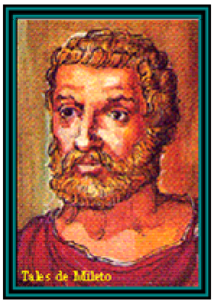
Capítulo 1:

Interacción Eléctrica

Tales de Mileto (624-543 A. C.)

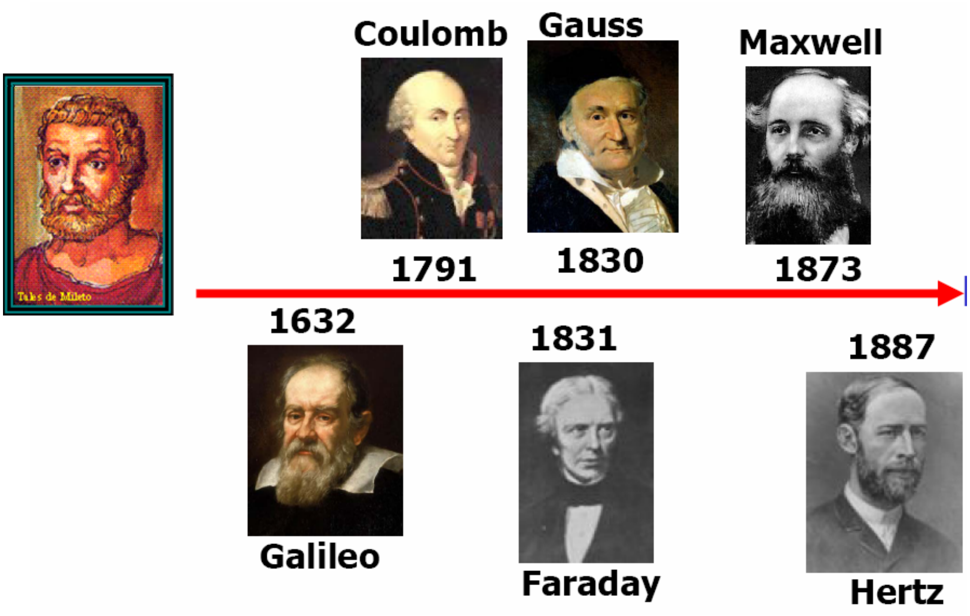


Observó que unas briznas de hierba seca eran atraídas por un trozo de ámbar que antes había frotado con su túnica.

ελεχτρον

(ámbar)

Electricidad por frotación

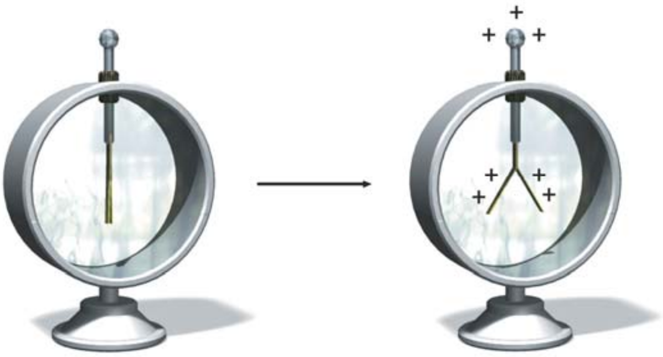


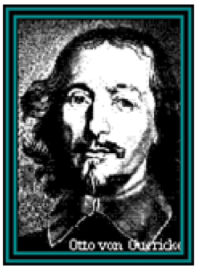
Willian Gilbert (1544-1603)

Aplico por primera vez el termino eléctrico (del griego elektron (ámbar)

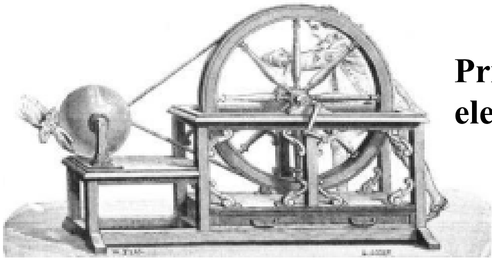
Clasifico los materiales en:
-Eléctricos (aislantes)
-Aneléctricos (conductores)

Ideó el primer electroscopio

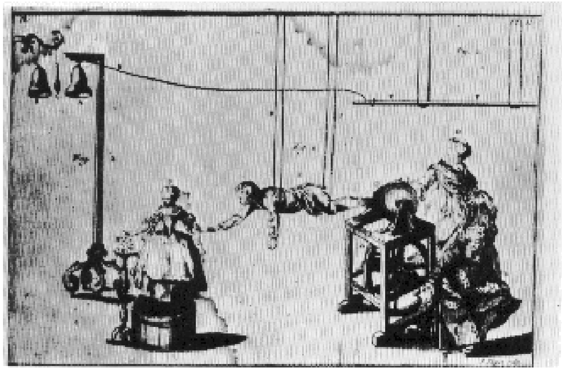




Otto von Guericke (1602-1686)



Primera máquina electrostática



Charles Du Fay (1698-1739)

Dos tipos de cargas:
-vitrea (+)
-resinosa (-)

TABLE 21-1

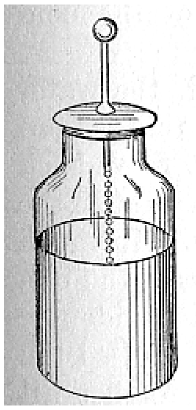
The Triboelectric Series

+ Positive End of Series
Asbestos
Glass
Nylon
Wool
Lead
Silk
Aluminum
Paper
Cotton
Steel
Hard rubber
Nickel and copper
Brass and silver
Synthetic rubber
Orlon
Saran
Polyethylene
Teflon
Silicone rubber
− Negative End of Series

Pieter van Musschenbroek (1692-1761)

Botella de Leiden
(Universidad de Leiden)

Permite almacenar cargas.



Capacitores modernos

Benjamín Franklin(1706-1790)

Electricidad atmosférica: planteó la naturaleza eléctrica de los rayos

Benjamín Franklin (1706-1790)

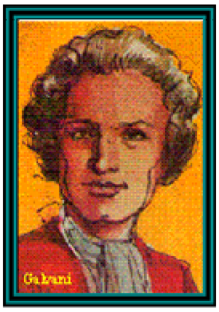
Desarrolló una teoría de fluido eléctrico

Substancias:
-positivas (exceso de fluido)
-negativas (defecto de fluido)

Al frotar se transfiere fluido (ahora carga) de un cuerpo al otro.

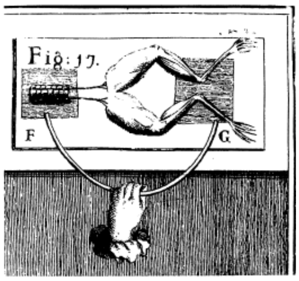
RELEVANTE:

Principio de conservación de la carga: En cualquier proceso que ocurra en un sistema aislado, la carga total o neta no cambia.

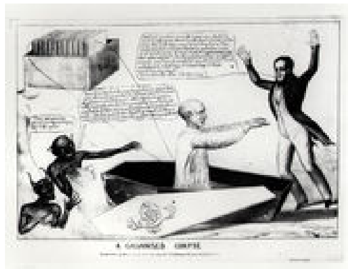


Luigi Galvani (1737-1798)

Electricidad animal



Galvanismo: teoría según la cual el cerebro de los animales produce electricidad que es transferida por los nervios, acumulada en los músculos y disparada para producir el movimiento de los miembros

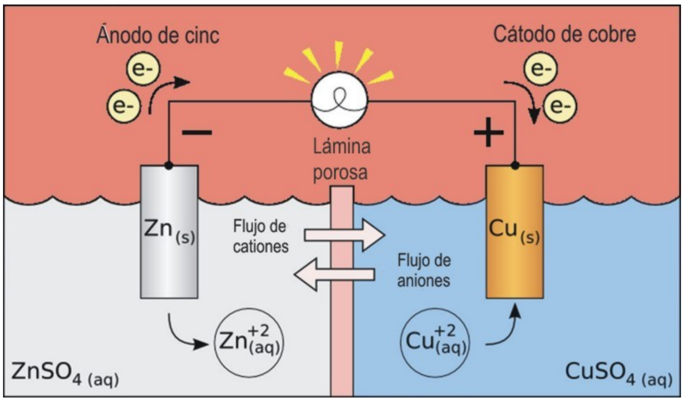


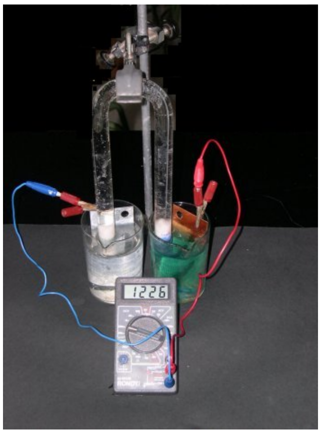
Mary W. Shelley (verano de 1816)

Frankenstein

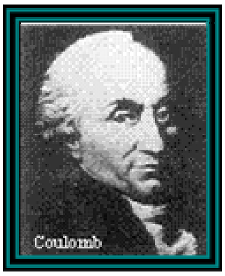
Alessandro Volta (1745-1827)

Pila de Volta: Apilamiento de discos de cinc y cobre, separados por discos de cartón humedecidos con un electrólito.





