\perp} = qv_{\perp}B$$

Movimiento circular en el plano perpendicular al campo

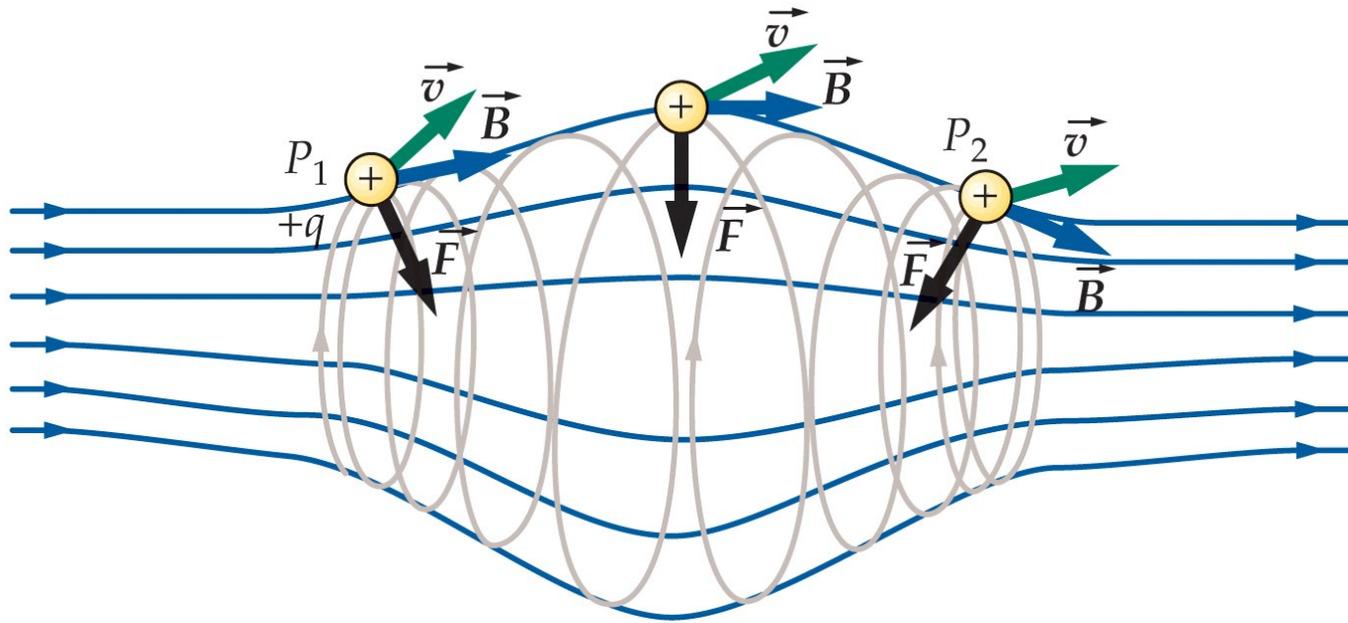


Hélice



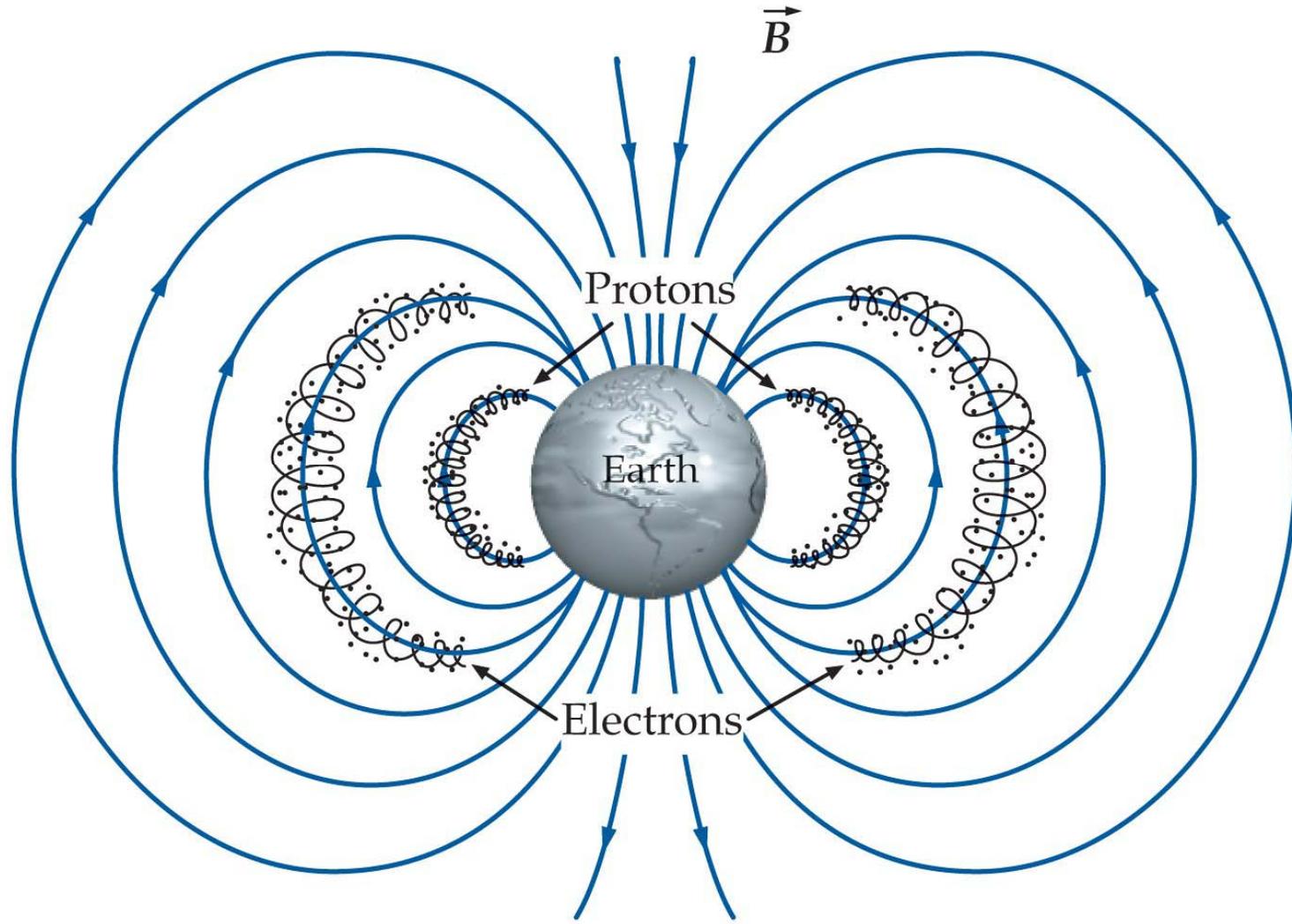


Fotografía en una cámara de niebla de la trayectoria helicoidal de un electrón



Botella magnética: cuando una partícula cargada se mueve en este campo, muy intenso en ambos extremos y débil en el centro, la partícula queda atrapada.

Partículas atrapadas en el campo magnético terrestre



Partículas chocan con los
átomos de la atmósfera

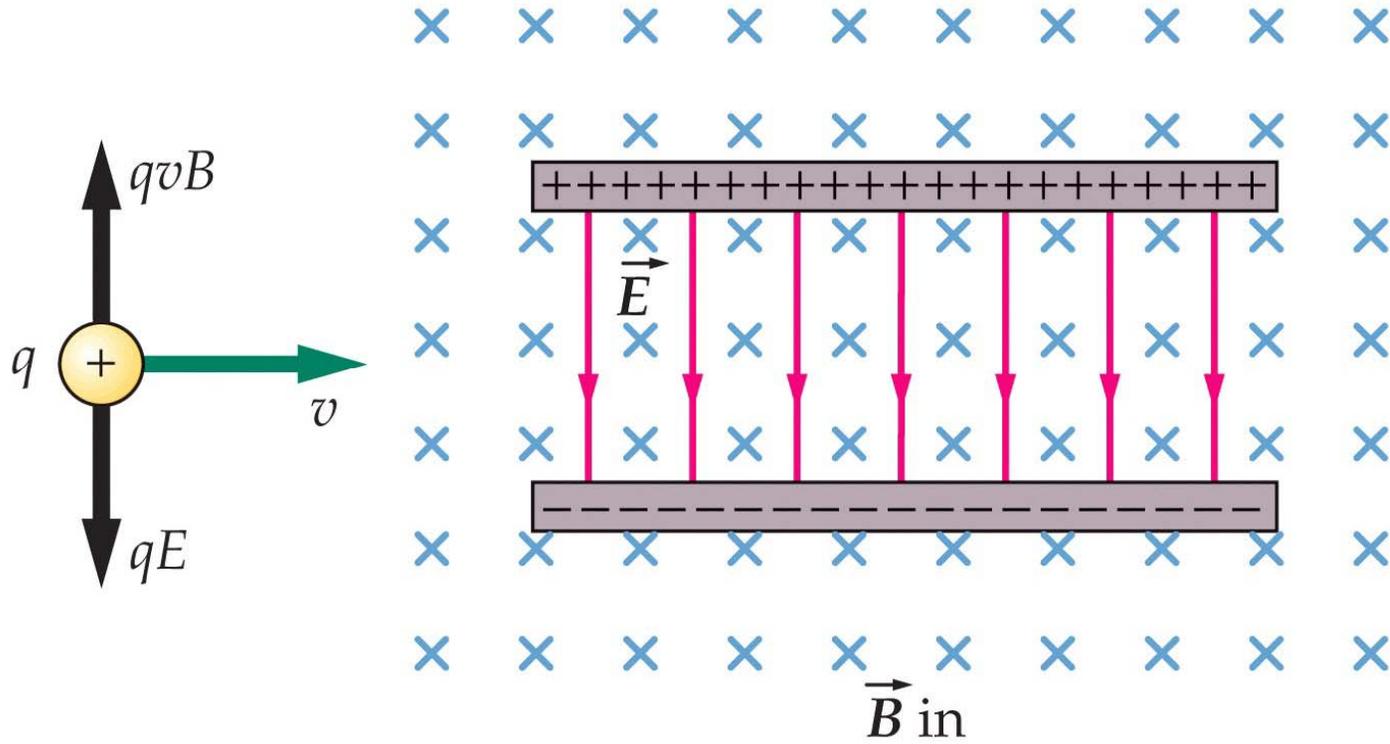


Emiten luz

Origen de las auroras boreales - australes

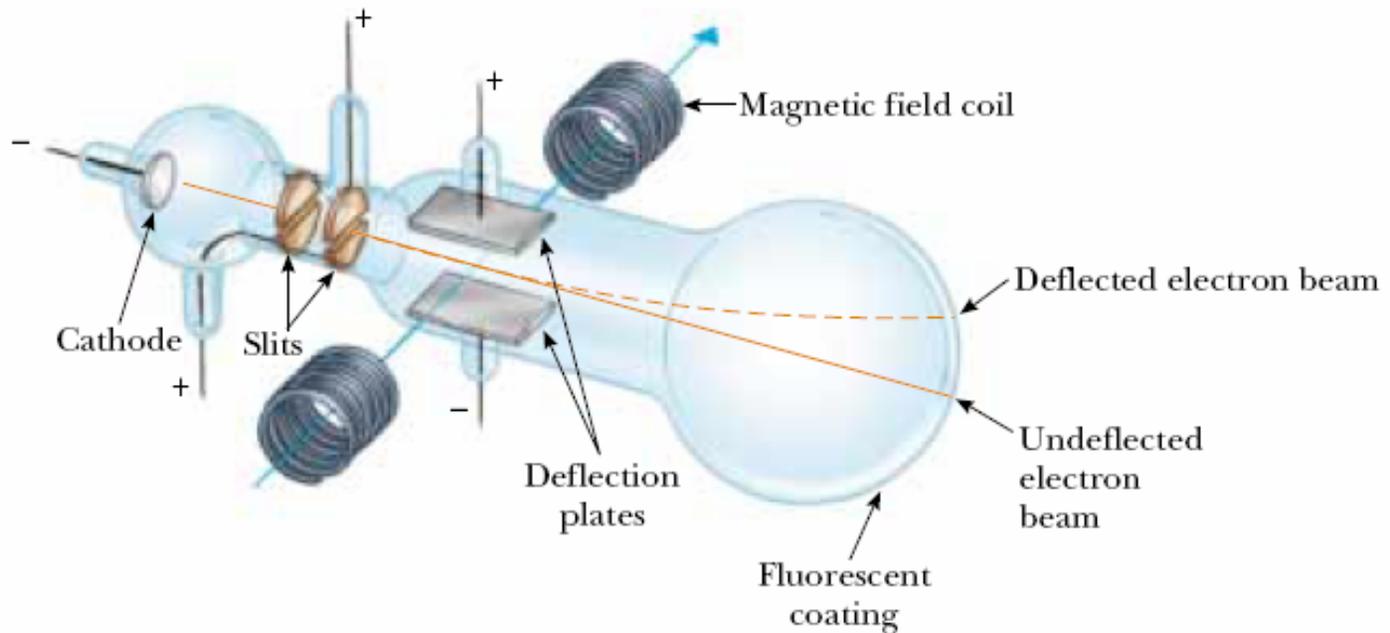
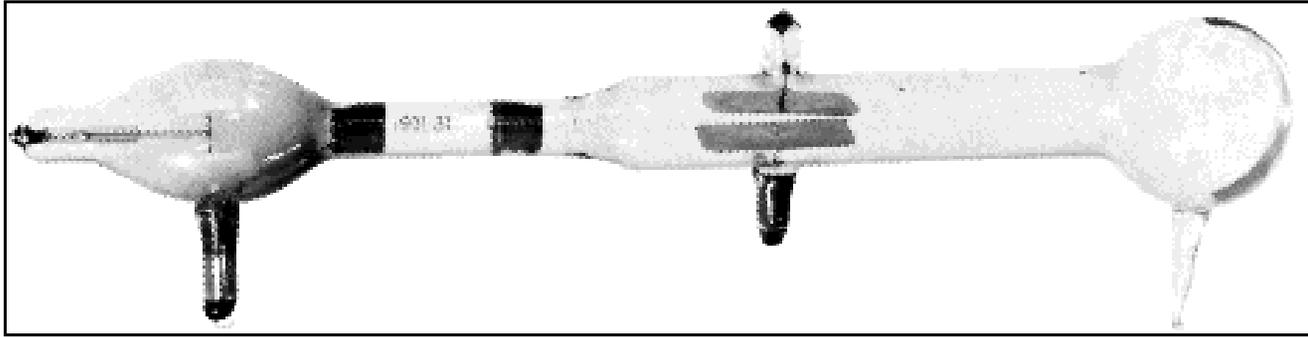


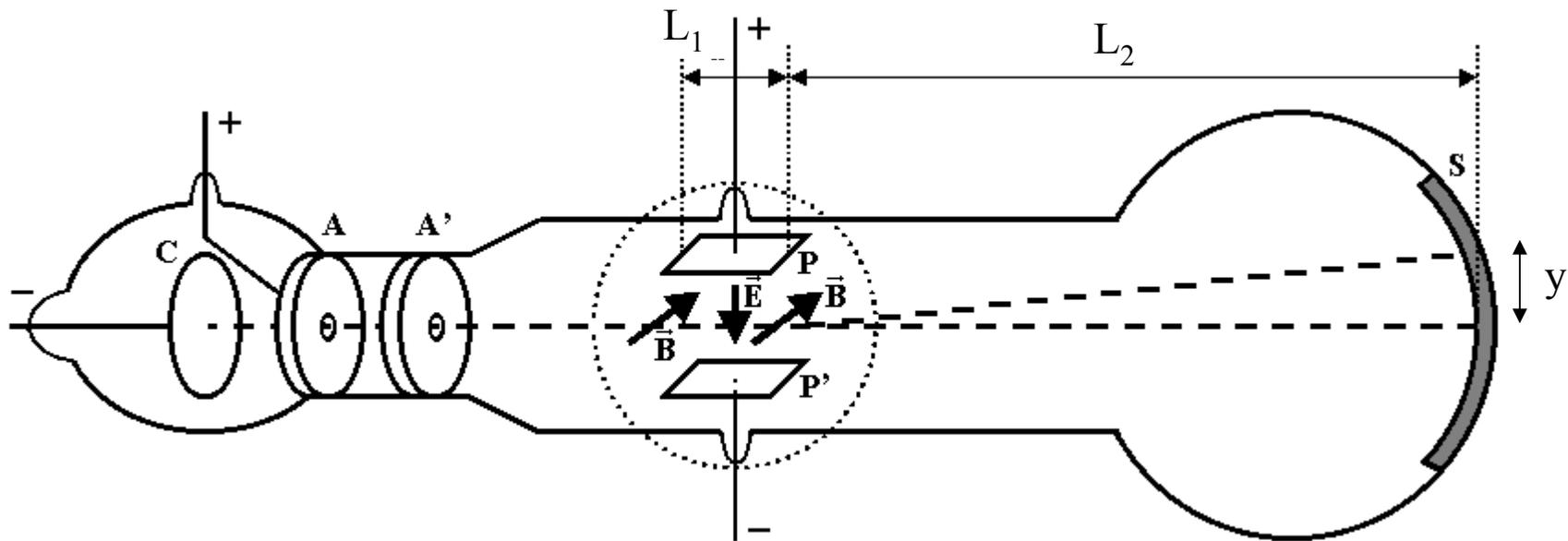
Selector de velocidades



$$v = \frac{E}{B}$$

Medicion de e/m - Experimento de J.J Thomson (1897)





$$B = 0 \quad \longrightarrow$$

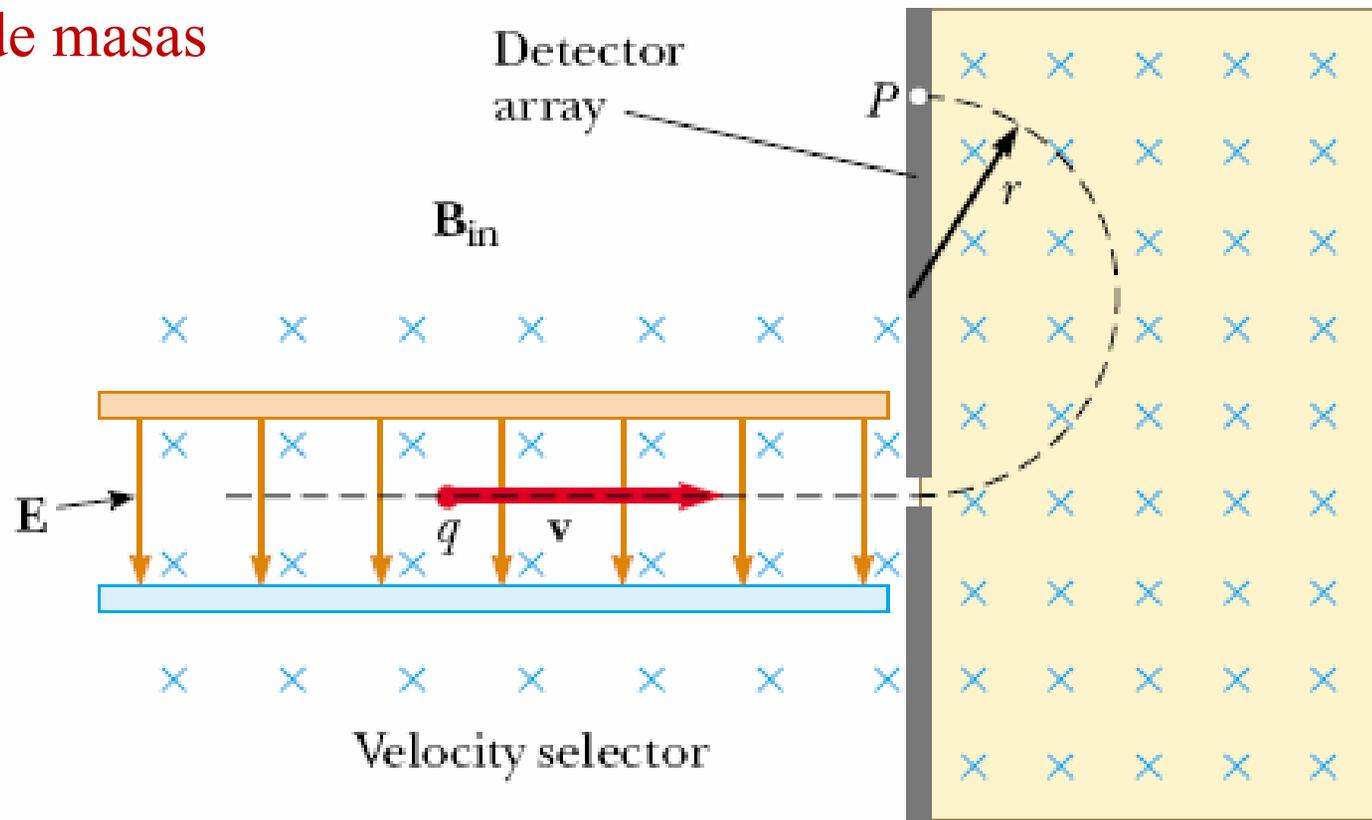
$$y = y_1 + y_2 = \frac{1}{2} \frac{eE_y L_1^2}{mv_0^2} + \frac{eE_y L_1 L_2}{mv_0^2} = \frac{eE_y L_1}{mv_0^2} \left(\frac{1}{2} L_1 + L_2 \right)$$

$$\text{Ajusto } B / y = 0 \quad \longrightarrow$$

$$F_E = F_B$$

$$q E = q v_0 B \quad \longrightarrow \quad v_0 = E/B$$

Espectrómetro de masas

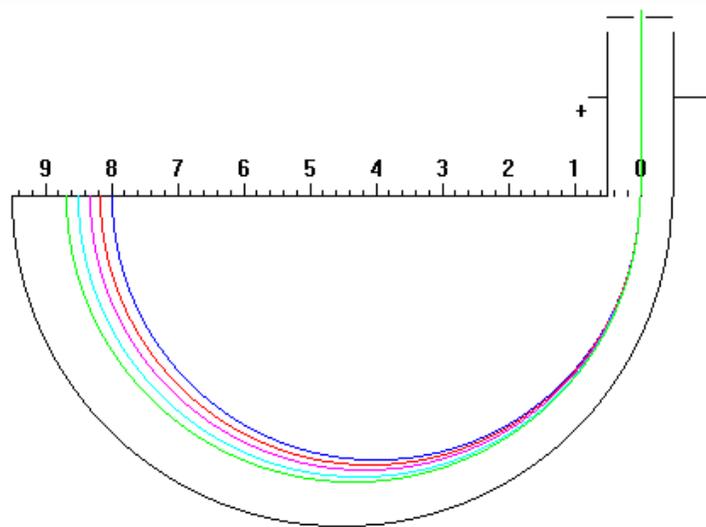


$$v = E / B$$

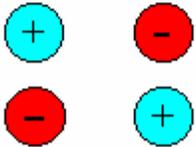
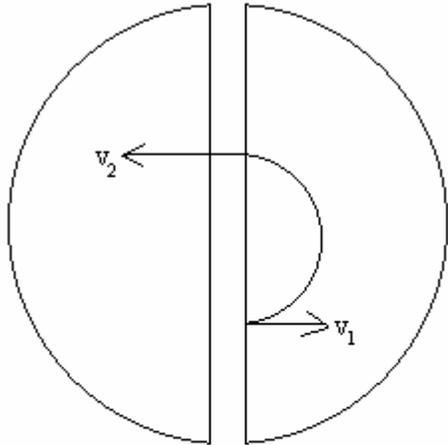
$$F_B = m a_c$$

$$q v B_0 = m v^2 / r$$

$$m = q B B_0 r / E$$

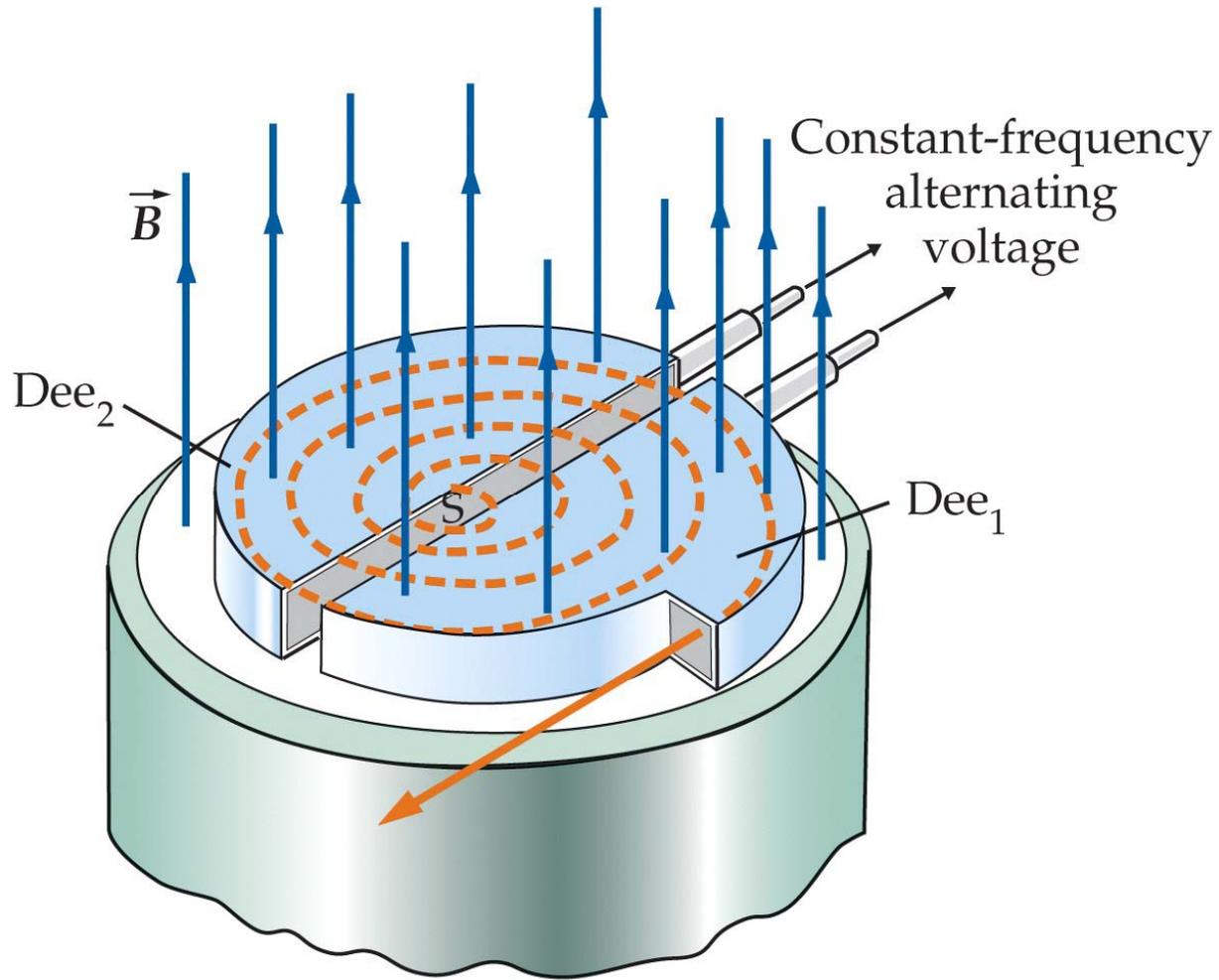


Ciclotrón



$$\omega = \frac{v}{r} = \frac{qB}{m}$$

$$r = \frac{mv}{qB}$$



$$K = \frac{1}{2}mv^2 = \frac{q^2 B^2 R^2}{2m}$$

La mayoría de los actuales aceleradores de partículas de alta energía descienden del primer ciclotrón de protones de 1 MeV construido por E. O. Lawrence y M. S. Livingstone en Berkeley (California). El artículo original fue publicado en la revista *Physical Review*, volumen 40, del 1 de abril de 1932, titulado "Producción de iones ligeros de alta velocidad sin el empleo de grandes voltajes"

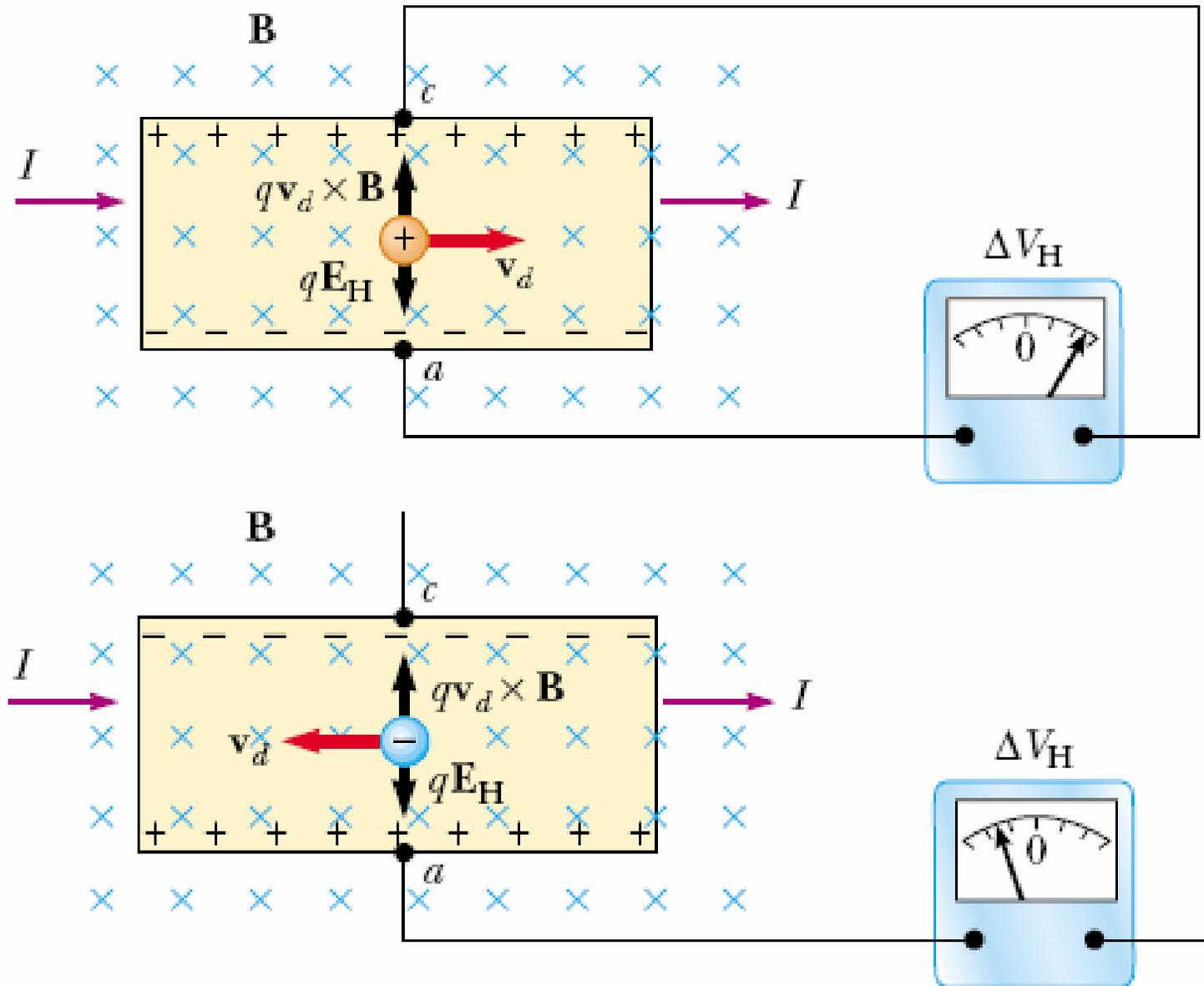


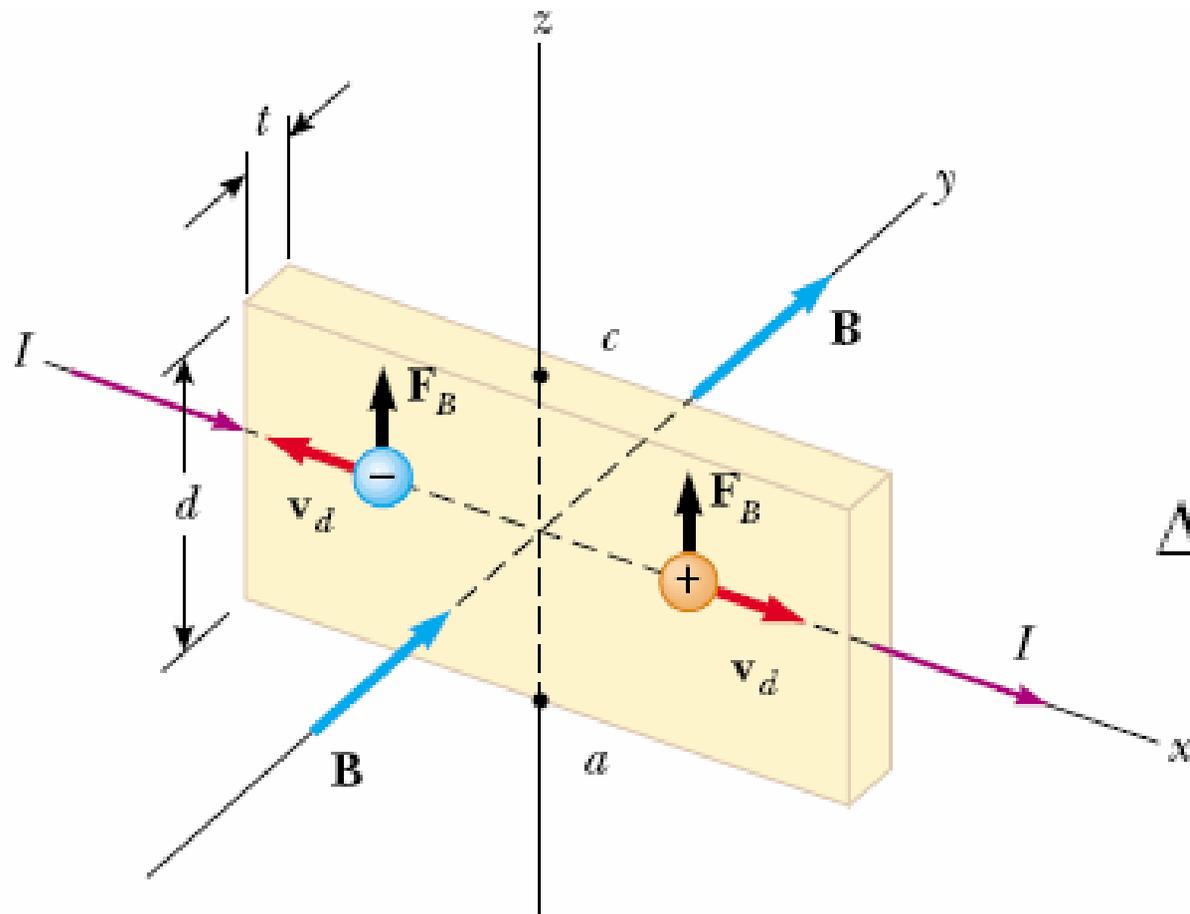
Esquema de un sincrotrón



En el sincrotrón ambos campos se hacen variar para mantener el camino de las partículas de forma constante, o sea, el radio no varía demasiado. La velocidad máxima a la que las partículas se pueden acelerar está dada por el punto en que la radiación sincrotrón emitida es igual a la energía inyectada