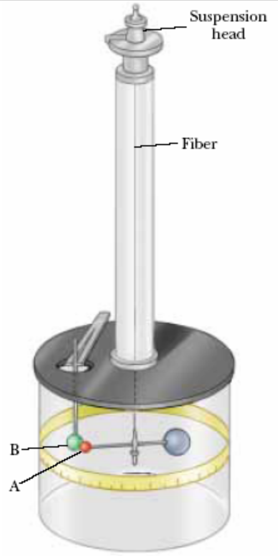
Pila y baterias modernas

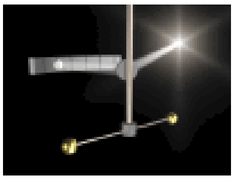


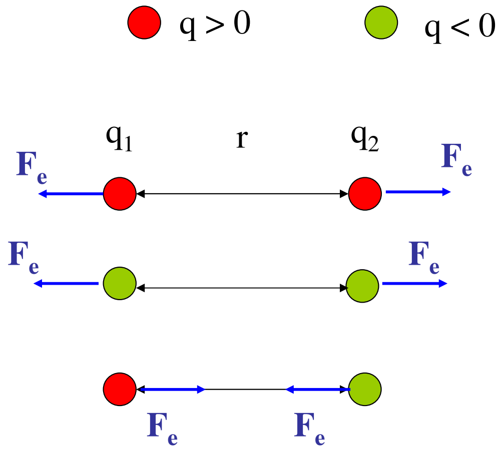
Charles Augustin de Coulomb (1736-1806)

Establecio leyes cuantitativas de la electrostática utilizando una balanza de torsión

Ley de Coulomb (1785)

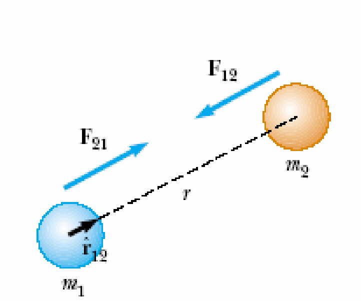






$$F_e = k_e \frac{|q_1||q_2|}{r^2}$$

Semejante con la ley de interacción gravitacional

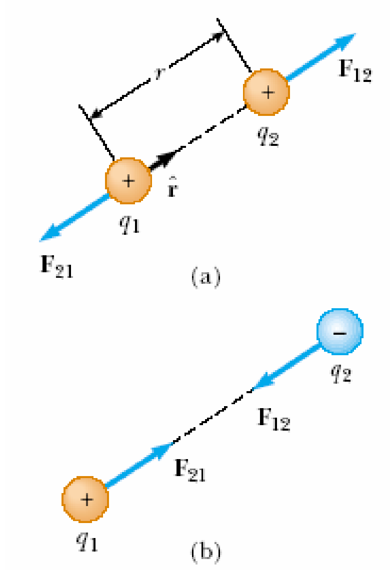


$$\mathbf{F}_{12} = -G\frac{m_1 m_2}{r^2}\,\hat{\mathbf{r}}_{12}$$

$$\mathbf{G\ m_1\ m_2} \Longleftrightarrow \mathbf{k_e\ q_1\ q_2}$$

Gravitatoria es atractiva

La escribimos en forma vectorial



$$\mathbf{F}_{12} = k_e \frac{q_1 q_2}{r^2}\hat{\mathbf{r}}$$

SI: [q] = C (Coulomb)

$$k_e = 8.987\,5 \times 10^9\ \mathrm{N \cdot m^2/C^2}$$

$$k_e = \frac{1}{4\pi\epsilon_0}$$

$$\epsilon_0 = 8.854\,2 \times 10^{-12}\ \mathrm{C^2/N \cdot m^2}$$

Permitividad del vacio

La carga libre más pequeña conocida en la naturaleza es la carga de un electrón o un protón

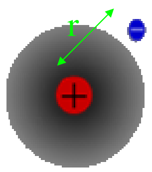
$$\mathbf{e = 1.60219 \times 10^{-19}\ C}$$

q = N e con N=1,2,3… e: cuanto de carga

Charge and Mass of the Electron, Proton, and Neutron

Particle	Charge (C)	Mass (kg)
Electron (e)	$-1.602\,191\,7 \times 10^{-19}$	$9.109\,5 \times 10^{-31}$
Proton (p)	$+1.602\,191\,7 \times 10^{-19}$	$1.672\,61 \times 10^{-27}$
Neutron (n)	0	$1.674\,92 \times 10^{-27}$

Ejemplo : átomo de hidrógeno



$$F_e = k_e \frac{|e||-e|}{r^2} = (8.99 \times 10^9\ \mathrm{N \cdot m^2/C^2}) \frac{(1.60 \times 10^{-19}\ \mathrm{C})^2}{(5.3 \times 10^{-11}\ \mathrm{m})^2}$$

$$= \boxed{8.2 \times 10^{-8}\ \mathrm{N}}$$

r = 0.53 Å = 0.53 10 $^{-10}$ m

$$F_g = G\frac{m_e m_p}{r^2}$$

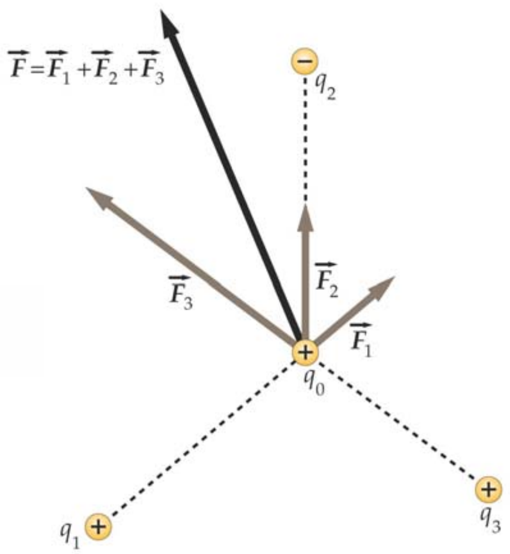
$$= (6.67 \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2})$$

$$\times \frac{(9.11 \times 10^{-31}\ \mathrm{kg})(1.67 \times 10^{-27}\ \mathrm{kg})}{(5.3 \times 10^{-11}\ \mathrm{m})^2}$$

$$= \boxed{3.6 \times 10^{-47}\ \mathrm{N}}$$

Principio de superposición

$$\mathbf{F}_1 = \mathbf{F}_{21} + \mathbf{F}_{31} + \mathbf{F}_{41}$$



Ejemplo

$$F_{13} = k_e \frac{|q_1||q_3|}{(\sqrt{2}a)^2}$$

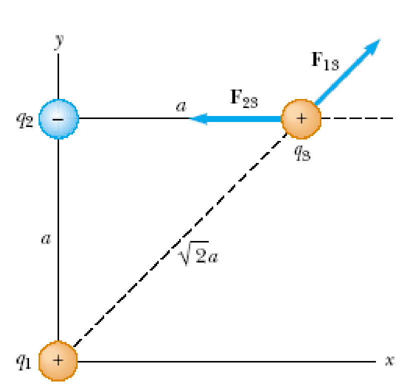
$$= (8.99 \times 10^9\ \mathrm{N \cdot m^2/C^2})\ \frac{(5.0 \times 10^{-6}\ \mathrm{C})(5.0 \times 10^{-6}\ \mathrm{C})}{2(0.10\ \mathrm{m})^2}$$

$$= 11\ \mathrm{N}$$

$$F_{23} = k_e \frac{|q_2||q_3|}{a^2}$$

$$= (8.99 \times 10^9\ \mathrm{N \cdot m^2/C^2})\ \frac{(2.0 \times 10^{-6}\ \mathrm{C})(5.0 \times 10^{-6}\ \mathrm{C})}{(0.10\ \mathrm{m})^2}$$

$$= 9.0\ \mathrm{N}$$



$q_1 = q_3 = 5.0\ \mu\mathrm{C}$,

$q_2 = -2.0\ \mu\mathrm{C}$,

$a = 0.10\ \mathrm{m}$.

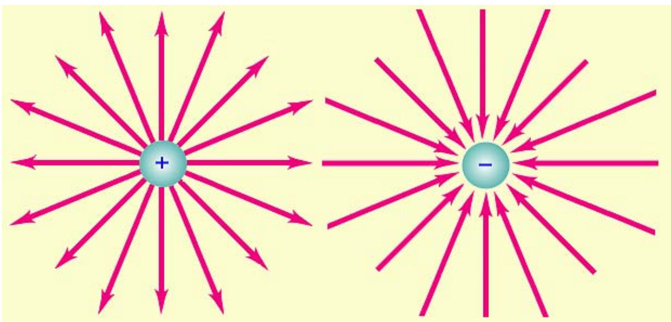
$$F_{13} \cos 45° = 7.9\ \mathrm{N}$$

$$F_{3x} = F_{13x} + F_{23x} = 7.9\ \mathrm{N} + (-9.0\ \mathrm{N}) = -1.1\ \mathrm{N}$$

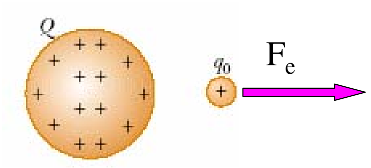
$$F_{3y} = F_{13y} + F_{23y} = 7.9\ \mathrm{N} + 0 = 7.9\ \mathrm{N}$$

$$\mathbf{F}_3 = \boxed{(-1.1\hat{\mathbf{i}} + 7.9\hat{\mathbf{j}})\ \mathrm{N}}$$

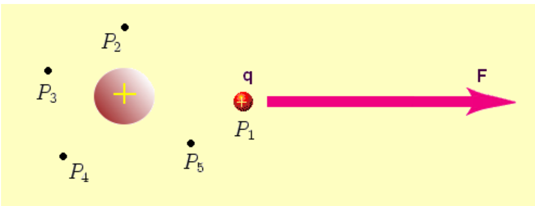
Campo Eléctrico



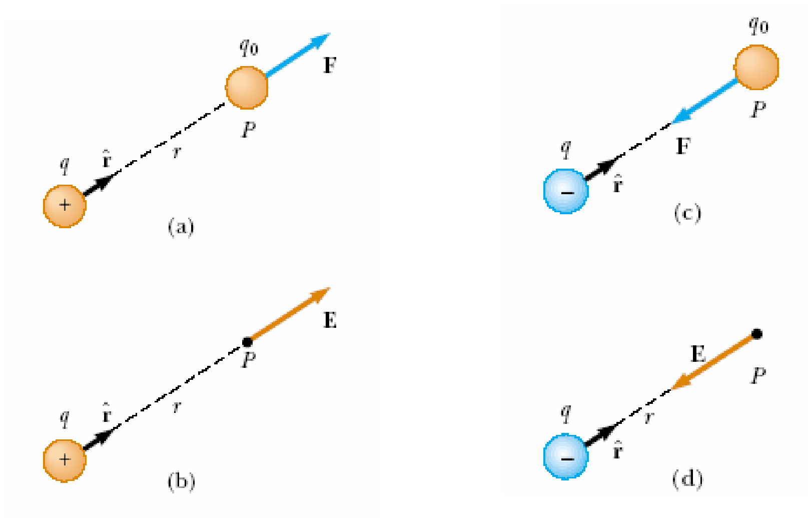
Definimos campo eléctrico en un punto del espacio como la fuerza eléctrica Fe que actua sobre una carga de prueba positiva situada en ese punto dividida por la magnitud de la carga de prueba.