Efecto Hall (Edwin Hall 1879)





$$qv_d B = qE_H$$

$$E_H = v_d B$$

$$\Delta V_H = E_H d = v_d B d$$

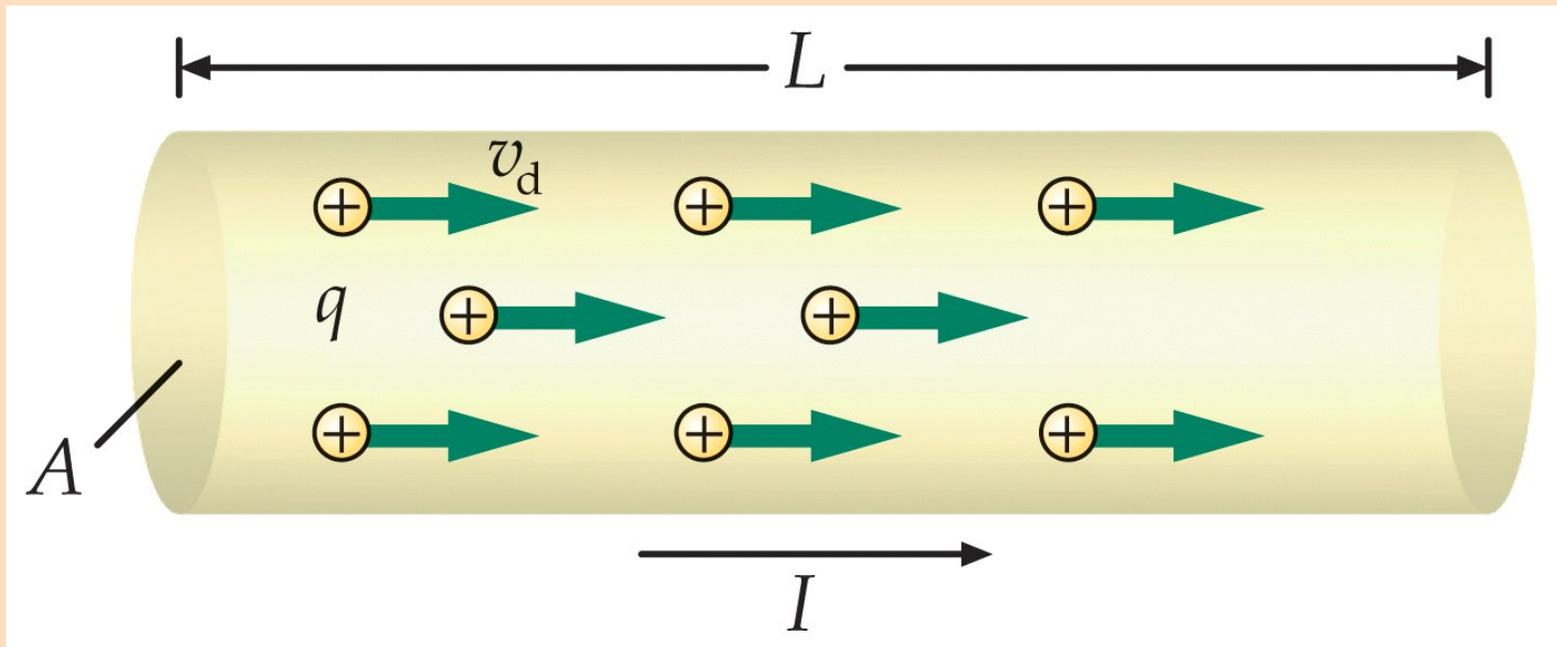
$$v_d = \frac{I}{nqA}$$

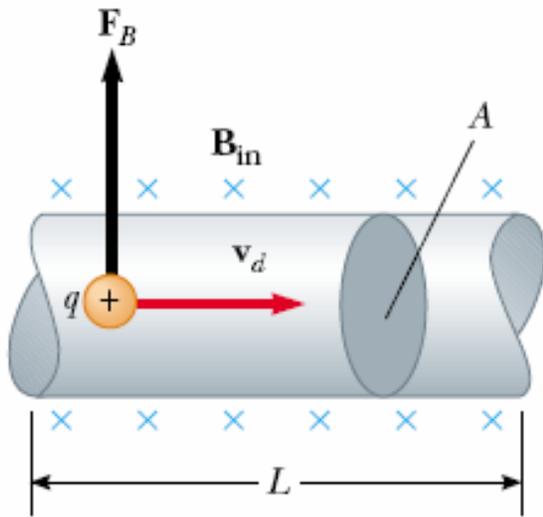
$$\Delta V_H = \frac{IBd}{nqA}$$

$$\Delta V_H = \frac{IB}{nqt} = \frac{R_H IB}{t}$$

$R_H = 1/nq$: Coeficiente Hall

Fuerza Magnética sobre un conductor de corriente

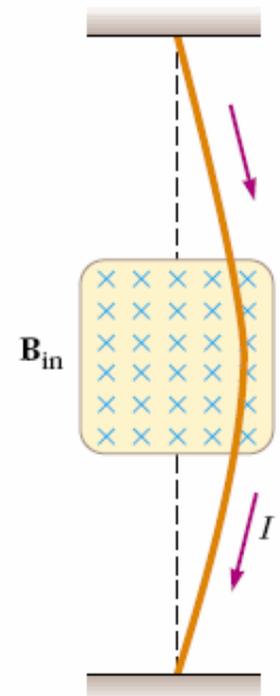
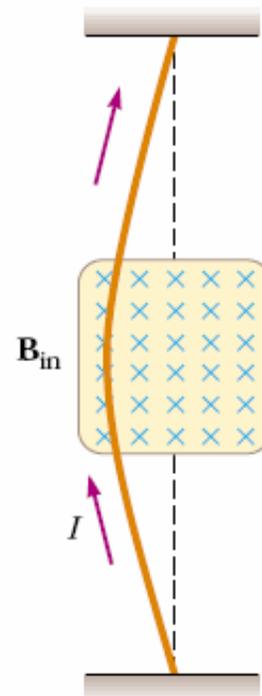
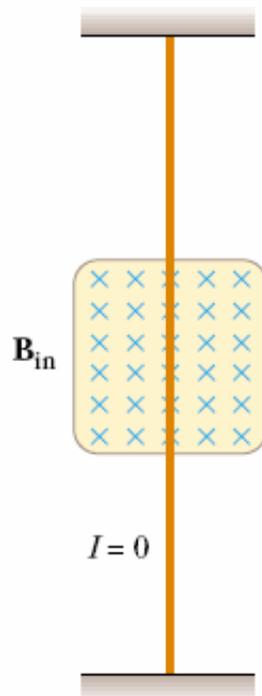
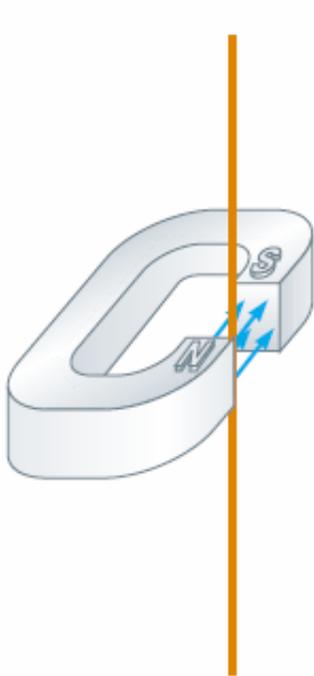


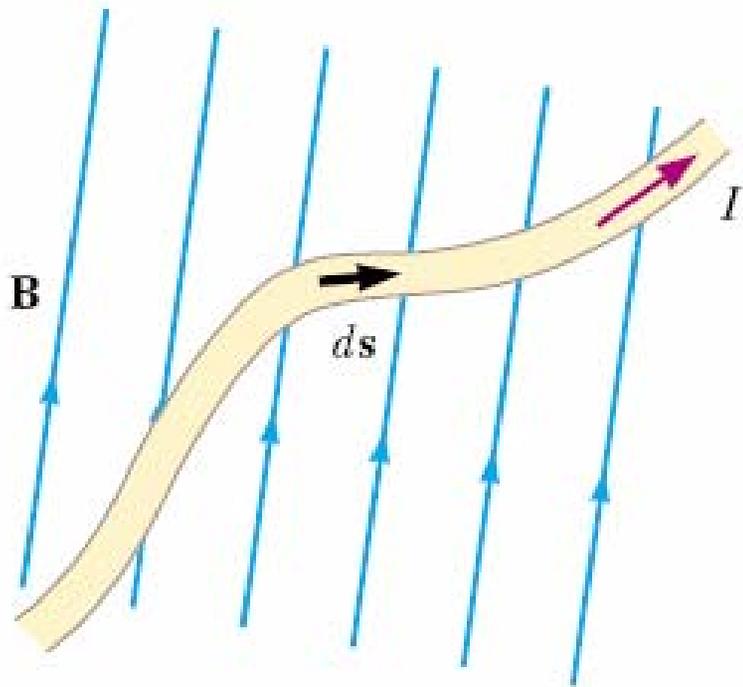
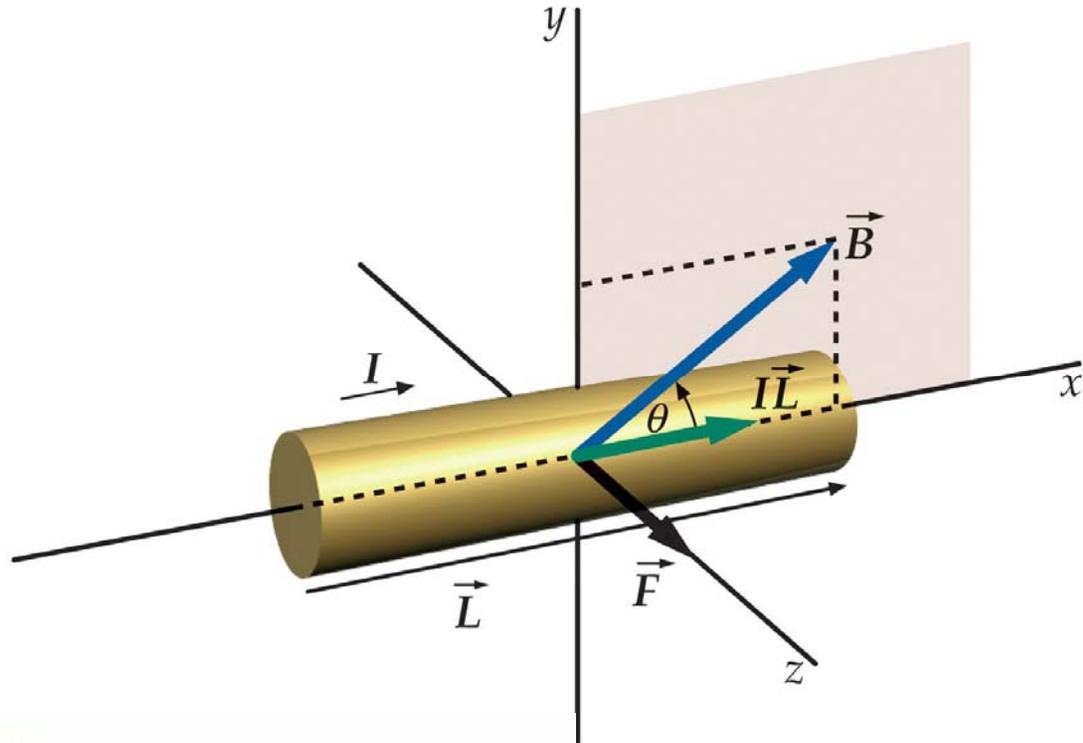


$$\mathbf{F}_B = (q\mathbf{v}_d \times \mathbf{B}) nAL$$

$$I = nqv_dA$$

$$\mathbf{F}_B = I\mathbf{L} \times \mathbf{B}$$

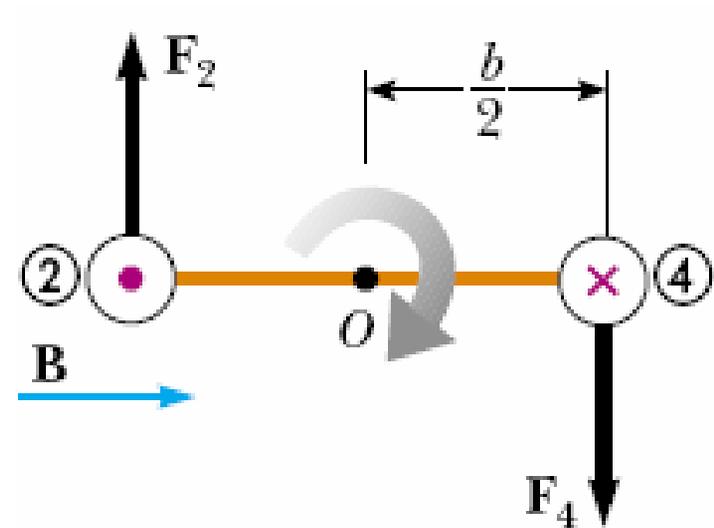
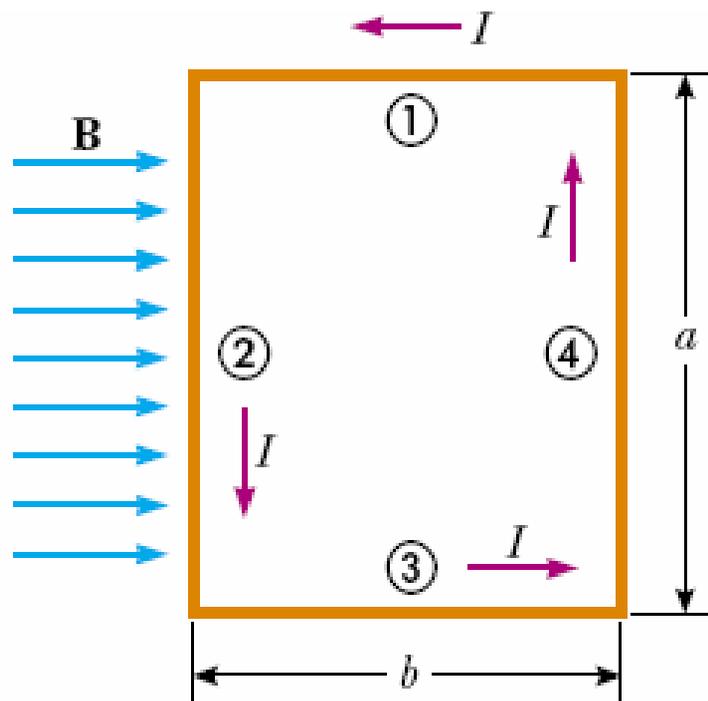




$$d\mathbf{F}_B = I d\mathbf{s} \times \mathbf{B}$$

$$\mathbf{F}_B = I \int_a^b d\mathbf{s} \times \mathbf{B}$$

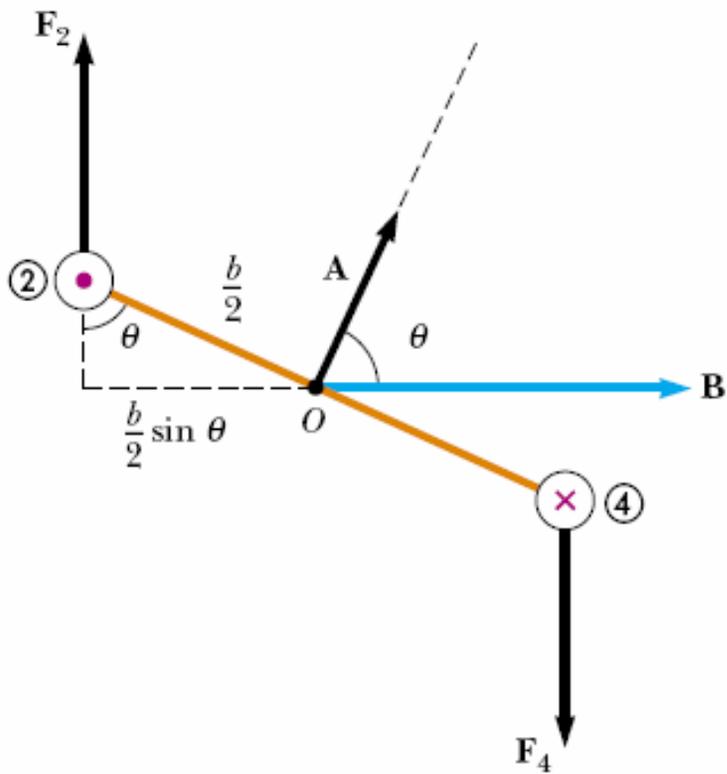
Torque sobre una espira



$$F_2 = F_4 = IaB$$

$$\tau_{\max} = F_2 \frac{b}{2} + F_4 \frac{b}{2} = (IaB) \frac{b}{2} + (IaB) \frac{b}{2} = IabB$$

$$\tau_{\max} = IAB$$



$$\tau = F_2 \frac{b}{2} \sin \theta + F_4 \frac{b}{2} \sin \theta$$

$$= IaB \left(\frac{b}{2} \sin \theta \right) + IaB \left(\frac{b}{2} \sin \theta \right) = IabB \sin \theta$$

$$= IAB \sin \theta$$

$$\boldsymbol{\tau} = I\mathbf{A} \times \mathbf{B}$$

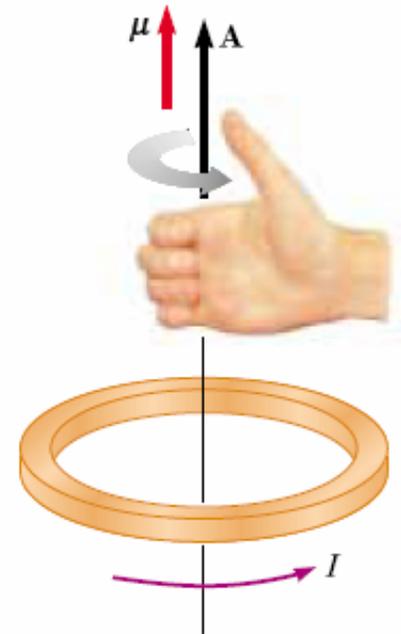
Definimos el momento dipolar magnético de la espira

$$\boldsymbol{\mu} = I\mathbf{A}$$

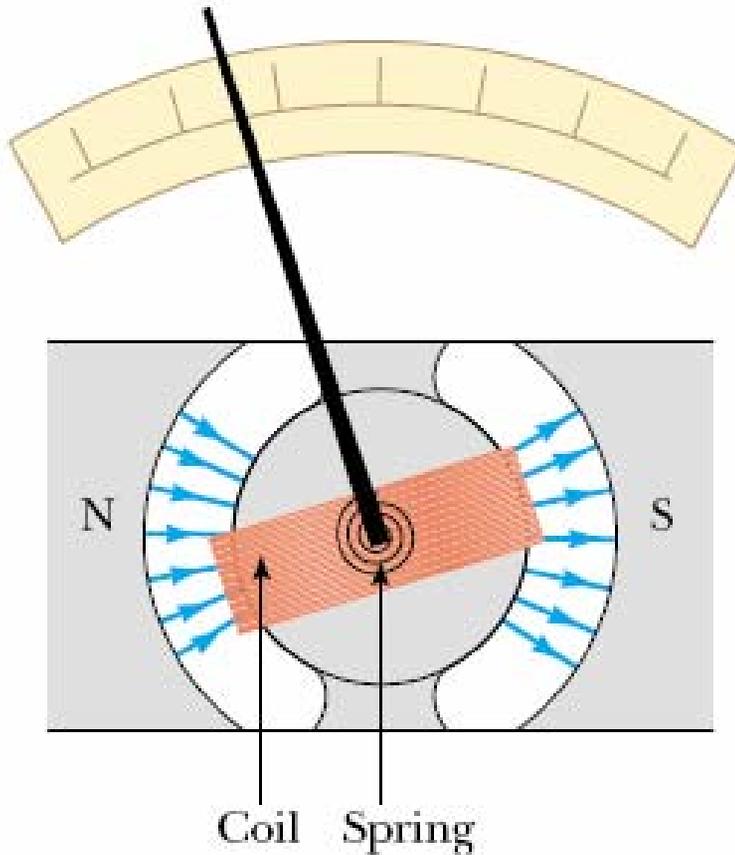
$$\boldsymbol{\tau} = \boldsymbol{\mu} \times \mathbf{B}$$

Se puede probar que:

$$U = -\boldsymbol{\mu} \cdot \mathbf{B}$$



Aplicación: Galvanómetro de bobina móvil

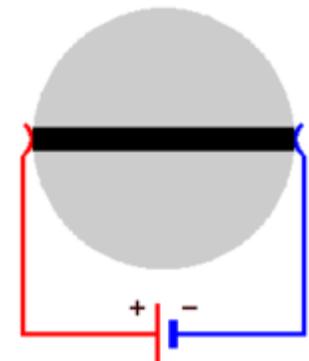
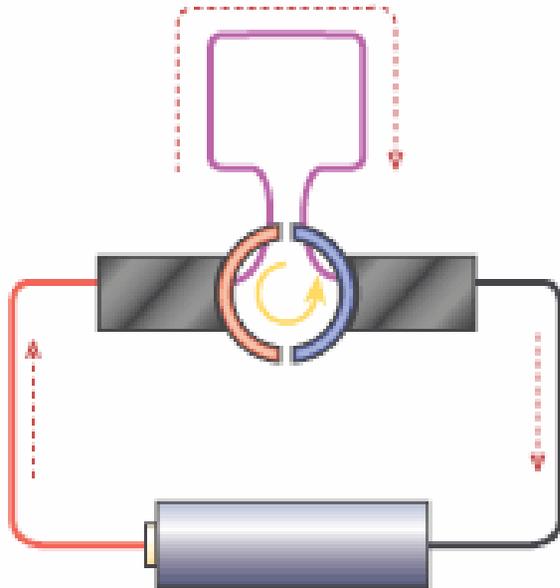
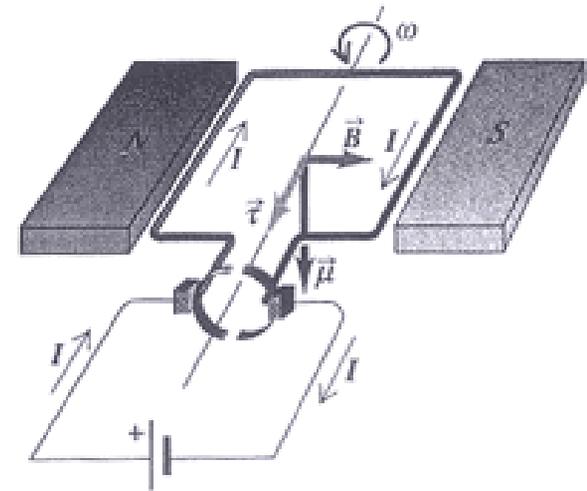
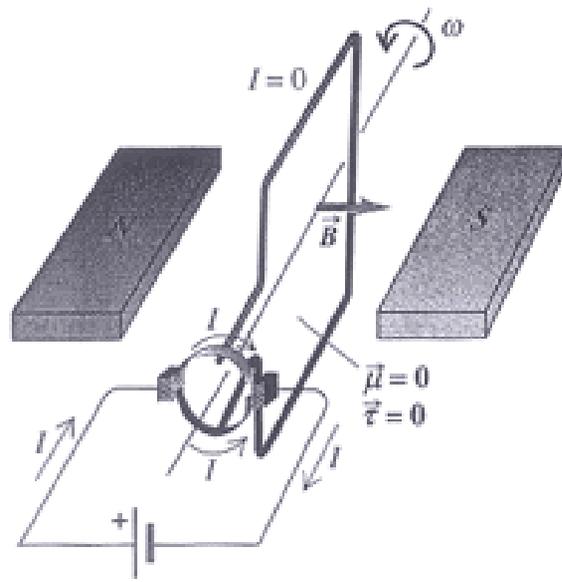
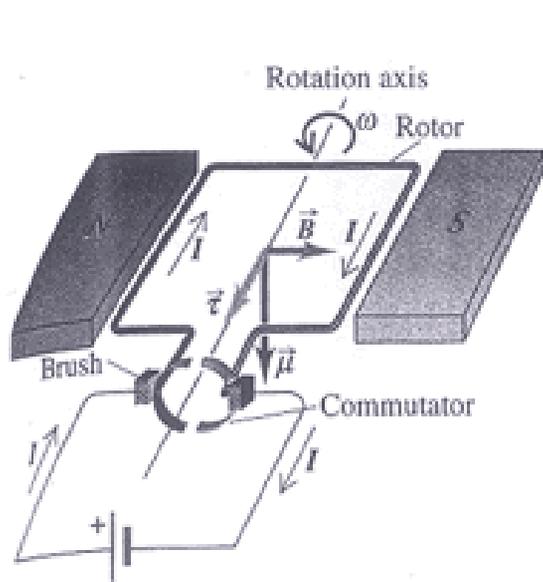


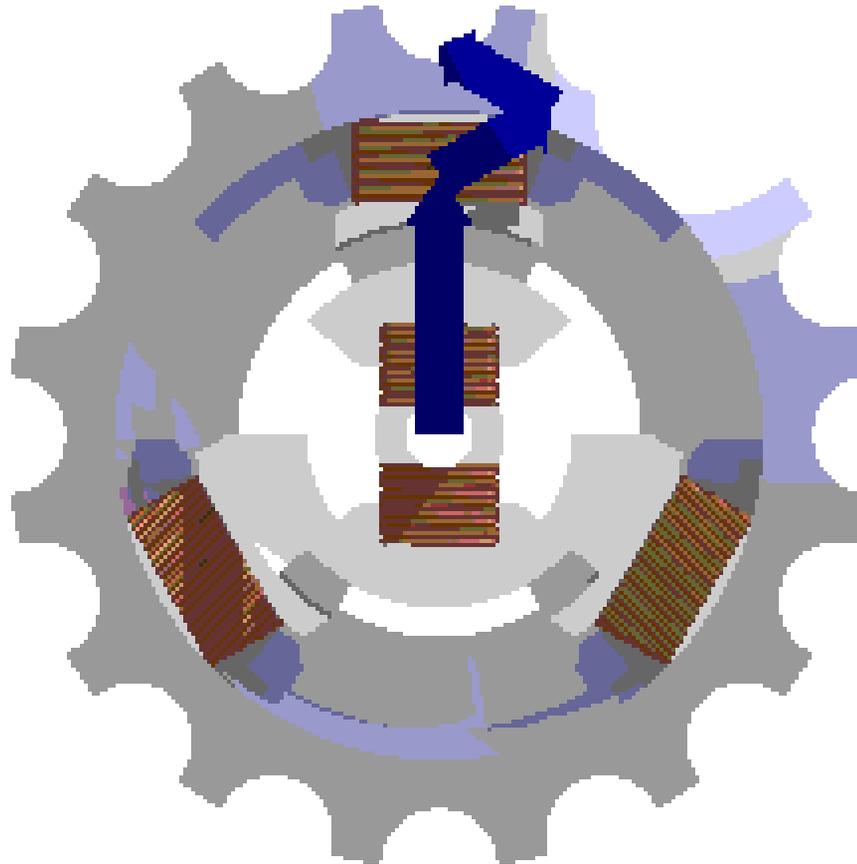
$$\tau = N I S B$$

$$\tau_{\text{res}} = K \alpha$$

$$I = K \alpha / N S B$$

Motor eléctrico

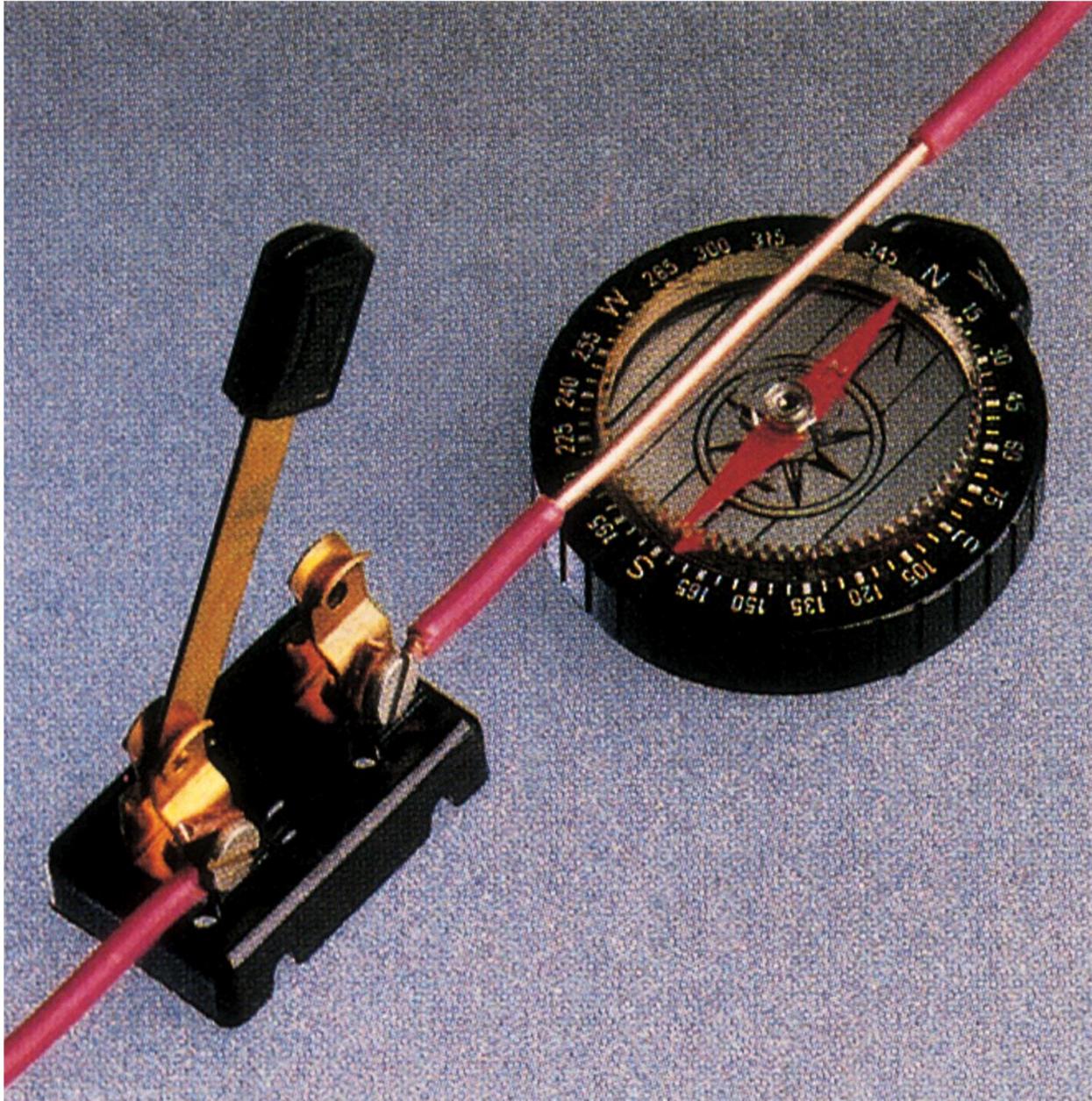


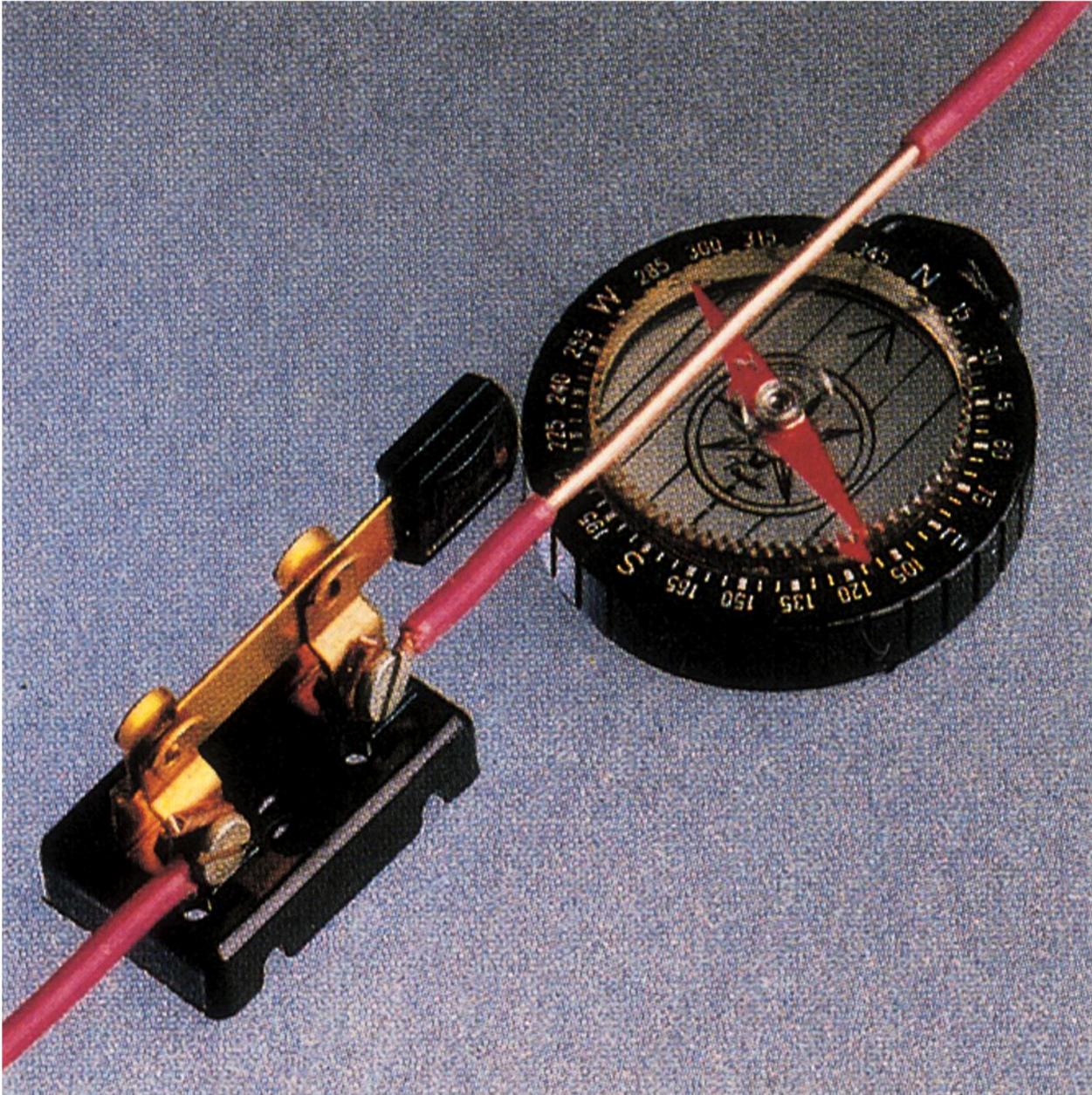


Campo magnético que rota como suma de vectores magnéticos a partir de 3 bobinas de la fase

Fuentes de campo magnético

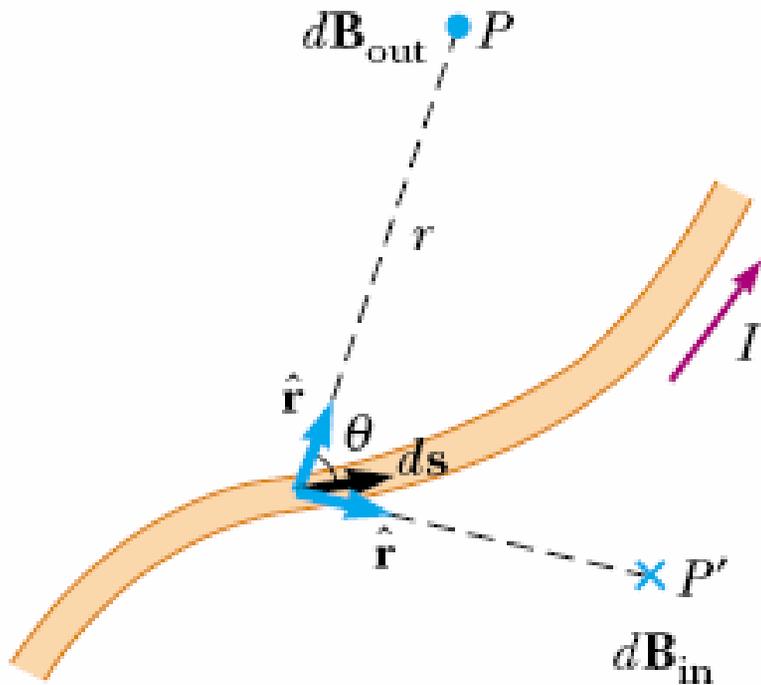






Hans Oersted (1820)

Una corriente eléctrica produce un campo magnético en el espacio que la rodea.



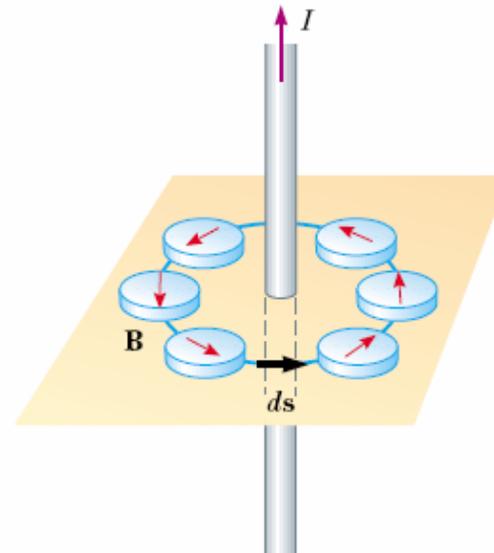
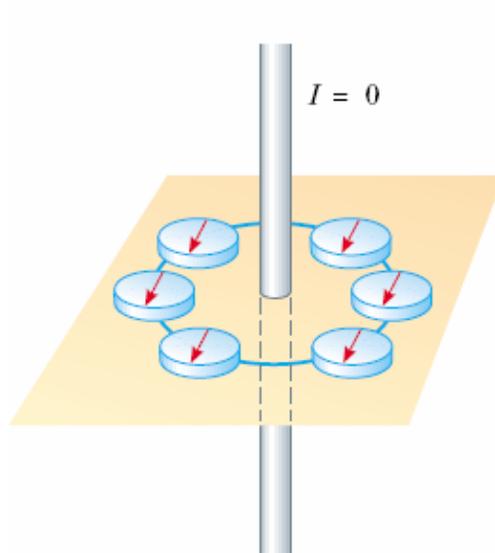
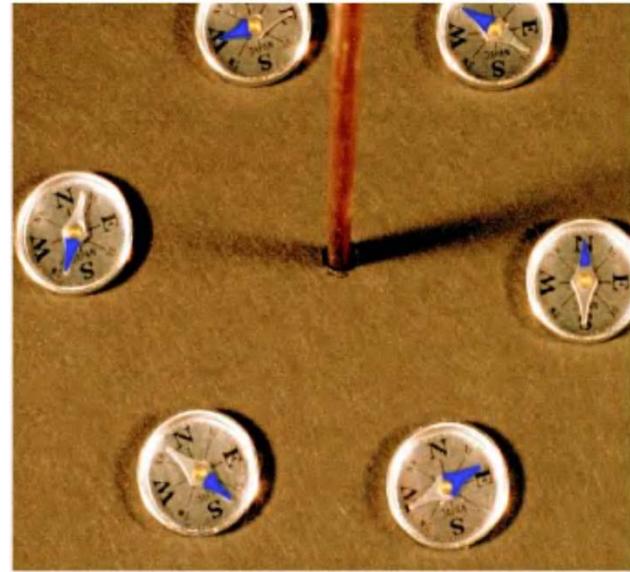
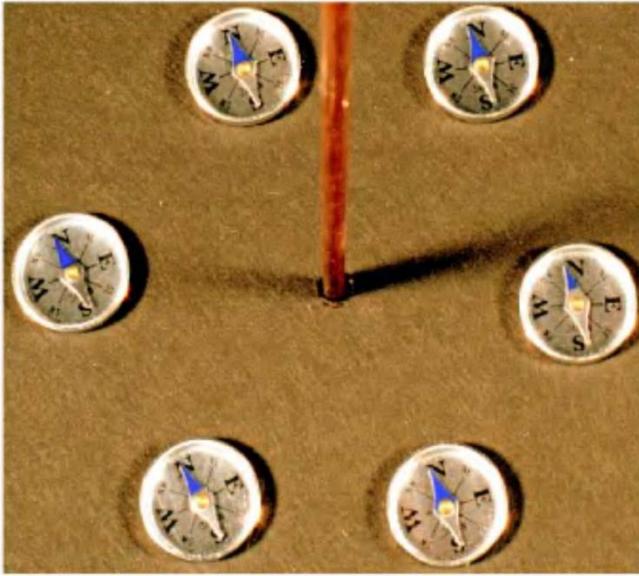
$\mu_0 = 4\pi \times 10^{-7} \text{ T}\cdot\text{m}/\text{A}$
Permeabilidad del vacío