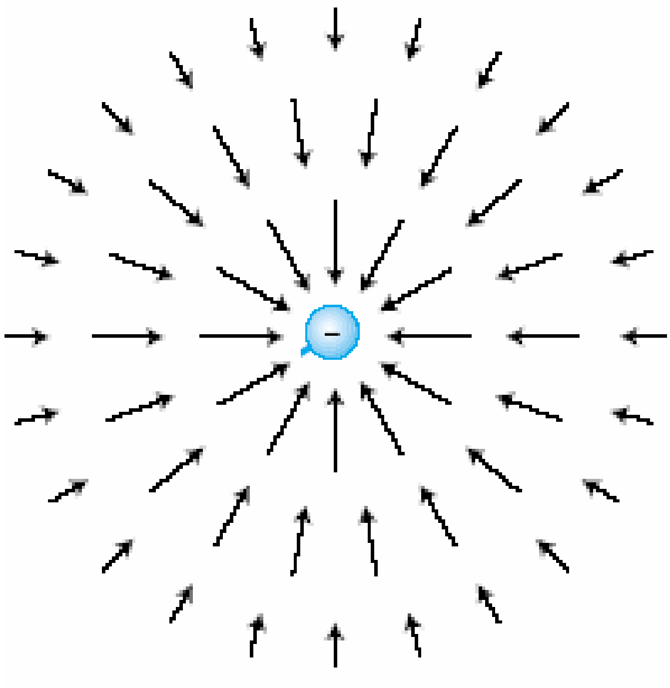
$$\mathbf{E} \equiv \frac{\mathbf{F}_e}{q_0}$$



$$\mathbf{F}_e = q\mathbf{E}$$

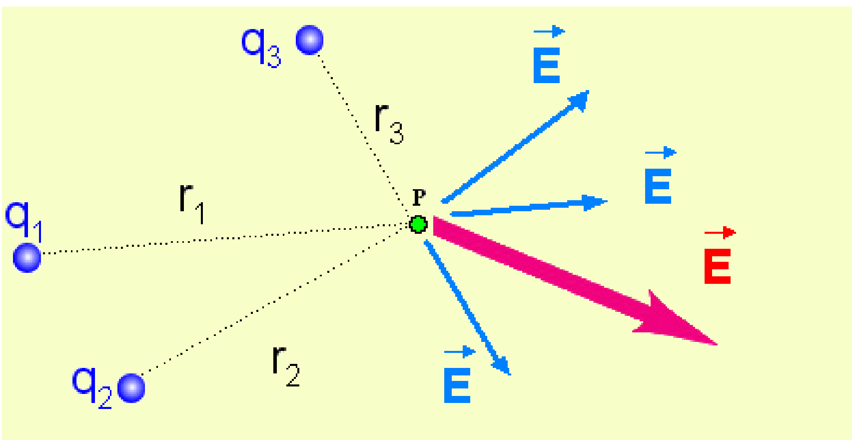


$$\mathbf{F}_e = k_e \frac{q q_0}{r^2}\,\hat{\mathbf{r}} \quad \Longrightarrow \quad \mathbf{E} = k_e \frac{q}{r^2}\,\hat{\mathbf{r}}$$

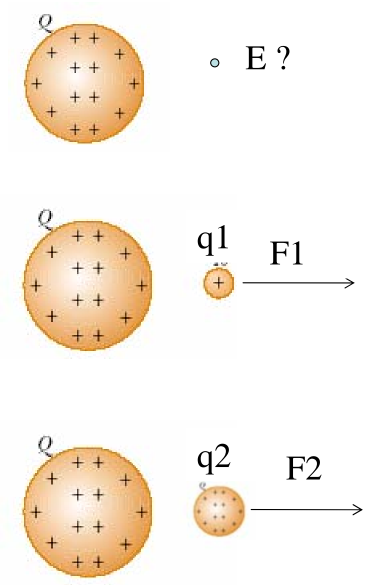


Varias cargas $\longrightarrow$ $\mathbf{E} = k_e \sum_i \frac{q_i}{r_i^2}\,\hat{\mathbf{r}}_i$

Principio de superposición



Comentario:

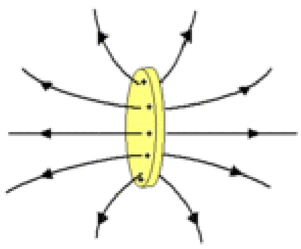


La carga de prueba debe ser lo suficientemente pequeña para que no interfiera en la distribución que genera el campo.

$$\mathbf{F1 / q1 \neq F2 \; / \; q2}$$

$$\boxed{\mathbf{E = \lim_{q \to 0} F / q}}$$

Líneas de Campo Eléctrico



Ayuda para visualizar patrones de campo eléctrico

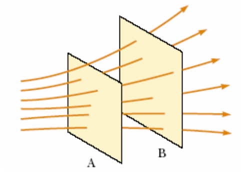
Criterios para dibujarlas

1. El vector E es tangente a la línea en cada punto.

2. Se adiciona un simbolo  **para indicar la dirección del campo**

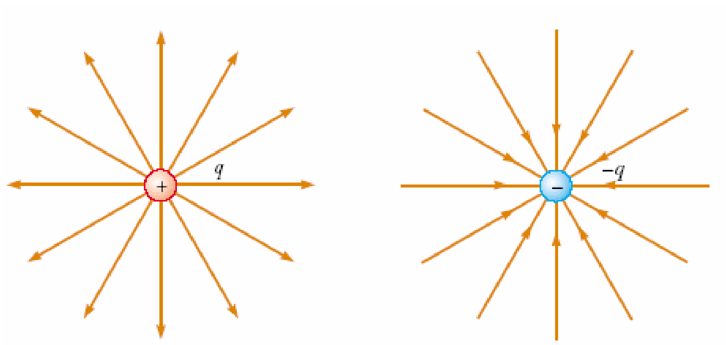
3. Las líneas de fuerza salen de las cargas positivas (fuentes) y entran en las cargas negativas (sumideros). Si no existen cargas positivas o negativas las líneas de campo empiezan o terminan en el infinito.

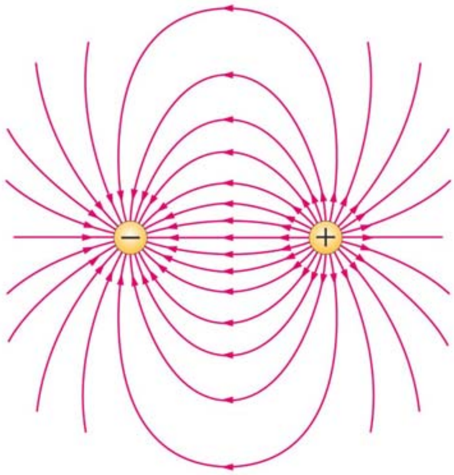
4. En cada punto del campo, el número de líneas por unidad de superficie perpendicular a ellas es proporcional a la intensidad de campo.

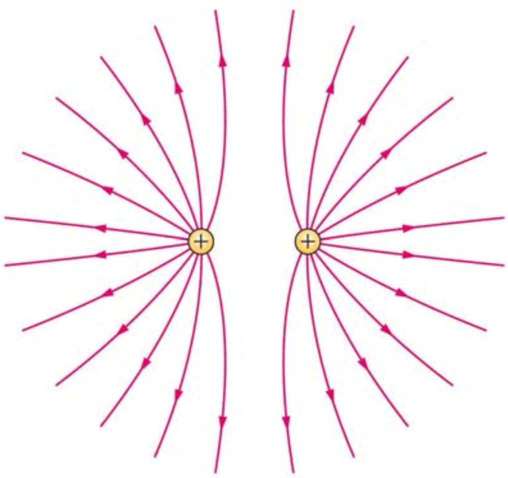
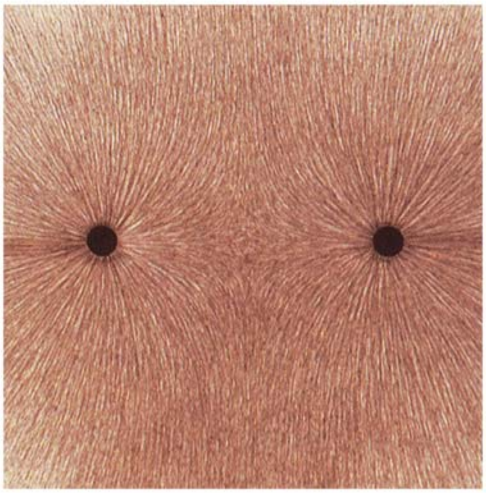