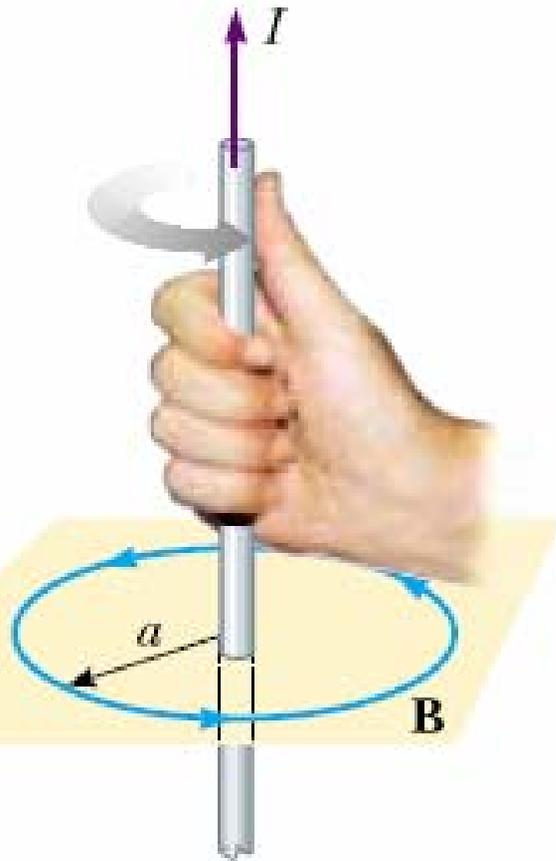
$$d\mathbf{B} = \frac{\mu_0}{4\pi} \frac{I d\mathbf{s} \times \hat{\mathbf{r}}}{r^2}$$

$$\mathbf{B} = \frac{\mu_0 I}{4\pi} \int \frac{d\mathbf{s} \times \hat{\mathbf{r}}}{r^2}$$

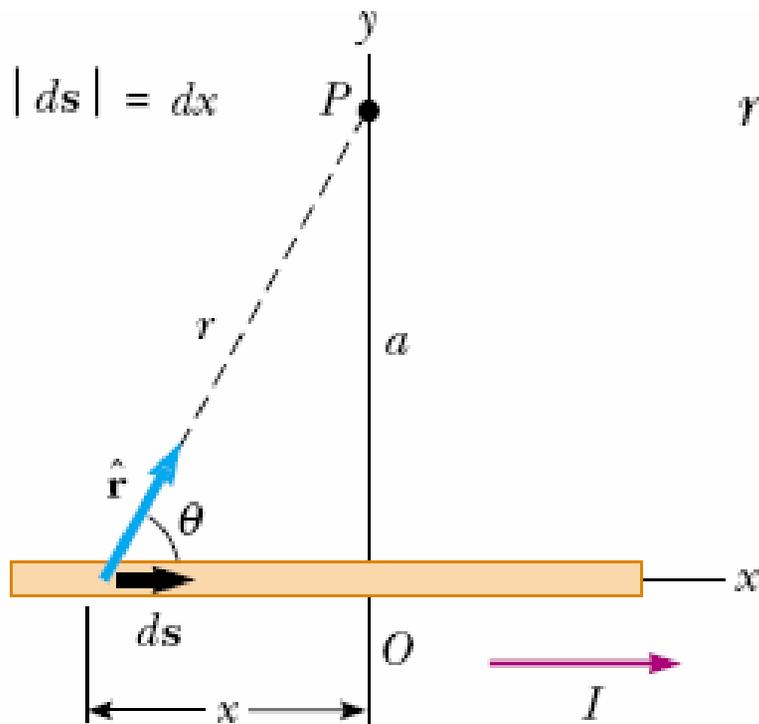
Ley de Biot y Sabart

Campo magnético producido por un conductor rectilíneo





Regla de la mano
derecha



$$|ds| = dx$$

$$r = \frac{a}{\sin \theta} = a \csc \theta \quad x = -a \cot \theta$$

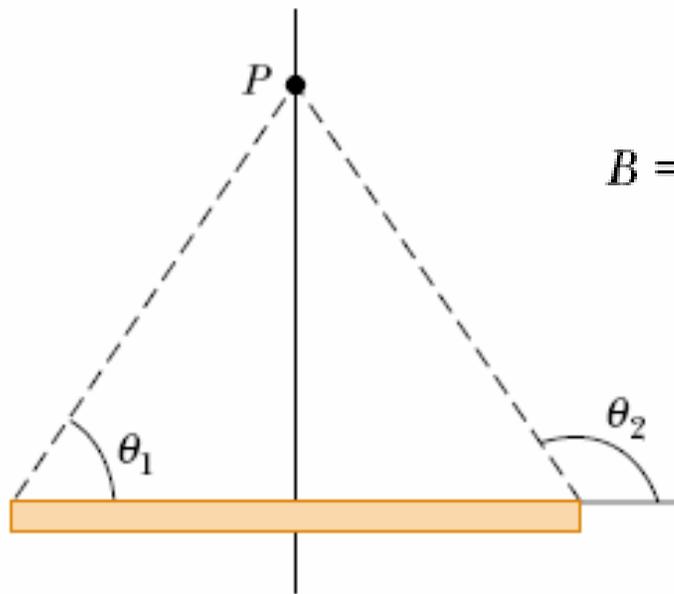
$$dx = a \csc^2 \theta d\theta$$

$$d\mathbf{B} = \frac{\mu_0}{4\pi} \frac{I ds \times \hat{\mathbf{r}}}{r^2}$$

$$ds \times \hat{\mathbf{r}} = |ds \times \hat{\mathbf{r}}| \hat{\mathbf{k}} = (dx \sin \theta) \hat{\mathbf{k}}$$

$$d\mathbf{B} = (dB) \hat{\mathbf{k}} = \frac{\mu_0 I}{4\pi} \frac{dx \sin \theta}{r^2} \hat{\mathbf{k}} \implies dB = \frac{\mu_0 I}{4\pi} \frac{dx \sin \theta}{r^2}$$

$$dB = \frac{\mu_0 I}{4\pi} \frac{a \csc^2 \theta \sin \theta d\theta}{a^2 \csc^2 \theta} = \frac{\mu_0 I}{4\pi a} \sin \theta d\theta$$

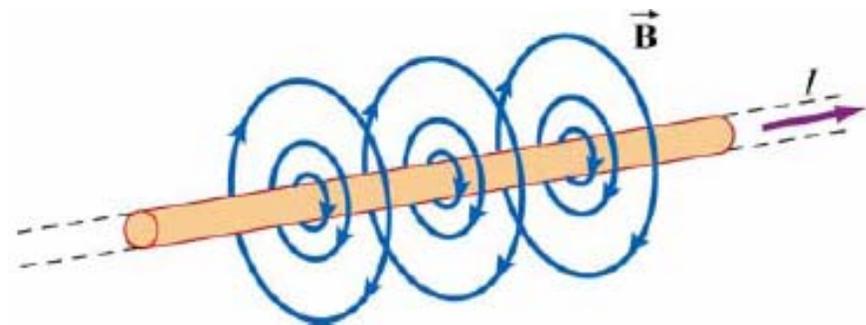


$$B = \frac{\mu_0 I}{4\pi a} \int_{\theta_1}^{\theta_2} \sin \theta \, d\theta = \frac{\mu_0 I}{4\pi a} (\cos \theta_1 - \cos \theta_2)$$

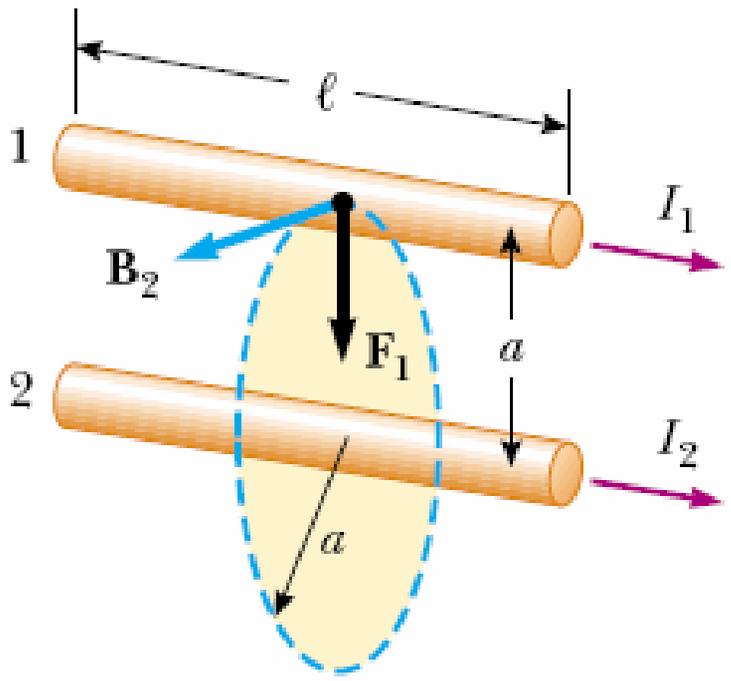
Para el caso de un conductor infinito:

$$(\cos \theta_1 - \cos \theta_2) = (\cos 0 - \cos \pi) = 2,$$

$$B = \frac{\mu_0 I}{2\pi a}$$



Fuerza magnética entre dos conductores paralelos



$$F_1 = I_1 \ell B_2 = I_1 \ell \left(\frac{\mu_0 I_2}{2\pi a} \right) = \frac{\mu_0 I_1 I_2}{2\pi a} \ell$$

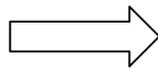
$$\frac{F_B}{\ell} = \frac{\mu_0 I_1 I_2}{2\pi a}$$

Caso particular:

$$I_1 = I_2$$

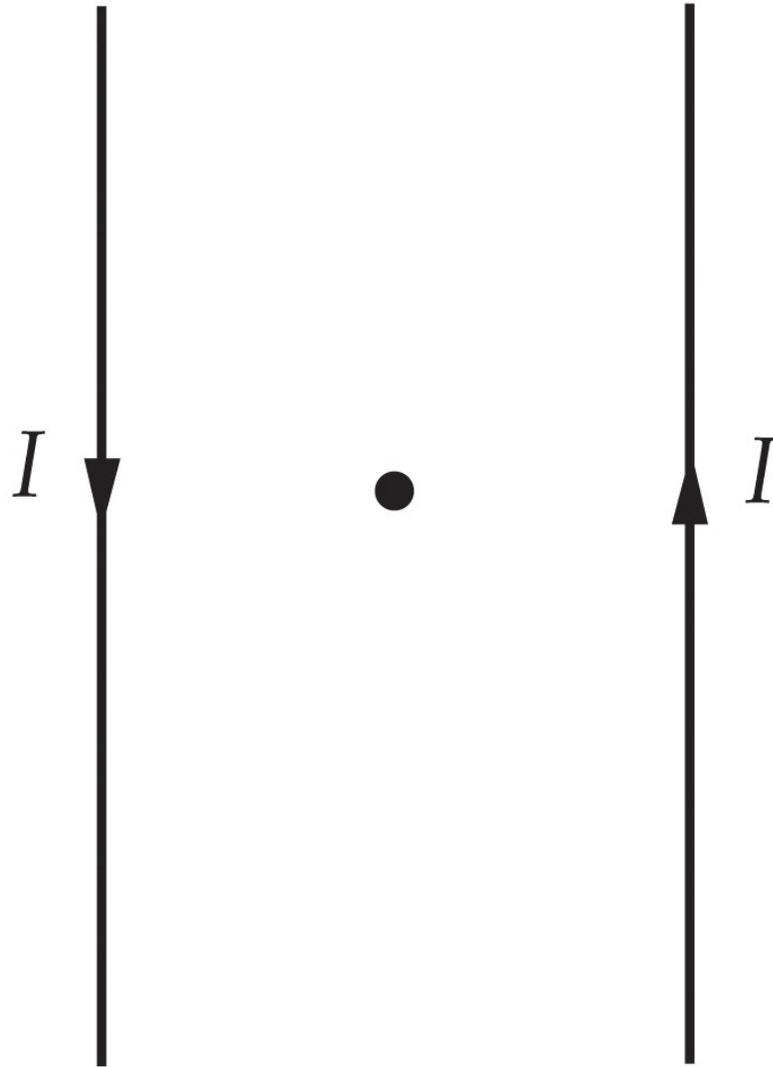
$$a = 1 \text{ m}$$

$$\text{Si } F_L = 2 \cdot 10^{-7} \text{ N/m}$$

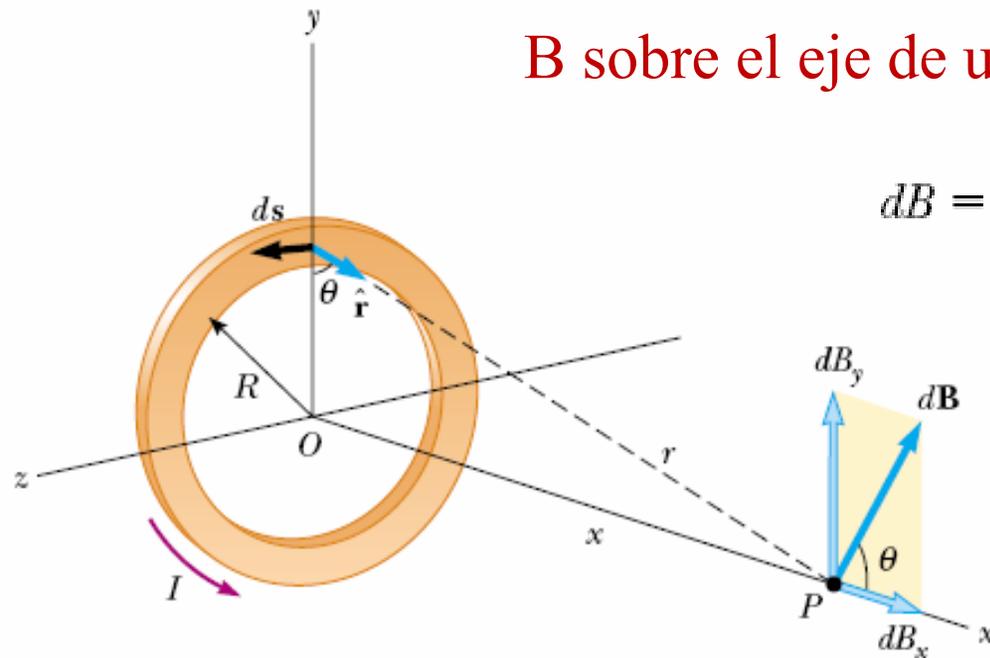


La corriente que circula por los alambres es 1 A.

Cómo es la fuerza entre los alambres si las corrientes son opuestas ?



B sobre el eje de un lazo de corriente circular



$$dB = \frac{\mu_0 I}{4\pi} \frac{|d\mathbf{s} \times \hat{\mathbf{r}}|}{r^2} = \frac{\mu_0 I}{4\pi} \frac{ds}{(x^2 + R^2)}$$

$$\mathbf{B} = B_x \hat{\mathbf{i}}$$

$$B_x = \oint dB \cos \theta = \frac{\mu_0 I}{4\pi} \oint \frac{ds \cos \theta}{x^2 + R^2}$$

$$r^2 = x^2 + R^2.$$

$$B_x = \frac{\mu_0 IR}{4\pi(x^2 + R^2)^{3/2}} \oint ds = \frac{\mu_0 IR^2}{2(x^2 + R^2)^{3/2}}$$

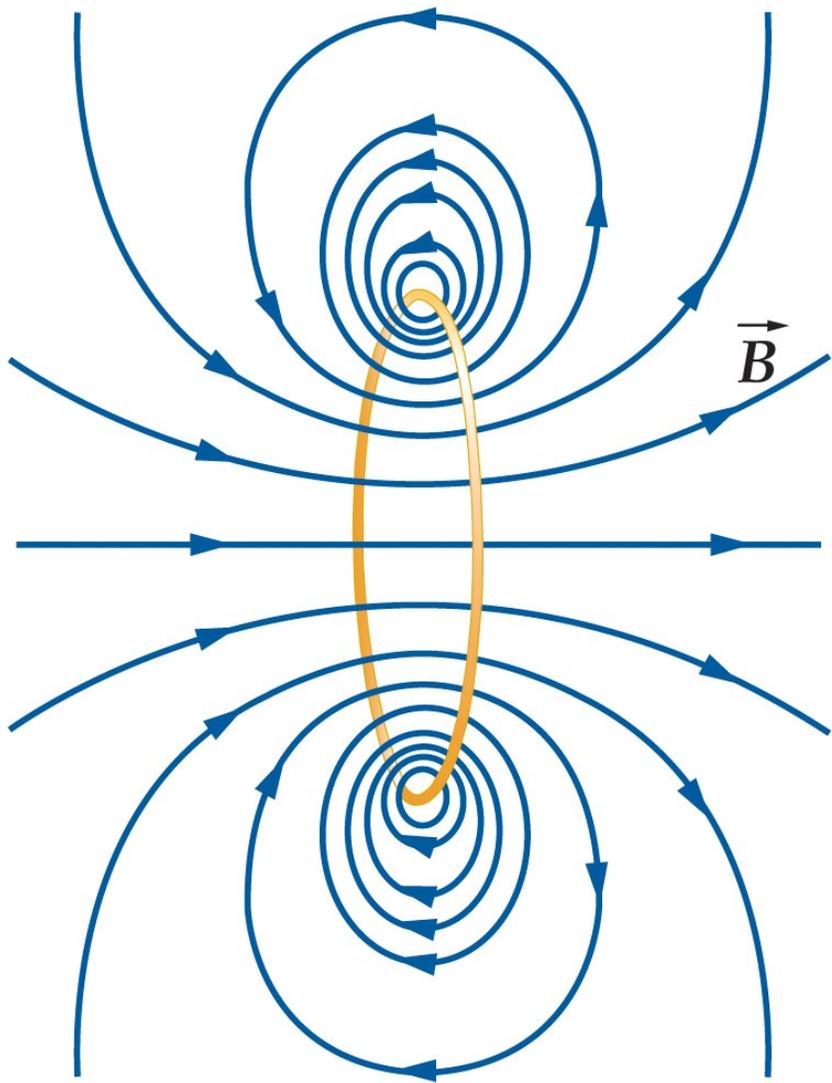
☀ $B = \frac{\mu_0 I}{2R}$ (at $x = 0$)

☀ $B \approx \frac{\mu_0 IR^2}{2x^3}$ (for $x \gg R$)

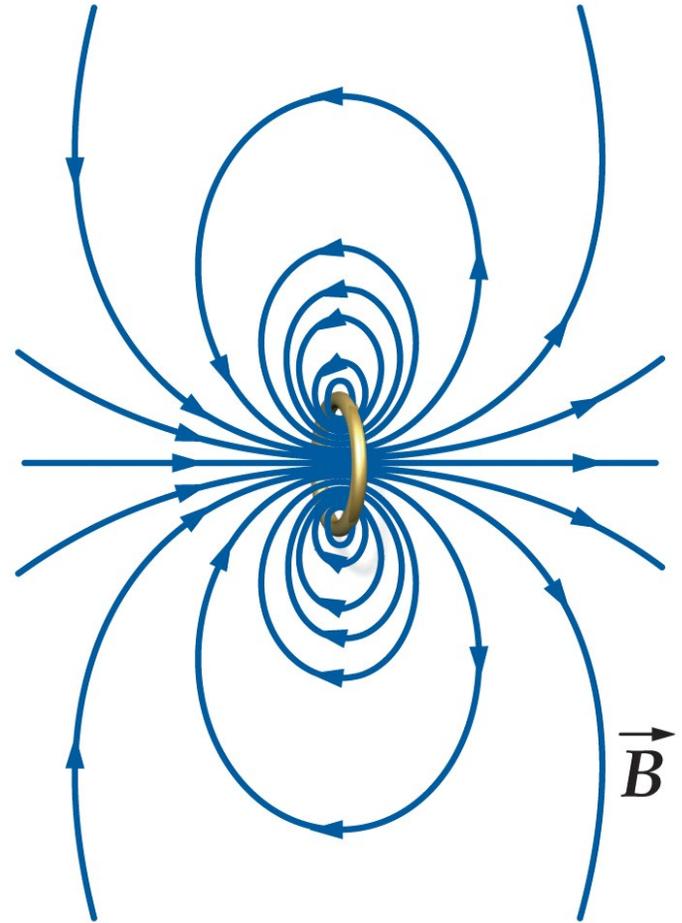
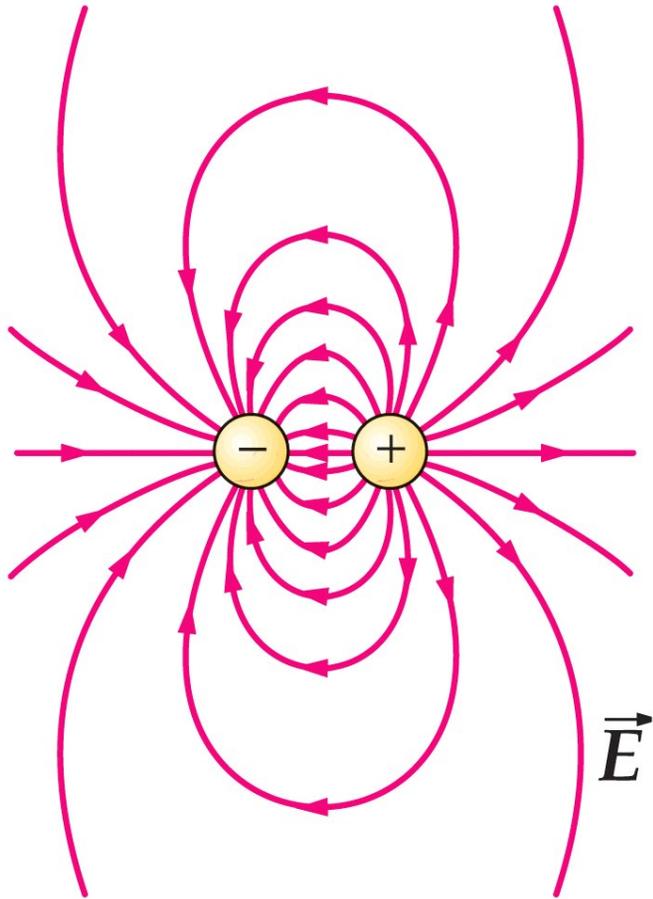
$$\longrightarrow$$

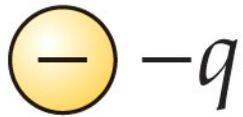
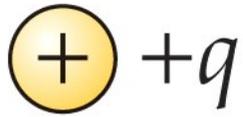
$$\mu = I(\pi R^2)$$

$$B \approx \frac{\mu_0}{2\pi} \frac{\mu}{x^3}$$

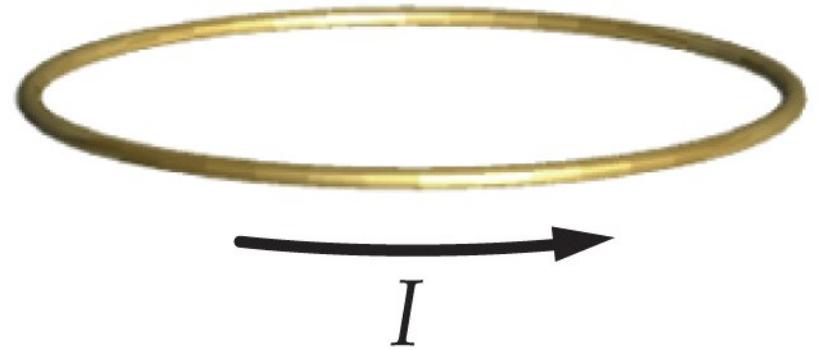


Analogía con el dipolo eléctrico



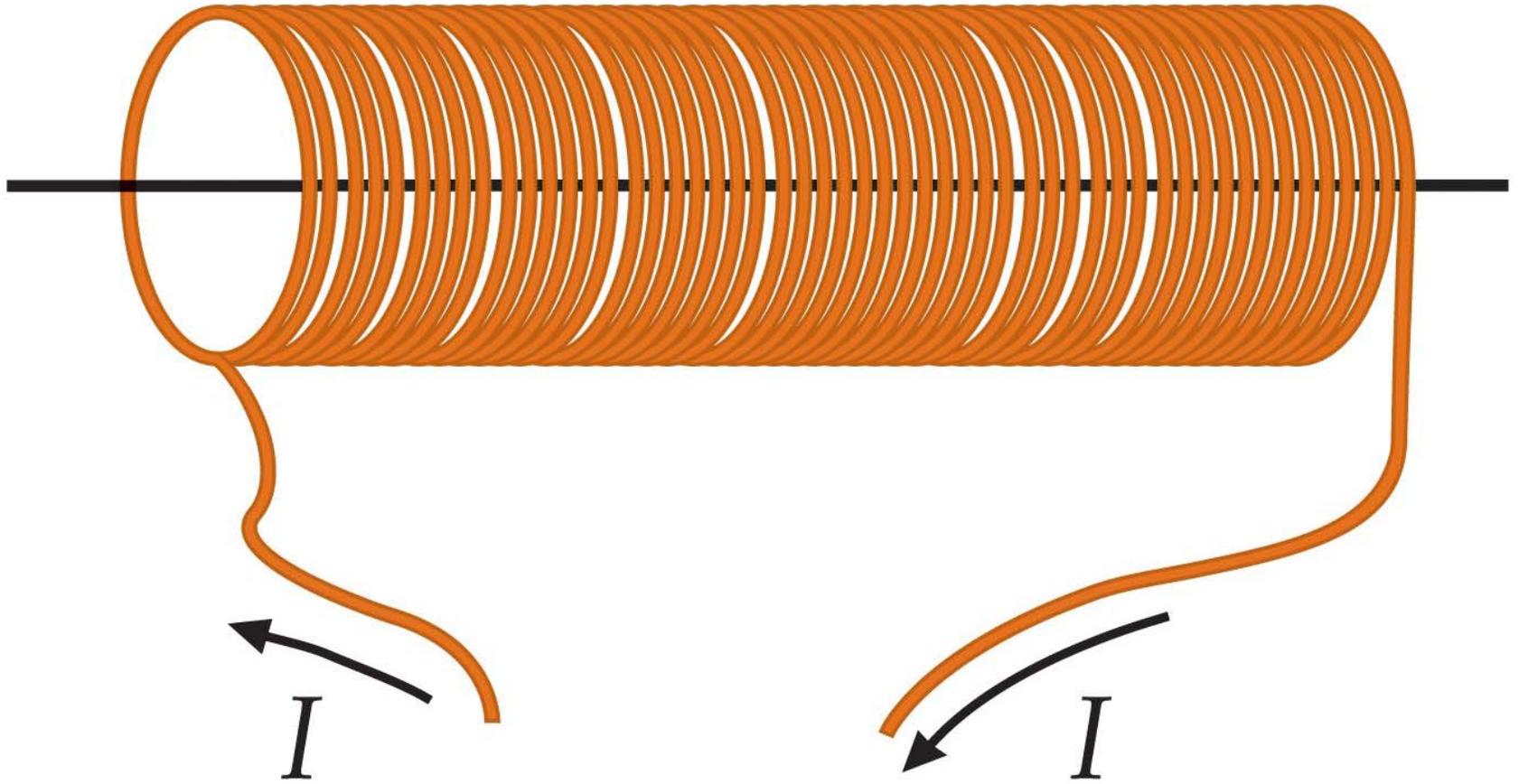


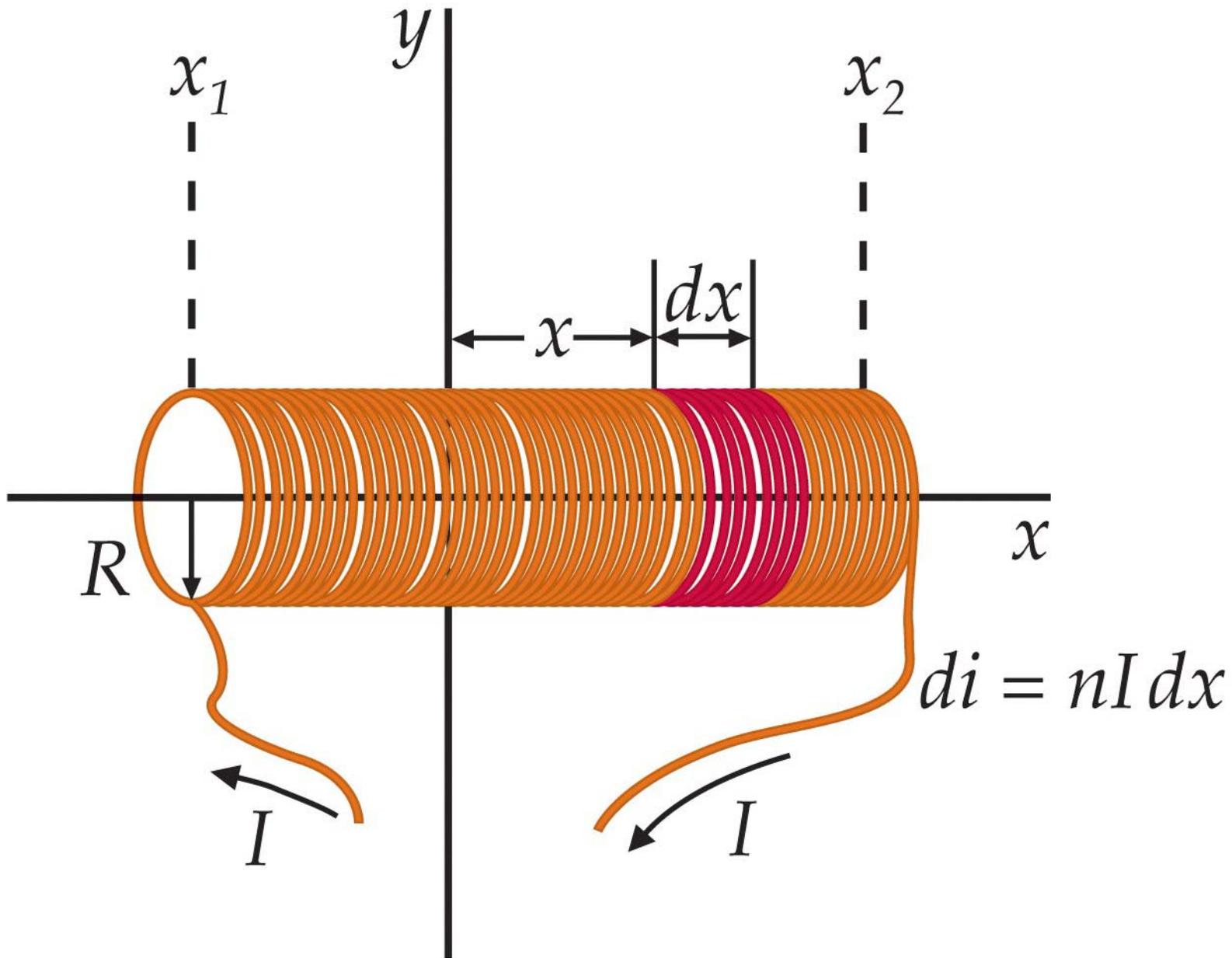
Electric dipole

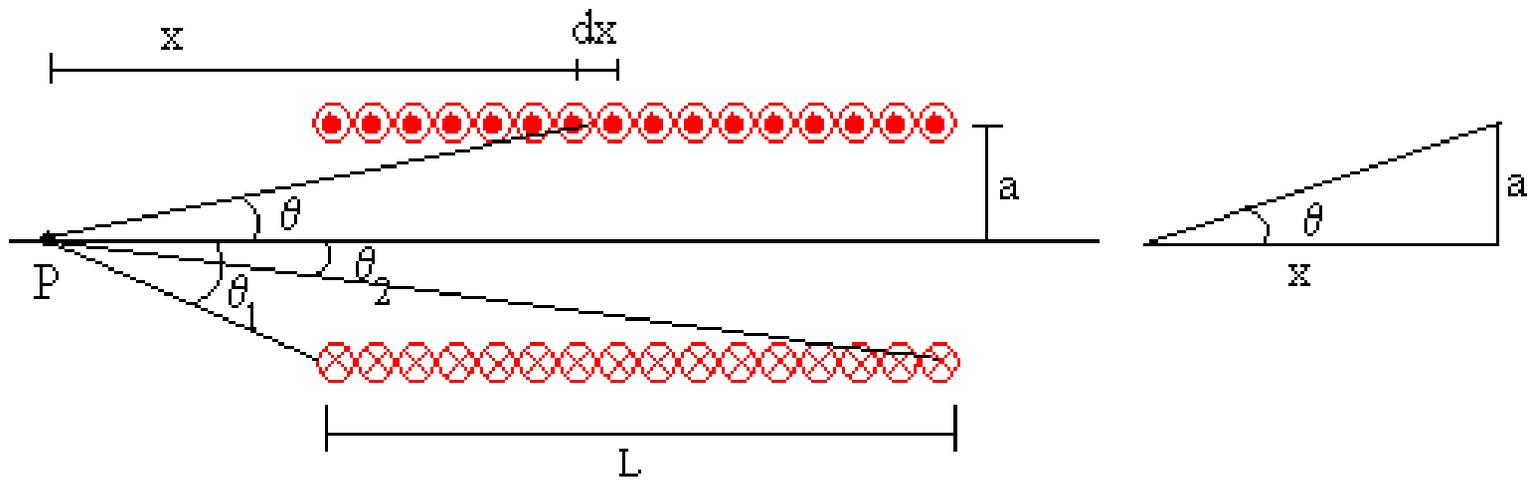


Magnetic dipole

Campo magnético a lo largo del eje de un solenoide







B producido por una espira de radio a en un punto P de su eje distante x

$$B = \frac{\mu_0 i a^2}{2 \left[\sqrt{a^2 + x^2} \right]^3}$$

B producido por el número dn de espiras

$$dB = \frac{\mu_0 i a^2}{2 \left[\sqrt{a^2 + x^2} \right]^3} \frac{N}{L} dx$$

$$a = x \cdot \tan \theta$$

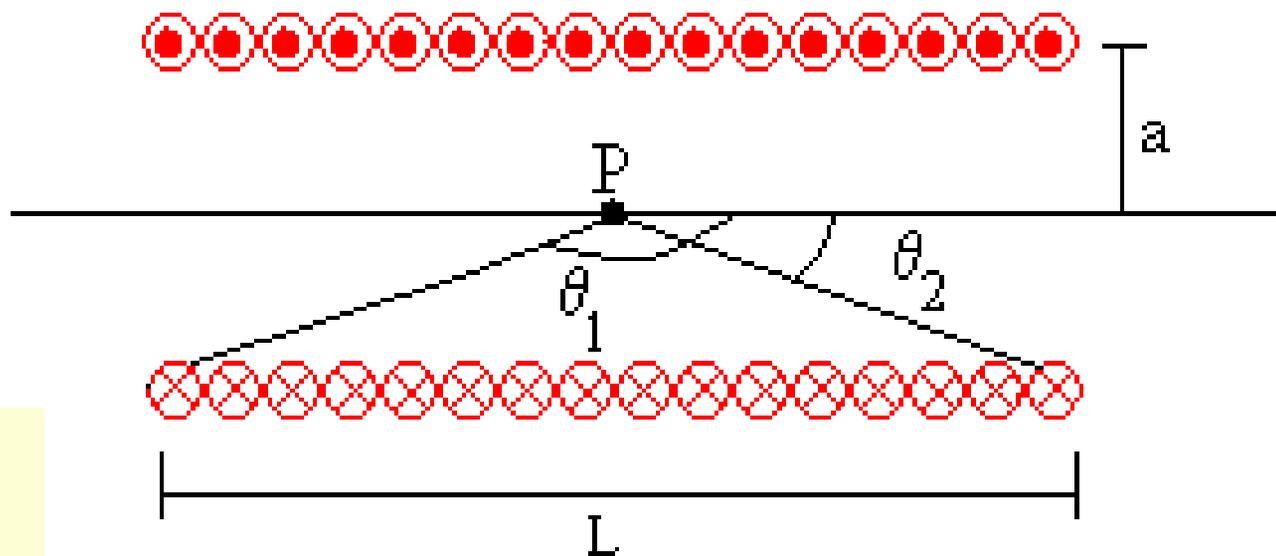
$$B = \frac{\mu_0 i N}{2L} \int_{\theta_1}^{\theta_2} -\sin \theta d\theta = \frac{\mu_0 i N}{2L} (\cos \theta_2 - \cos \theta_1)$$