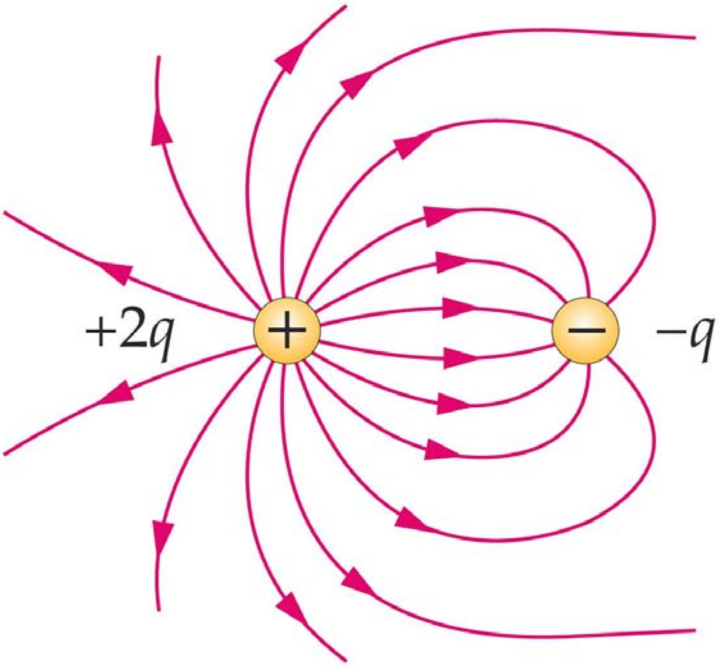
$$E_A > E_B$$

5. Dos líneas de fuerza nunca pueden cortarse.

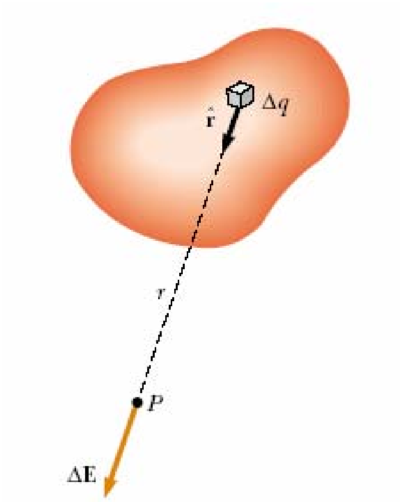






Distribuciones continuas de carga



$$\Delta \mathbf{E} = k_e \frac{\Delta q}{r^2} \hat{\mathbf{r}}$$

$$\mathbf{E} \approx k_e \sum_i \frac{\Delta q_i}{r_i^{\,2}} \hat{\mathbf{r}}_i$$

$$\mathbf{E} = k_e \lim_{\Delta q_i \to 0} \sum_i \frac{\Delta q_i}{r_i^{\,2}} \hat{\mathbf{r}}_i = k_e \int \frac{dq}{r^2} \hat{\mathbf{r}}$$

volumétrica

$$\rho \equiv \frac{Q}{V}$$

$$dq = \rho \, dV$$

superficial

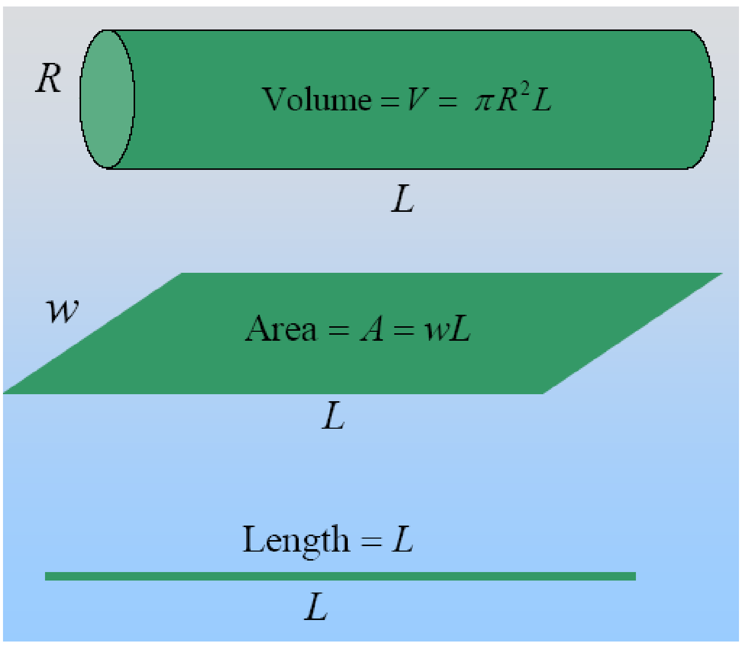
$$\sigma \equiv \frac{Q}{A}$$

$$dq = \sigma \, dA$$

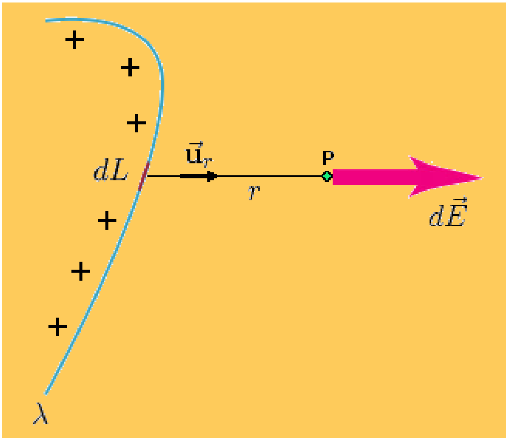
lineal

$$\lambda \equiv \frac{Q}{\ell}$$

$$dq = \lambda \, d\ell$$

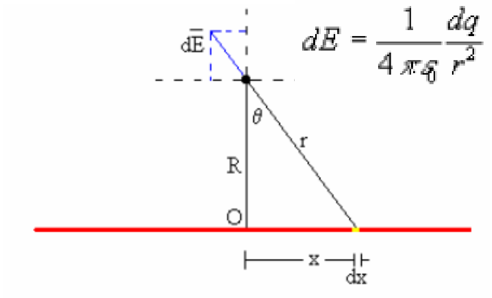


Distribución lineal arbitraria

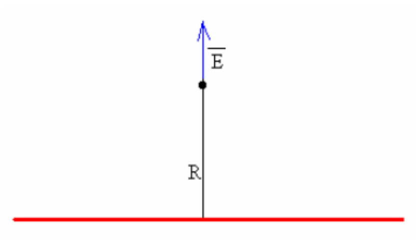


$$\vec{E} = \int_L d\vec{E} = \int_L \frac{1}{4\pi\epsilon_0}\frac{dq}{r^2}\vec{u_r} = \int_L \frac{1}{4\pi\epsilon_0}\frac{\lambda dL}{r^2}\vec{u_r}$$

Distribución rectilínea infinita



$$dE = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2}$$

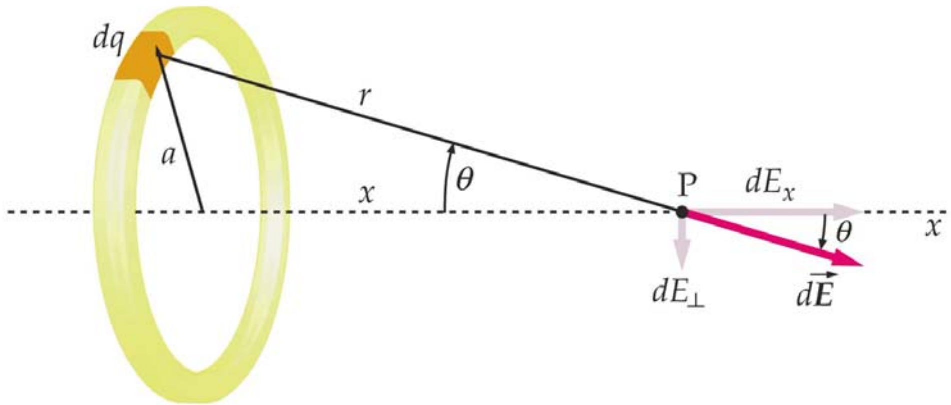


$$dE_y = dE\cos\theta$$

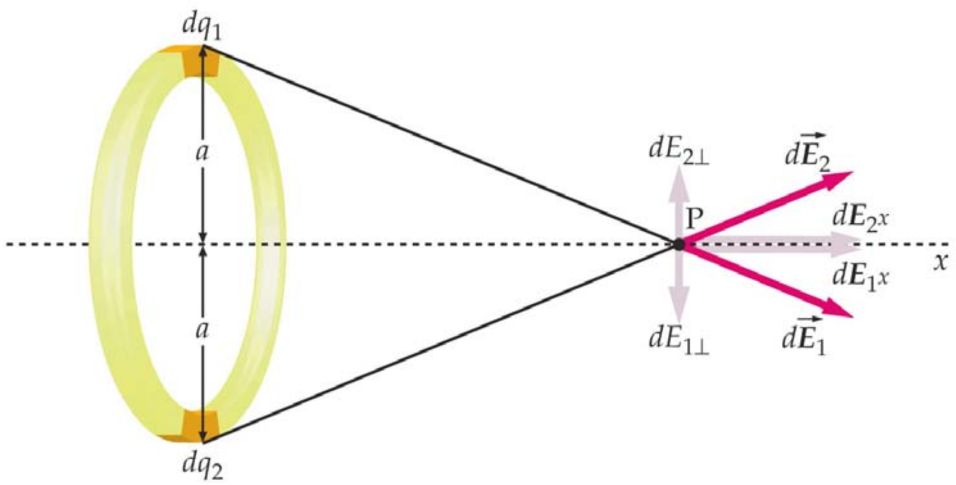
$$E = \int_{-\infty}^{\infty} dE_y = \int_{-\infty}^{\infty} \frac{1}{4\pi\varepsilon_0}\frac{\lambda dx}{r^2}\cos\theta = \int_{-\pi/2}^{\pi/2} \frac{1}{4\pi\varepsilon_0}\frac{\lambda\dfrac{Rd\theta}{\cos^2\theta}}{\left(\dfrac{R}{\cos\theta}\right)^2}\cos\theta$$

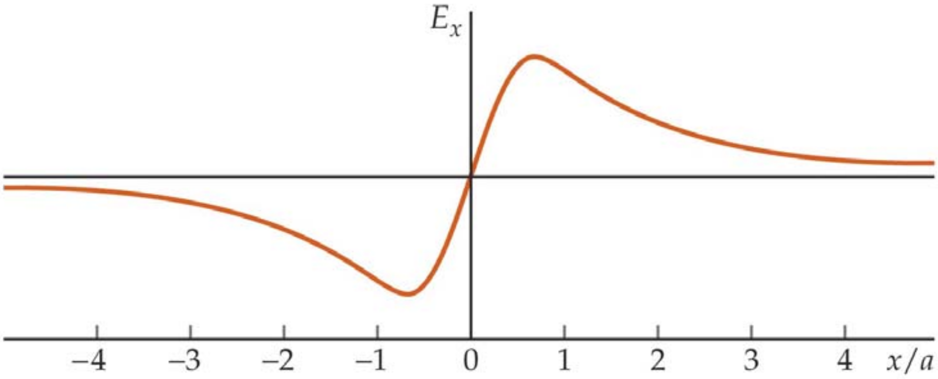
$$= \frac{\lambda}{4\pi\varepsilon_0 R}\int_{-\pi/2}^{\pi/2}\cos\theta\, d\theta = \frac{\lambda}{2\pi\varepsilon_0 R}$$

Campo eléctrico en el eje de un anillo cargado

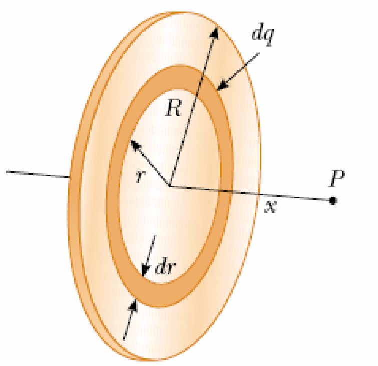


Podemos representar las características de la solución sin resolver la ecuación ?





Distribución superficial de carga:
campo eléctrico en el eje de un disco uniformemente cargado



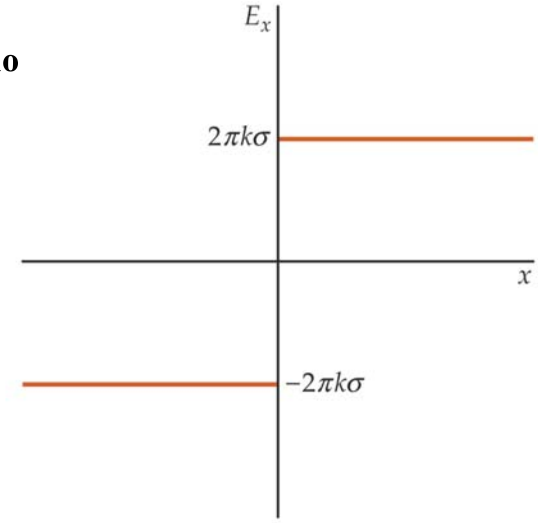
$$dE_x = \frac{k_e x}{(x^2 + r^2)^{3/2}}(2\pi\sigma r\, dr)$$

$$\begin{aligned} E_x &= k_e x \pi\sigma \int_0^R \frac{2r\, dr}{(x^2+r^2)^{3/2}} \\ &= k_e x \pi\sigma \int_0^R (x^2+r^2)^{-3/2}\, d(r^2) \\ &= k_e x \pi\sigma \left[\frac{(x^2+r^2)^{-1/2}}{-1/2}\right]_0^R \\ &= 2\pi k_e \sigma \left(1 - \frac{x}{(x^2+R^2)^{1/2}}\right) \end{aligned}$$

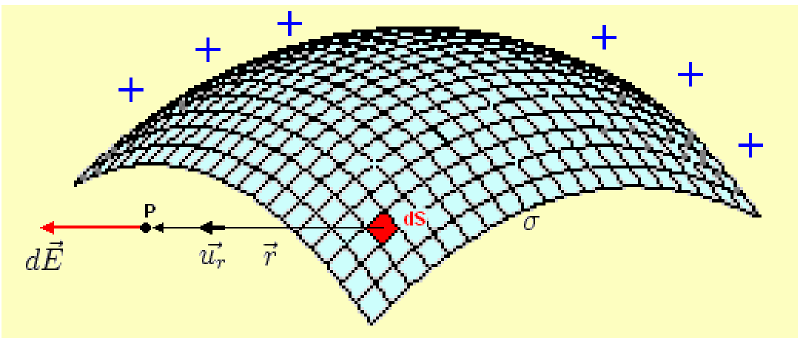
$R >> x$

$$E_x = 2\pi k_e \sigma = \frac{\sigma}{2\epsilon_0}$$

Campo eléctrico producido por un plano infinito

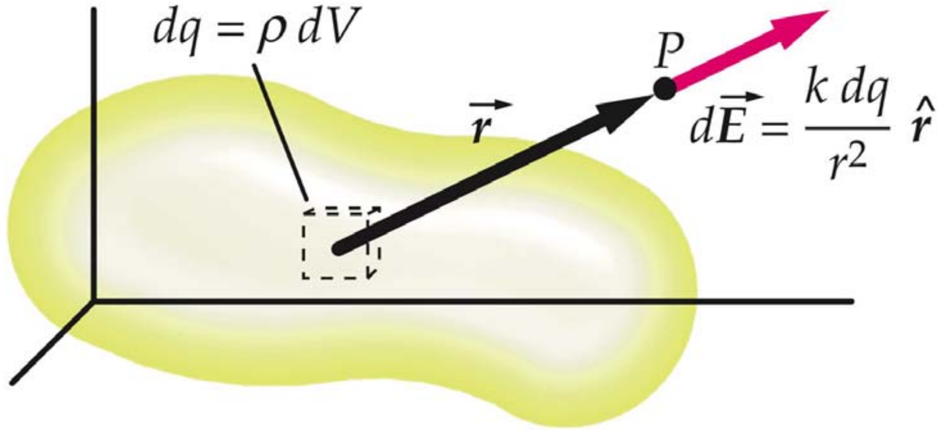


Distribución superficial arbitraria



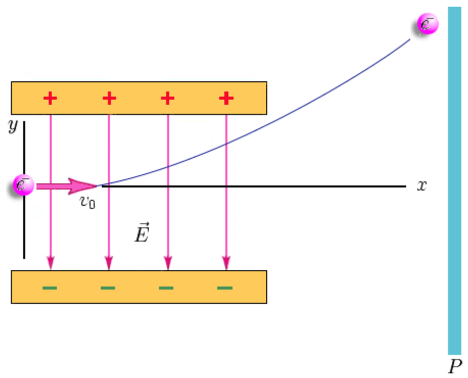
$$\vec{E} = \int_S d\vec{E} = \int_S \frac{1}{4\pi\epsilon_0} \frac{dq}{r^2} \vec{u_r} = \int_S \frac{1}{4\pi\epsilon_0} \frac{\sigma dS}{r^2} \vec{u_r}$$

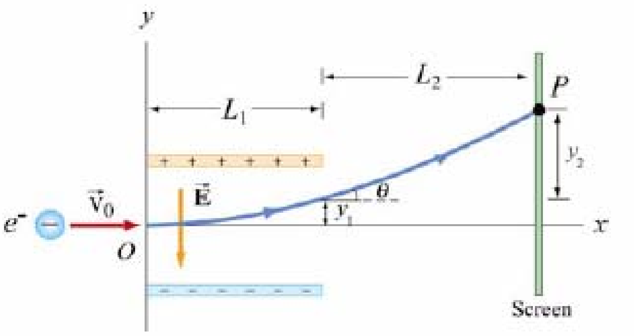
Distribución volumétrica arbitraria



$$\mathbf{E} = k_e \int \frac{dq}{r^2}\,\hat{\mathbf{r}}$$

Movimento de Partículas Puntuales en Campos Eléctricos



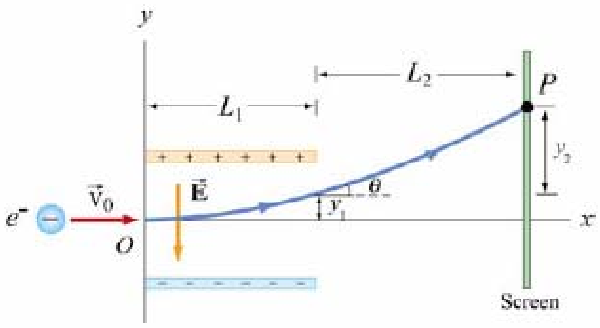


$$\vec{\mathbf{a}} = \frac{q\vec{\mathbf{E}}}{m} = -\frac{qE_y}{m}\hat{\mathbf{j}} = \frac{eE_y}{m}\hat{\mathbf{j}}$$

$$\vec{\mathbf{v}} = v_x\,\hat{\mathbf{i}} + v_y\,\hat{\mathbf{j}} = v_0\,\hat{\mathbf{i}} + a_y\,t_1\,\hat{\mathbf{j}} = v_0\,\hat{\mathbf{i}} + \left(\frac{eE_y}{m}\right)t_1\,\hat{\mathbf{j}} = v_0\,\hat{\mathbf{i}} + \left(\frac{eE_yL_1}{mv_0}\right)\hat{\mathbf{j}}$$

$$y_1 = \frac{1}{2}a_y\,t_1^{\,2} = \frac{1}{2}\left(\frac{eE_y}{m}\right)\left(\frac{L_1}{v_0}\right)^2$$