$$1 + \tan^2 \theta = 1 / \cos^2 \theta$$

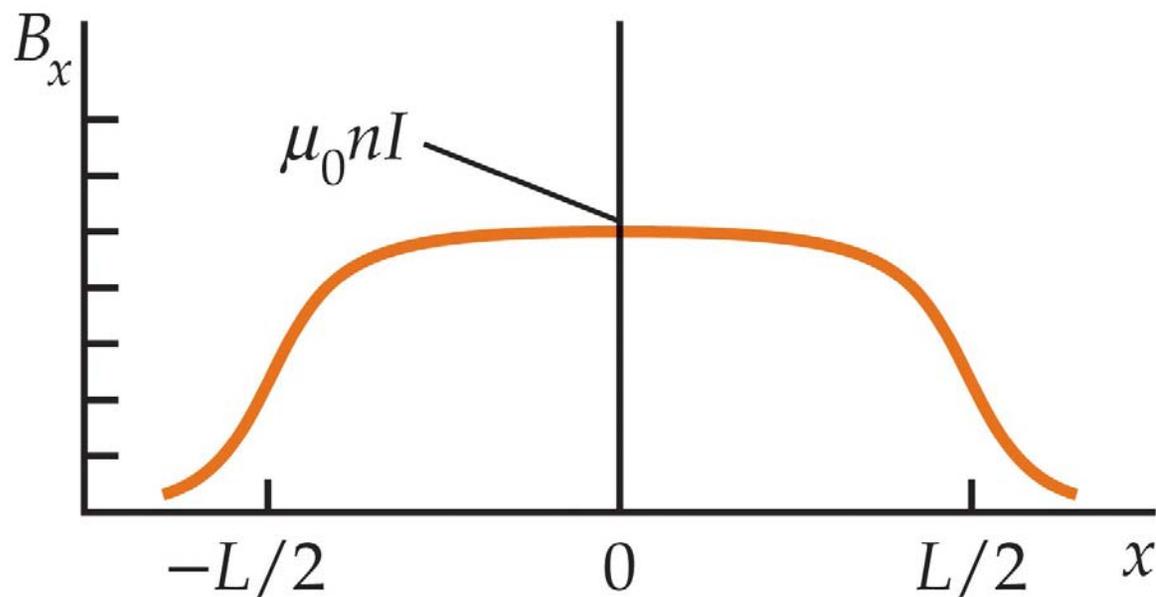
Si el solenoide
es muy largo

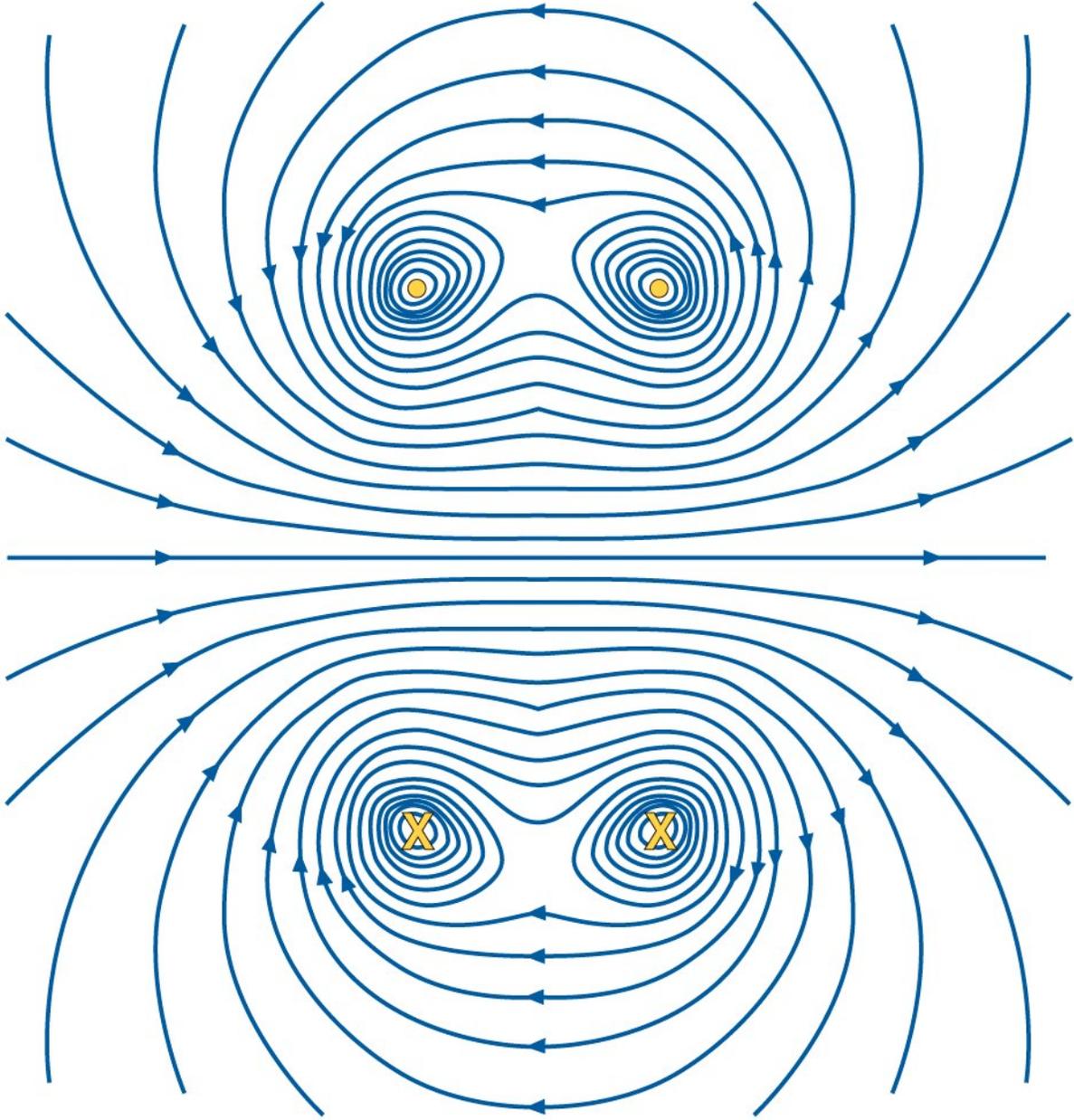
$$\theta_1 \rightarrow \pi$$

$$\theta_2 \rightarrow 0$$

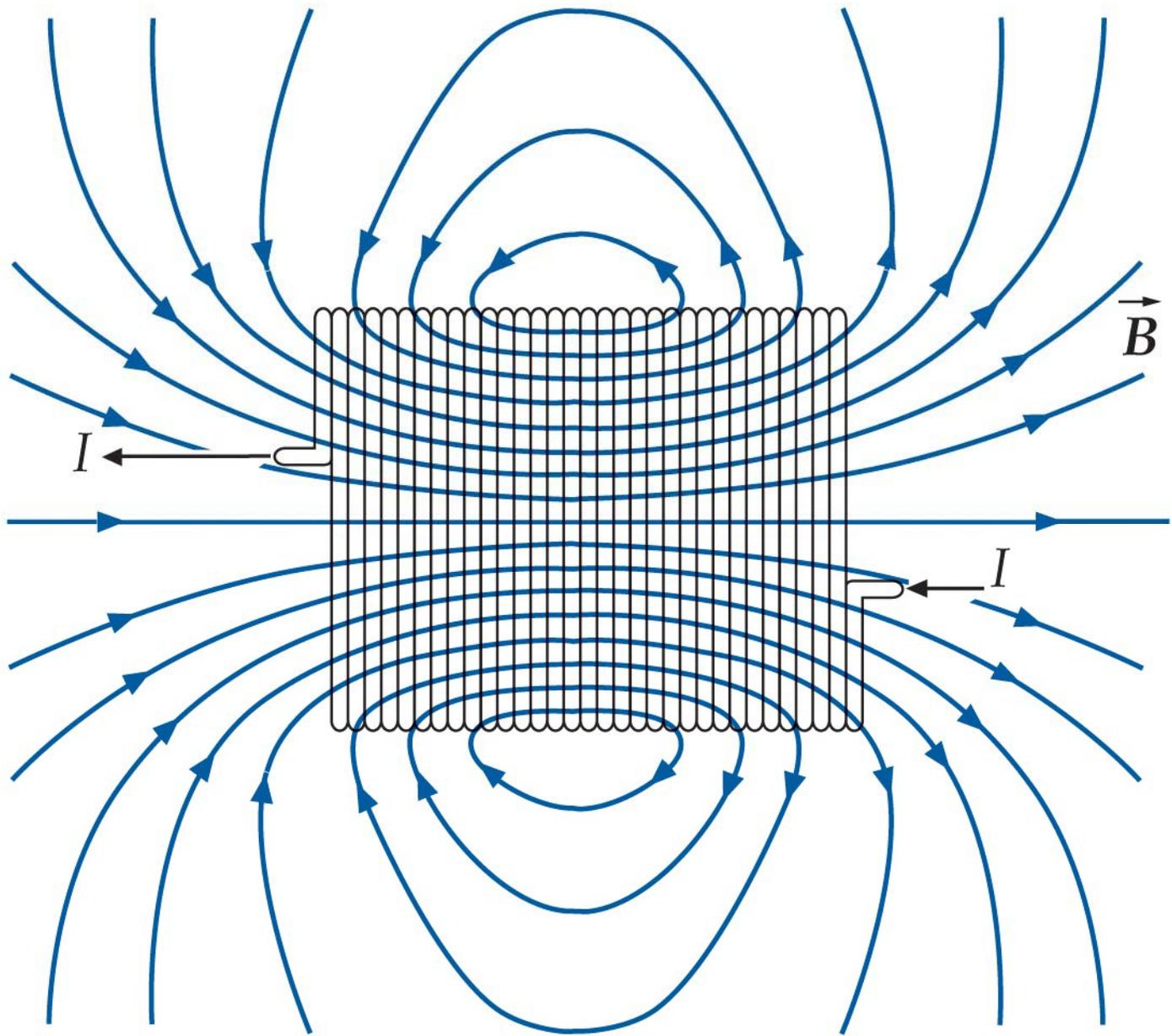


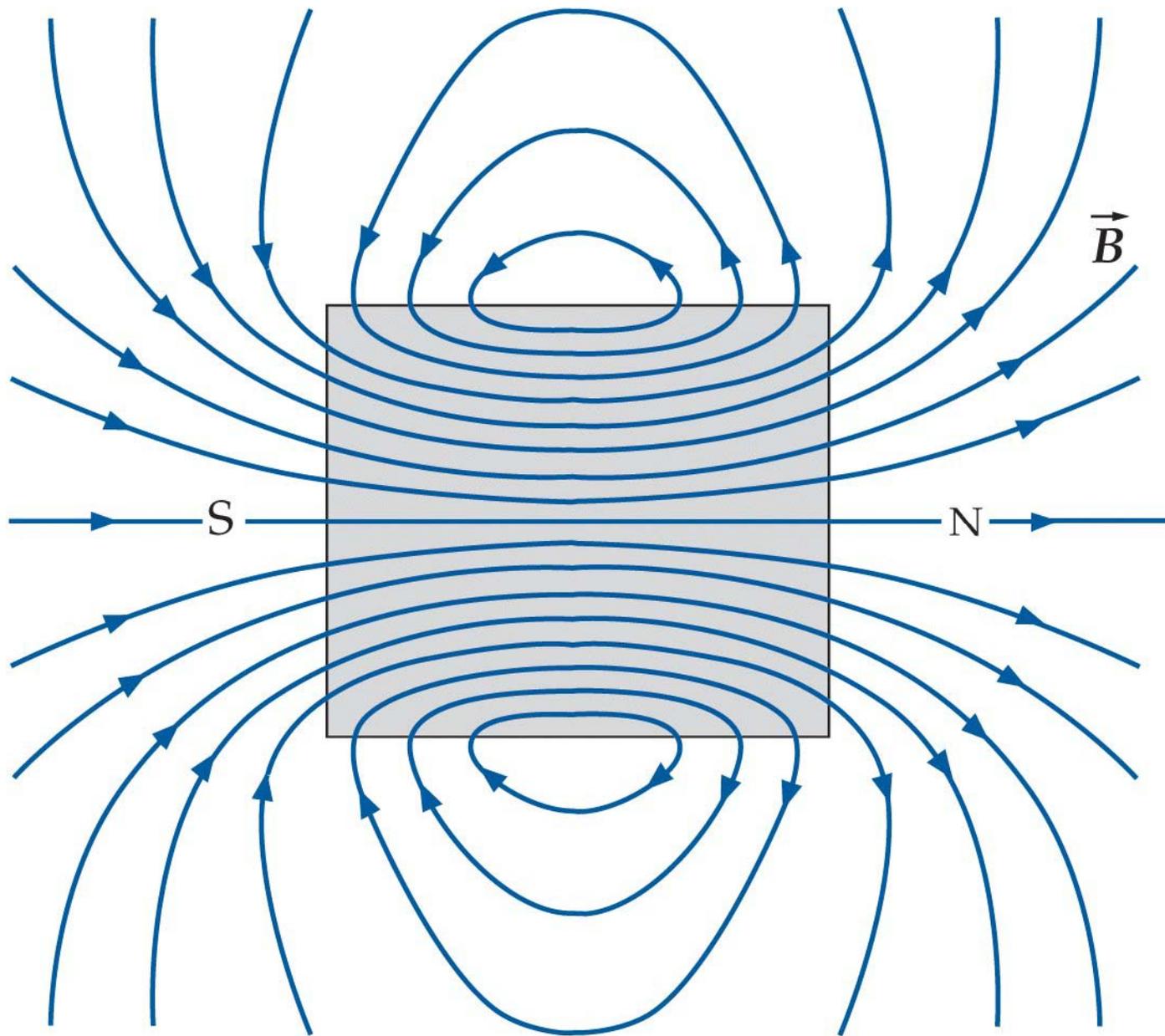
$$B_x = \mu_0 n I$$

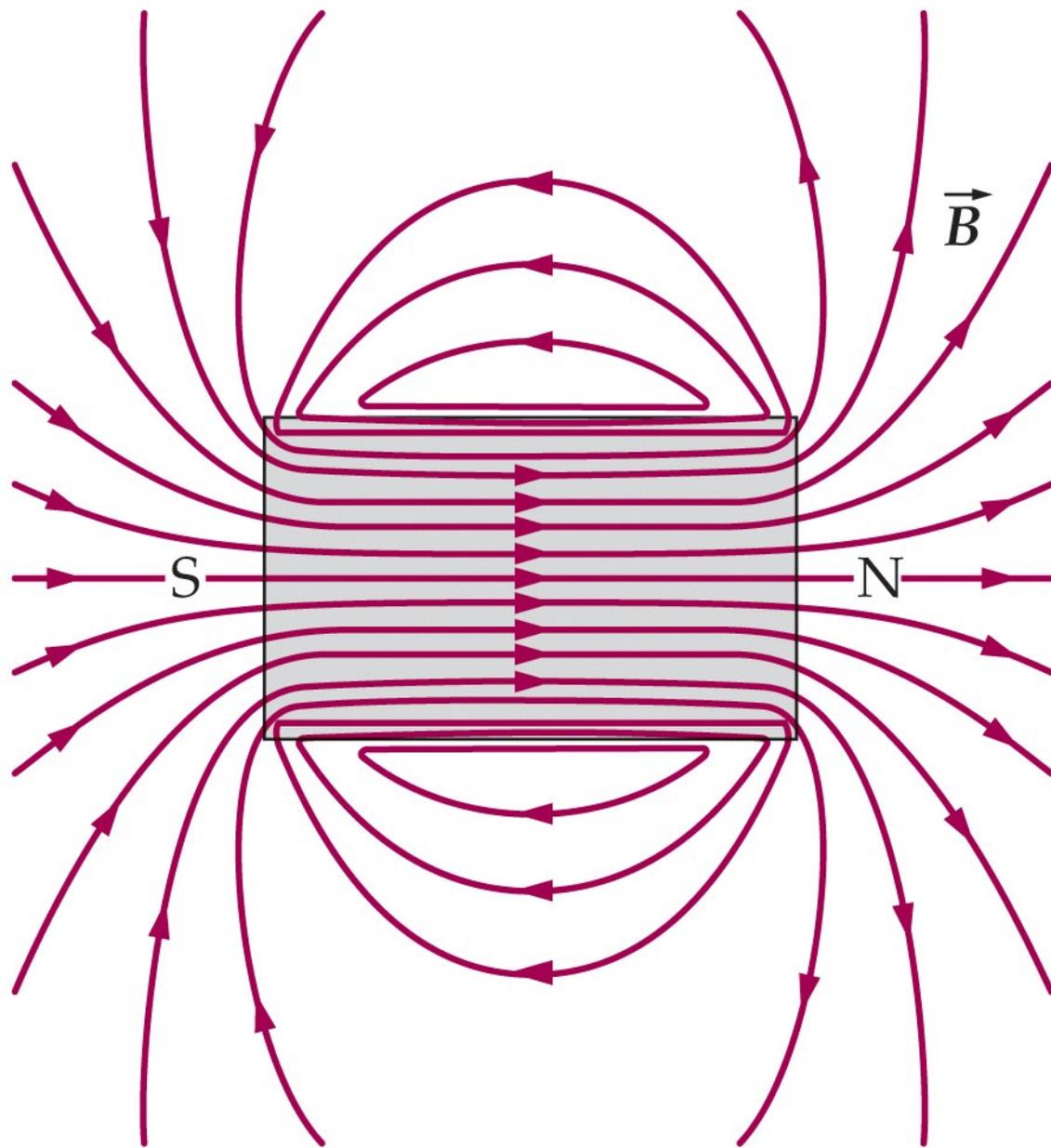




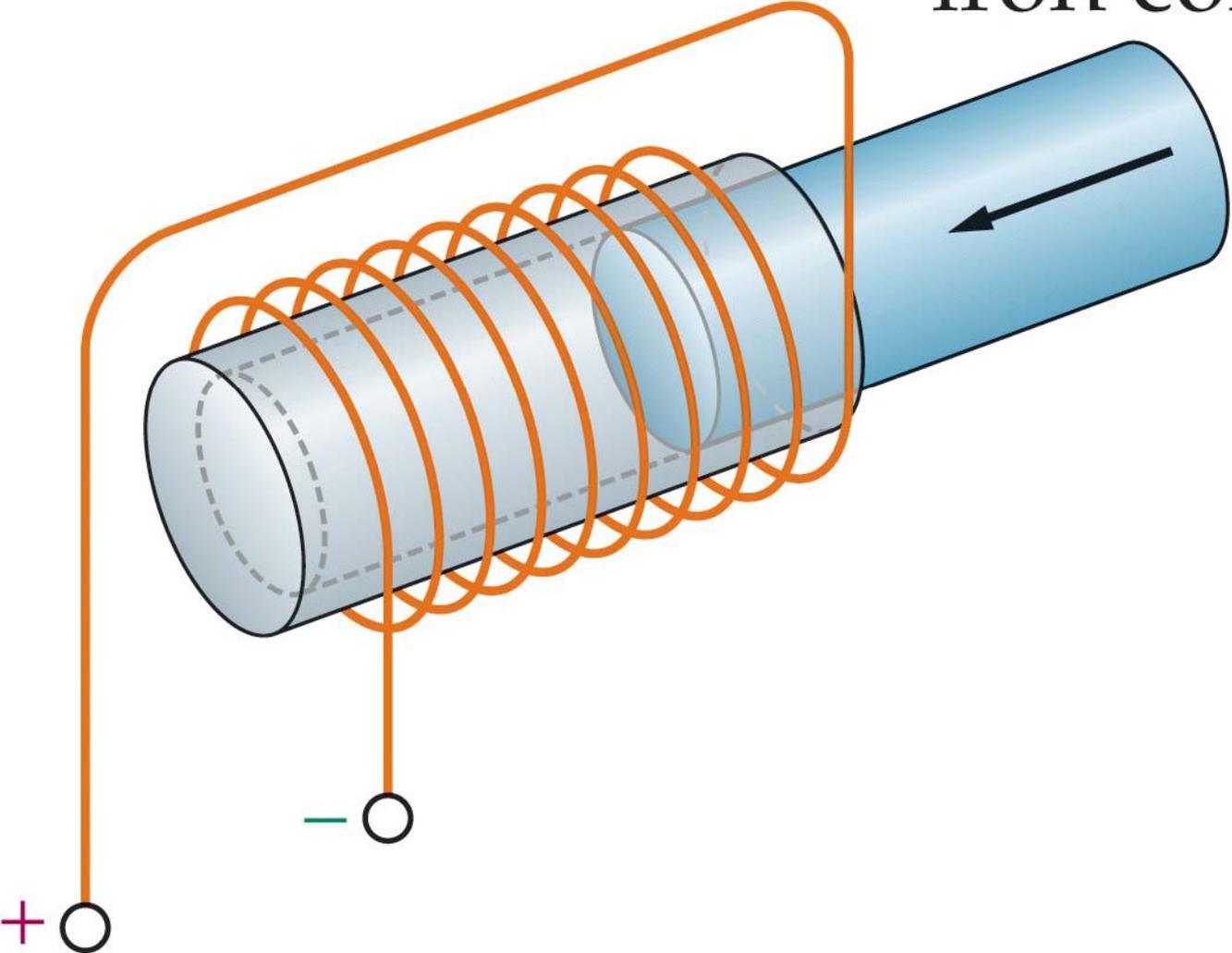


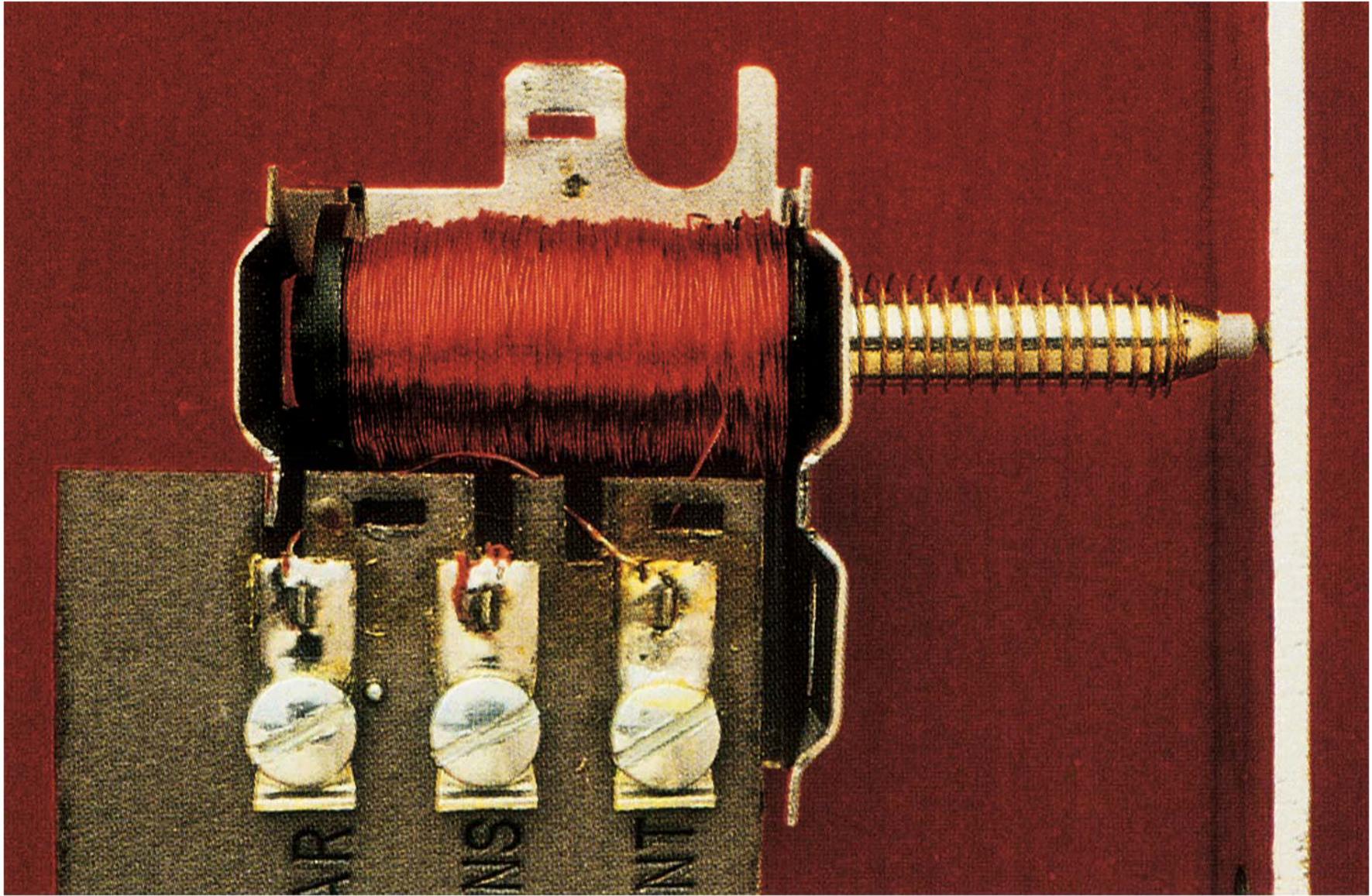




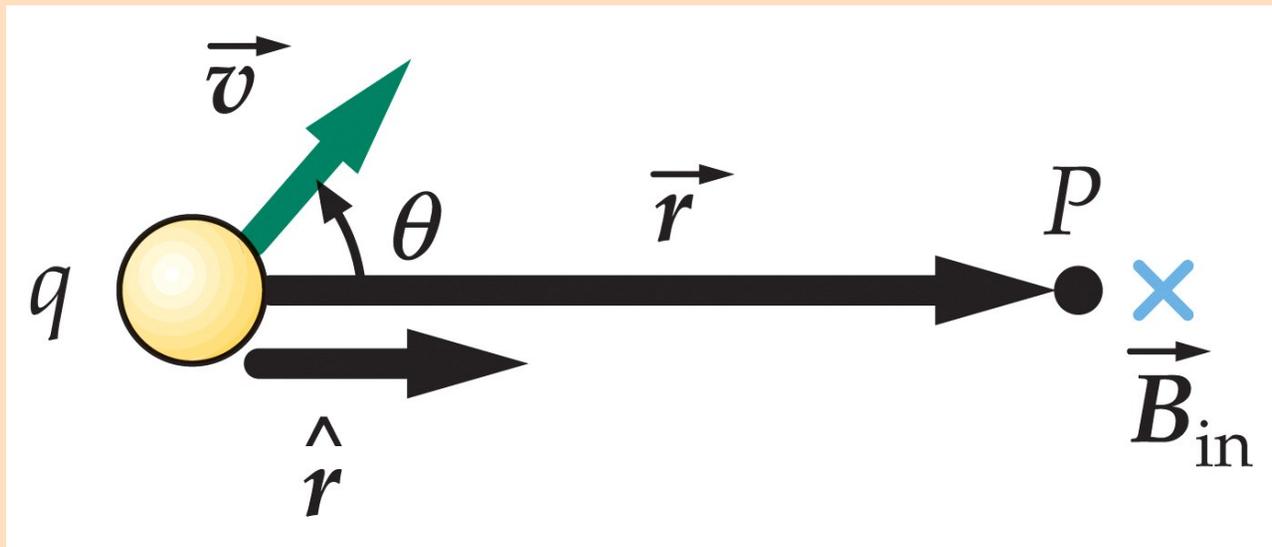


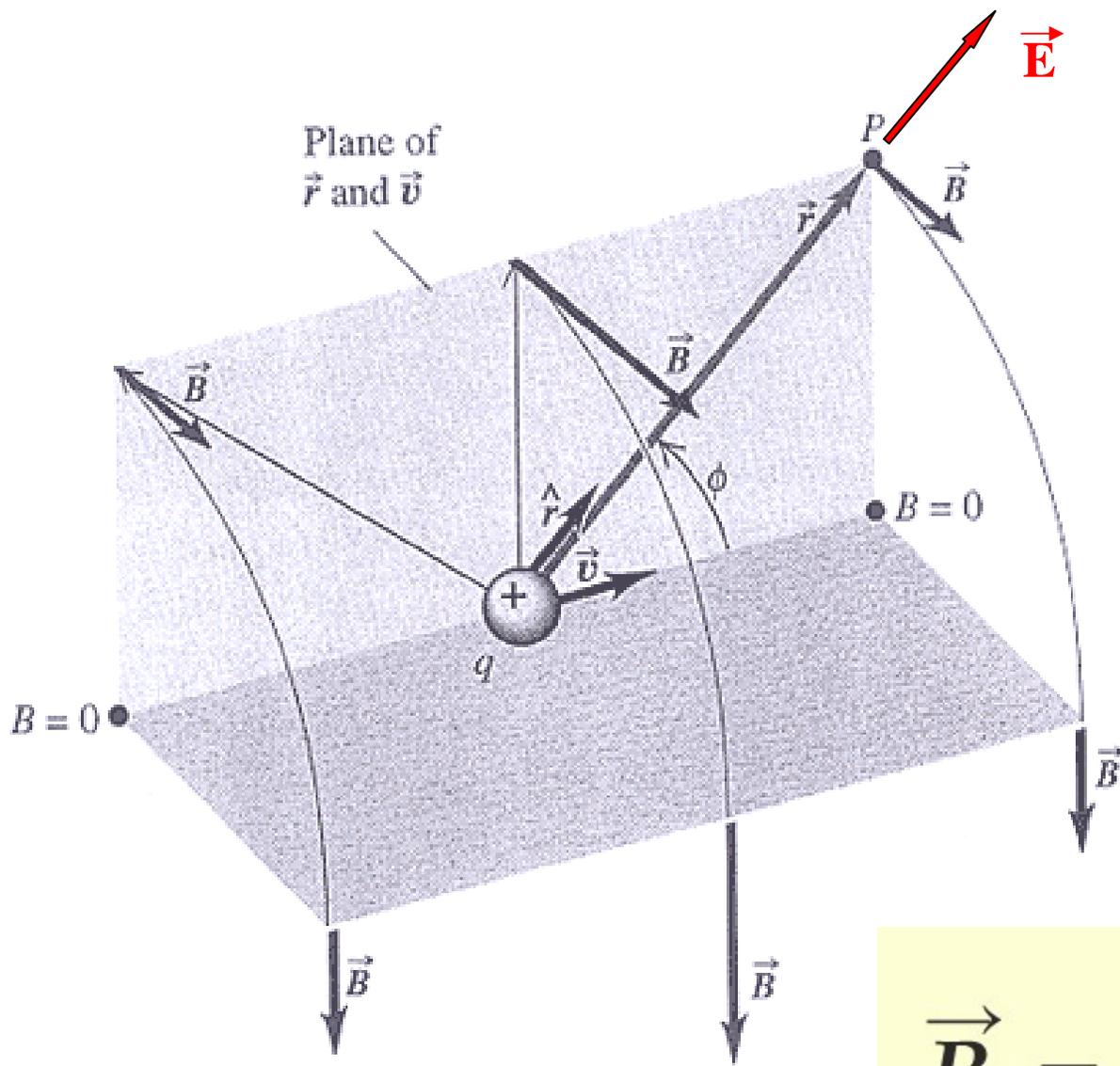
Movable
iron core



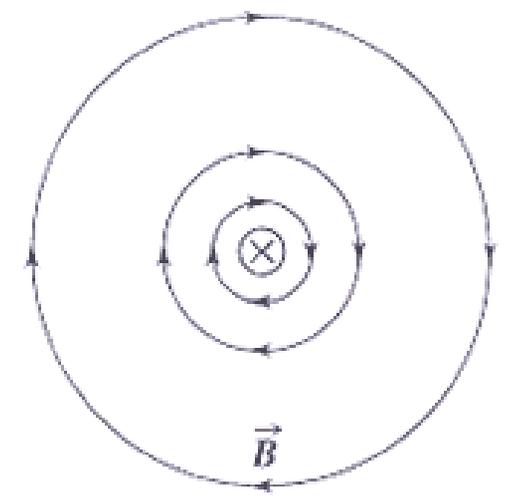


Campo producido por una carga en movimiento



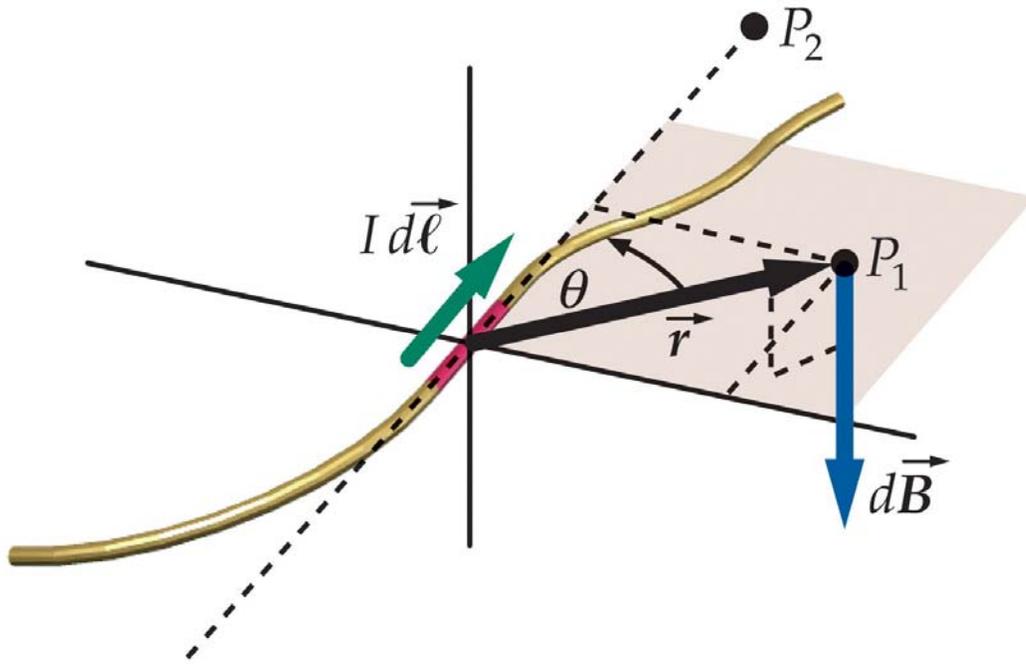


(a)



(b)

$$\vec{B} = \frac{\mu_0}{4\pi} \frac{q\vec{v} \times \hat{r}}{r^2}$$