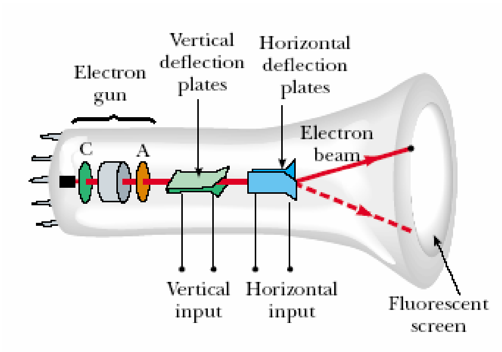
$$\tan\theta = \frac{v_y}{v_x} = \frac{(eE_y/m)(L_1/v_0)}{v_0} = \frac{eE_y\,L_1}{mv_0^{\,2}}$$

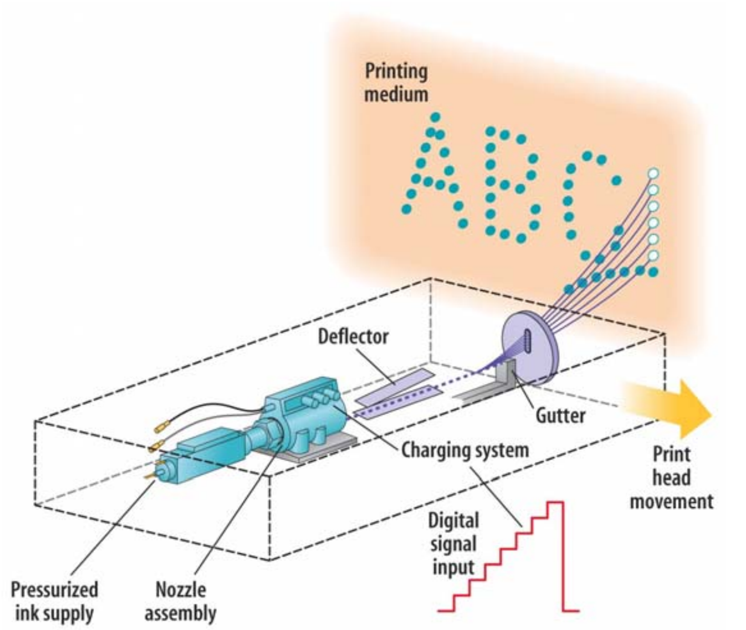


$$\boxed{y = y_1 + y_2 = \frac{1}{2}\frac{eE_y\,L_1^{\,2}}{mv_0^{\,2}} + \frac{eE_y\,L_1L_2}{mv_0^{\,2}} = \frac{eE_y\,L_1}{mv_0^{\,2}}\left(\frac{1}{2}L_1 + L_2\right)}$$

Puedo medir e/m $\Longrightarrow$ relación carga-masa

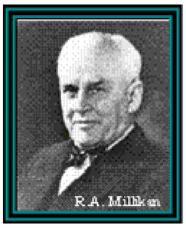
Tubo de rayos catódicos

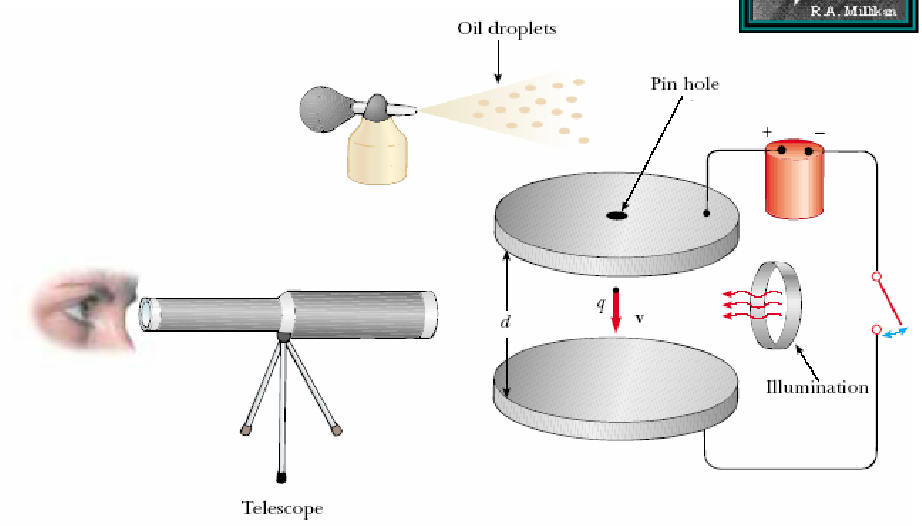


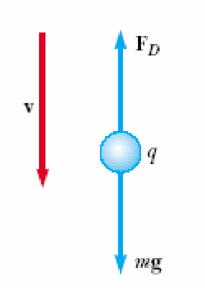


Experimento de la gota de aceite de Millikan

(Premio Nobel 1923)





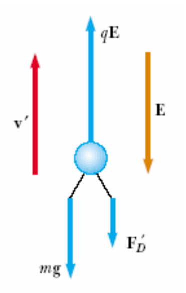


$F_D = mg \qquad v = cte$

$m = 4/3\ \pi\ r^3\ \rho_{ac}$ $\qquad$ ρ_{ac}: densidad aceite

$F_D = 6\ \pi\ r\ v\ \eta$ $\qquad$ η: viscosidad aire

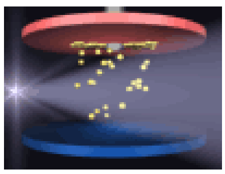
$F_D = mg \Longrightarrow r = (9\ \eta\ v/2\ \rho_{ac} g)^{1/2}$



$F_D' + mg = q\ E$

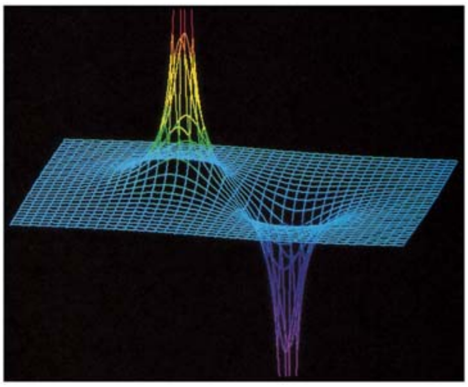
$6\ \pi\ r\ v'\ \eta + 6\ \pi\ r\ v\ \eta = q\ E$

$q = [6\ \pi\ r\ (v' + v)\ \eta] / E$



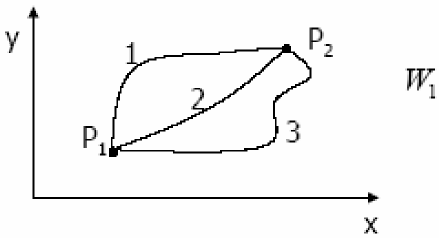
$q = n\ e \qquad n = 0,1.2,3,\ldots.$ $\qquad$ $e = 1.60 \times 10^{-19}$ C

Potencial Eléctrico

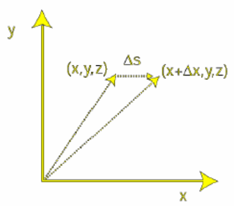


Repaso: Trabajo realizado por fuerzas conservativas

$$W_{12} = \int_{Path1} \vec{F} \bullet d\vec{s} = \int_{Path2} \vec{F} \bullet d\vec{s} = \int_{Path3} \vec{F} \bullet d\vec{s}$$



$$W_{11} = \oint_{Any} \vec{F} \bullet d\vec{s} = 0$$



$ds = dx\ \mu_x$ $\Longrightarrow$ $W_c = \int_{x_i}^{x_f} F_x\, dx = -\Delta U$

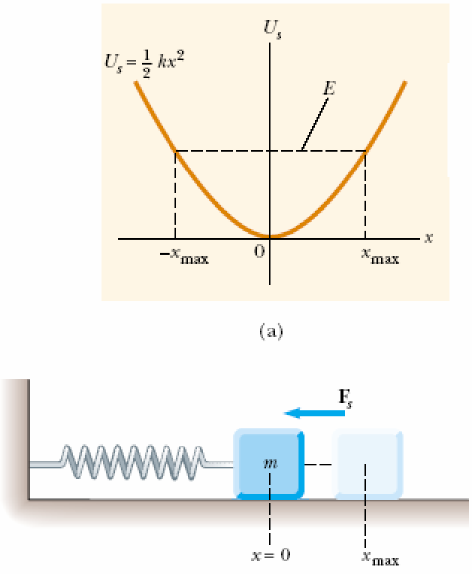
$$\Delta U = U_f - U_i = -\int_{x_i}^{x_f} F_x\, dx$$

$$dU = -F_x\, dx$$

$$F_x = -\frac{dU}{dx}$$

$$\boxed{\mathbf{F} = -\nabla U.}$$

Ejemplo: fuerza elástica de un resorte



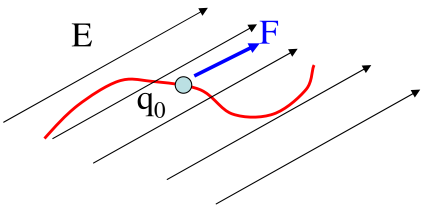
(a)

(b)

$$F_s = -\frac{dU_s}{dx} = -\frac{d}{dx}\left(\frac{1}{2}kx^2\right) = -kx$$

Ejemplificar con la gravedad !!!

Caso eléctrico



$$\Delta U = -q_0 \int_A^B \mathbf{E}\cdot d\mathbf{s}$$

Potencial eléctrico $\Rightarrow$ $V = \frac{U}{q_0}$

$$\Delta V \equiv \frac{\Delta U}{q_0} = -\int_A^B \mathbf{E}\cdot d\mathbf{s}$$

$$dV = -\mathbf{E}\cdot d\mathbf{s} \qquad E_x = -\frac{\partial V}{\partial x} \qquad E_y = -\frac{\partial V}{\partial y} \qquad E_z = -\frac{\partial V}{\partial z}$$

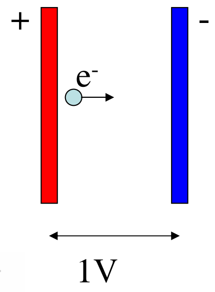
$$\boxed{\mathbf{E} = -\nabla V = -\left(\hat{\mathbf{i}}\frac{\partial}{\partial x} + \hat{\mathbf{j}}\frac{\partial}{\partial y} + \hat{\mathbf{k}}\frac{\partial}{\partial z}\right)V}$$

[V] = [U] / [q] = J/C = V (volt)

[E] = N/C = J/mC = V/m

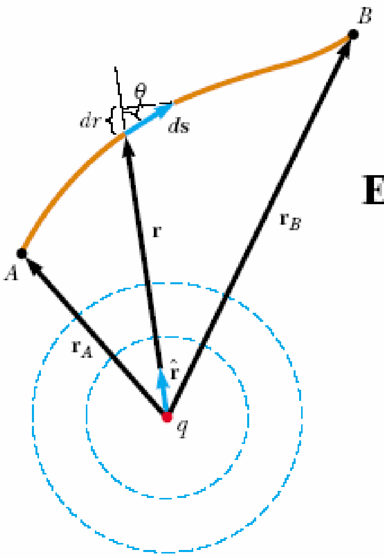
eV es una unidad de energía que se utiliza en Física Atomica. eV quiere decir electron-volt--esto es, la cantidad de energía necesaria para mover un electrón a través de una diferencia de potencial de un voltio.

$$\Delta U = e\ \Delta V$$



$$1\ \mathrm{eV} = 1.60 \times 10^{-19}\ \mathrm{C \cdot V} = 1.60 \times 10^{-19}\ \mathrm{J}$$

Potencial Eléctrico entre Cargas Puntuales



$$V_B - V_A = -\int_A^B \mathbf{E}\cdot d\mathbf{s}$$

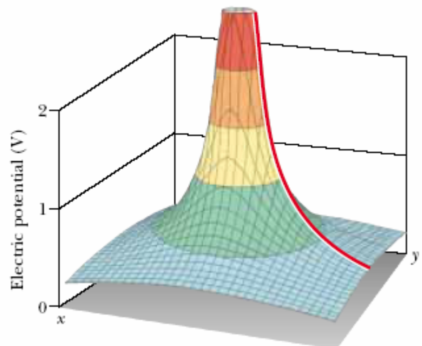
$$\mathbf{E} = k_e q\hat{\mathbf{r}}/r^2 \qquad \mathbf{E}\cdot d\mathbf{s} = k_e \frac{q}{r^2}\,\hat{\mathbf{r}}\cdot d\mathbf{s}$$

$$V_B - V_A = -k_e q\int_{r_A}^{r_B}\frac{dr}{r^2} = \frac{k_e q}{r}\Bigg]_{r_A}^{r_B}$$

$$V_B - V_A = k_e q\left[\frac{1}{r_B} - \frac{1}{r_A}\right]$$

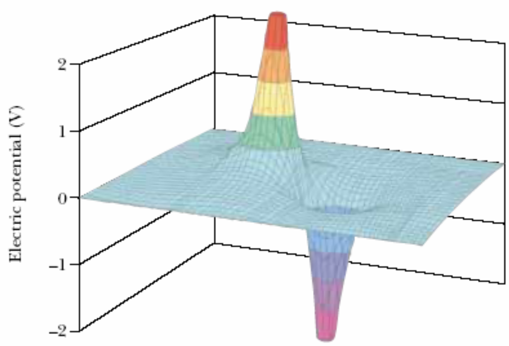
$$V = k_e\frac{q}{r} + \mathrm{C}$$

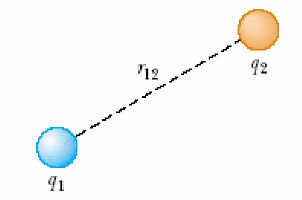
$$\mathbf{V(r\to\infty) = 0} \Rightarrow \boxed{V = k_e\frac{q}{r}}$$



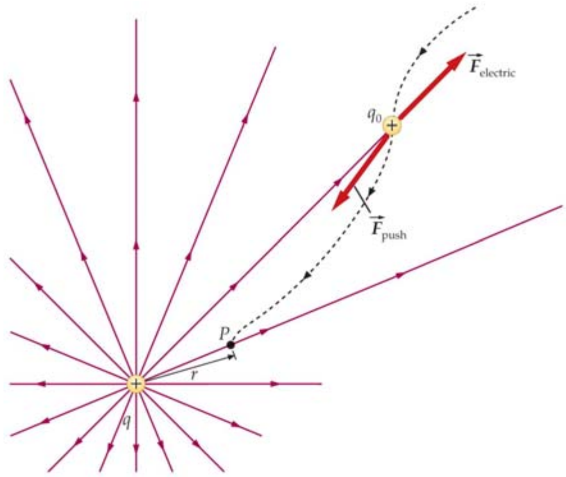
$$V = k_e\frac{q}{r}$$

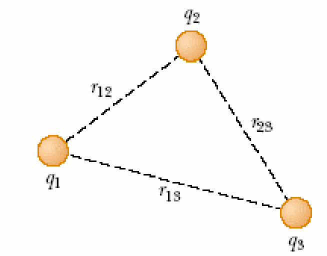
$$V = k_e\sum_i\frac{q_i}{r_i}$$





$$U = k_e \frac{q_1 q_2}{r_{12}}$$

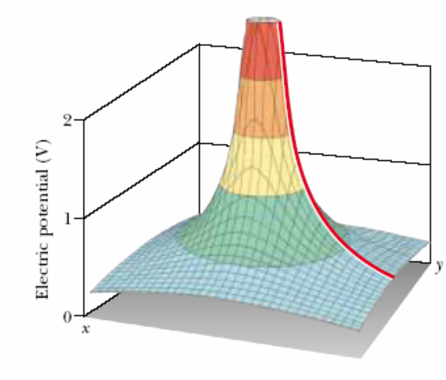


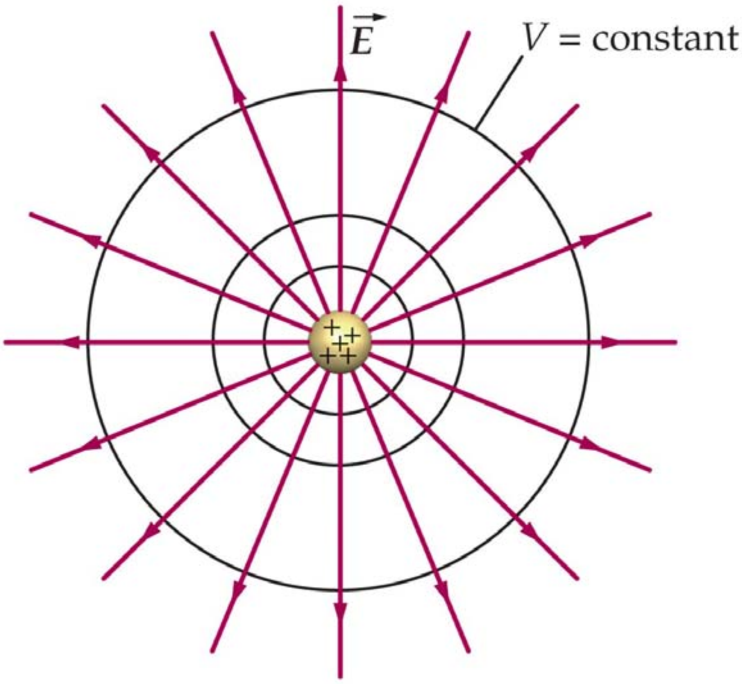


$$U = k_e \left(\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}} \right)$$

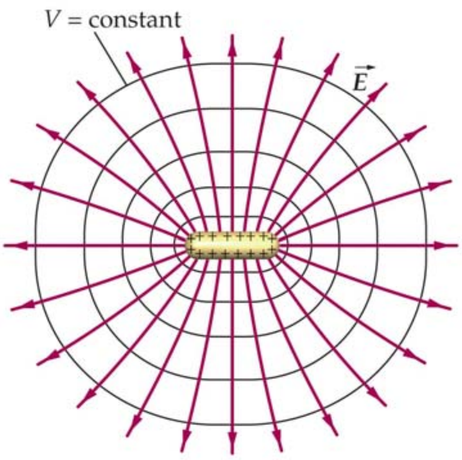
Superficies Equipotenciales

$$V(\mathbf{r}) = V_0$$

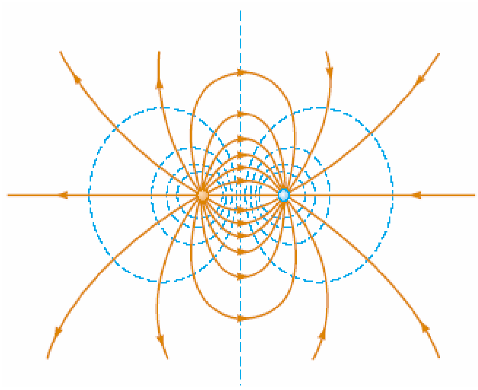




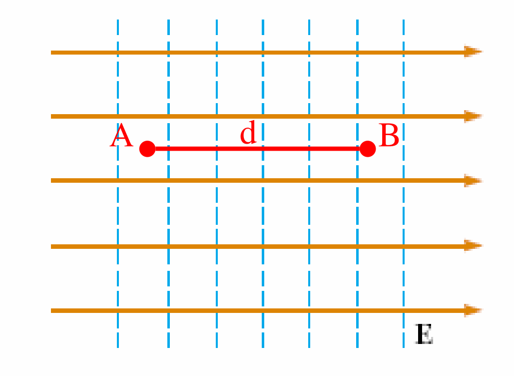
$\vec{E}$
V = constant



V = constant
$\vec{E}$



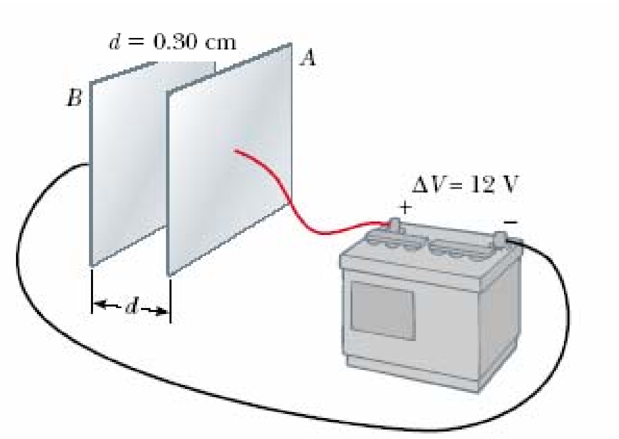
Campo eléctrico uniforme



$$V_B - V_A = \Delta V = -\int_A^B \mathbf{E} \cdot d\mathbf{s} = -\int_A^B (E \cos 0°)\, ds = -\int_A^B E\, ds$$