$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I d\vec{\ell} \times \hat{r}}{r^2}$$

$$S dl = dV$$

$$I = J S$$

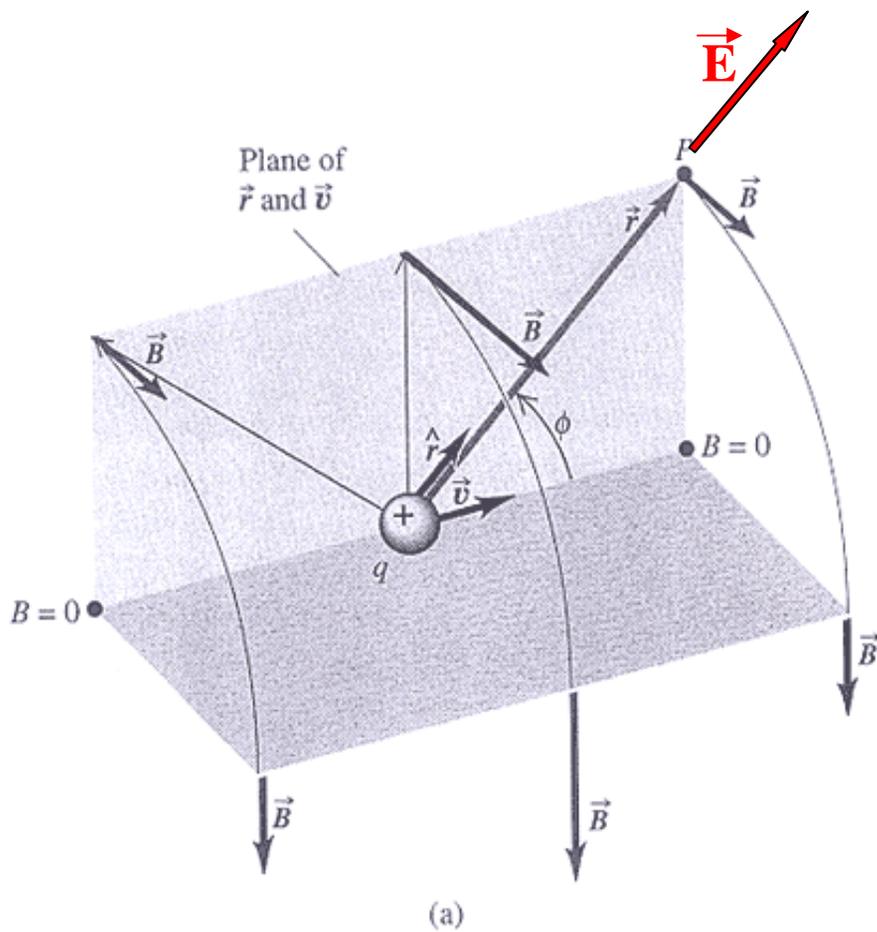
$$J = n q v$$

$$I d\vec{\ell} = J S dl \vec{u}_T = J dV = n q v dV$$

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{q \vec{v} \times \hat{r}}{r^2} n dV$$

\Rightarrow
 para
 una
 carga

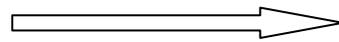
$$\vec{B} = \frac{\mu_0}{4\pi} \frac{q \vec{v} \times \hat{r}}{r^2}$$



$$\vec{B} = \frac{\mu_0}{4\pi} \frac{q\vec{v} \times \hat{r}}{r^2}$$

$$\vec{E}_{i,P} = \frac{kq_i}{r_{i,P}^2} \hat{r}_{i,P}$$

**E y B son
perpendiculares**



$$\vec{B} = 1/c^2 \vec{v} \times \vec{E}$$

Determine el valor y las unidades de la constante c !!!!