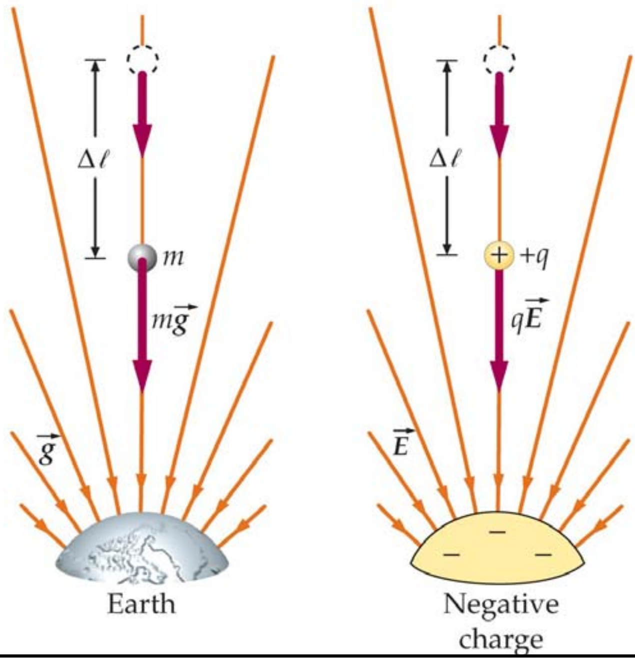
$$\Delta V = -E \int_A^B ds = -Ed \qquad V_B < V_A$$

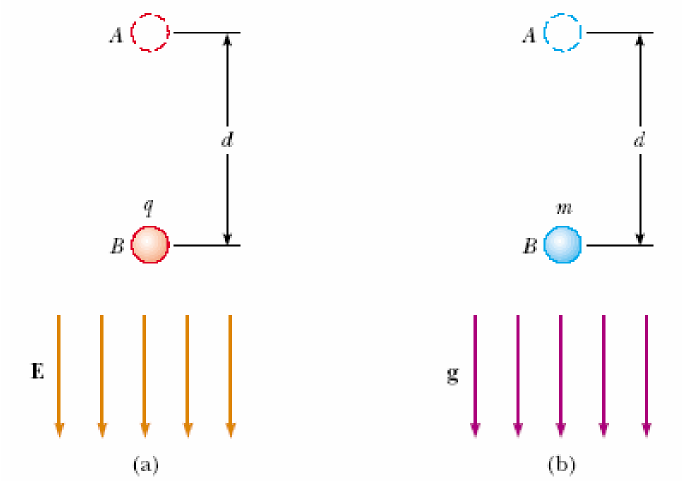
Ejemplo práctico



$$E = \frac{|V_B - V_A|}{d} = \frac{12\ \text{V}}{0.30 \times 10^{-2}\ \text{m}} = \boxed{4.0 \times 10^3\ \text{V/m}}$$

Analogía con la gravedad





$V_A > V_B$

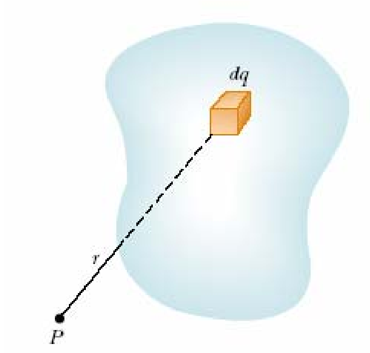
$U_A > U_B$ si $q > 0$

$U_A < U_B$ si $q < 0$

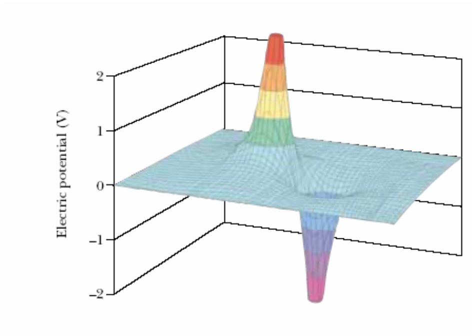
Principio de conservación de la energía

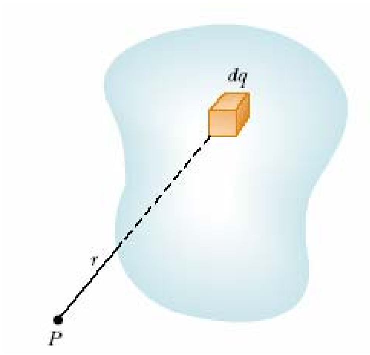
U(A) + Ec(A) = U(B) + Ec(B)

Cálculo de *V* para un distribución de carga



$$V = k_e \sum_i \frac{q_i}{r_i}$$

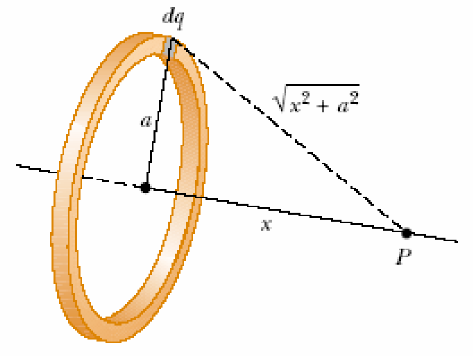




$$dV = k_e \frac{dq}{r}$$

$$V = k_e \int \frac{dq}{r}$$

Ejemplo



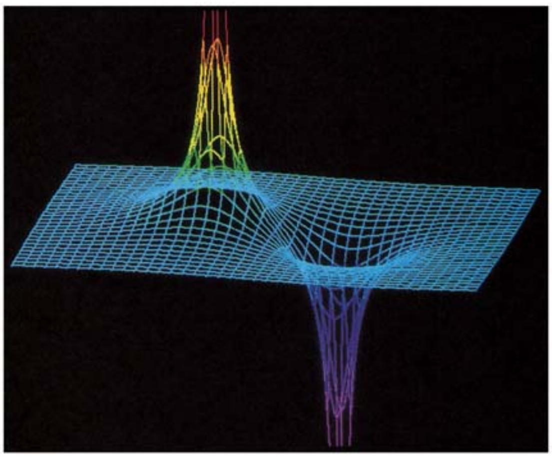
$$V = k_e \int \frac{dq}{r} = k_e \int \frac{dq}{\sqrt{x^2 + a^2}}$$

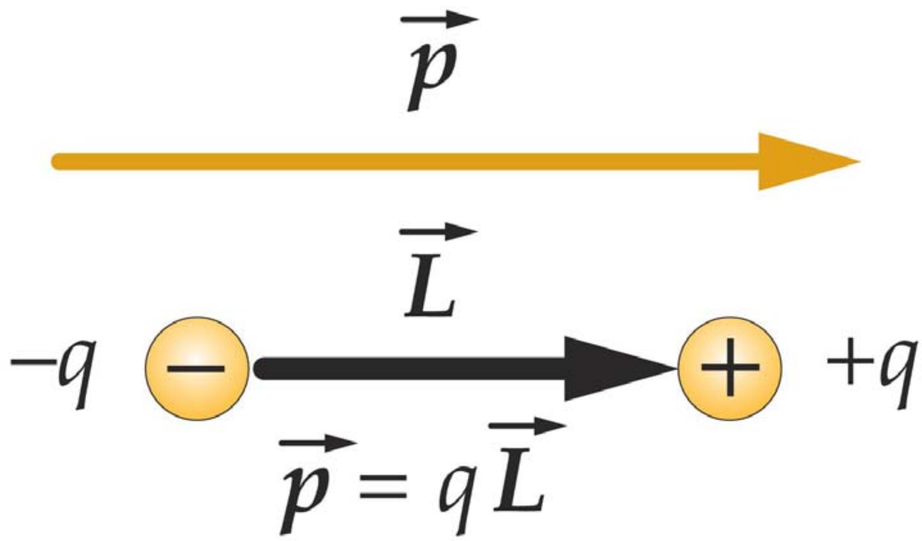
$$V = \frac{k_e}{\sqrt{x^2 + a^2}} \int dq = \boxed{\frac{k_e Q}{\sqrt{x^2 + a^2}}}$$

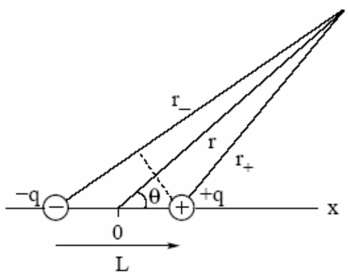
$$\begin{aligned} E_x &= -\frac{dV}{dx} = -k_e Q \frac{d}{dx} (x^2 + a^2)^{-1/2} \\ &= -k_e Q(-\tfrac{1}{2})(x^2 + a^2)^{-3/2}(2x) \end{aligned}$$

$$E_x = \boxed{\frac{k_e Q x}{(x^2 + a^2)^{3/2}}}$$

Dipolo Eléctrico





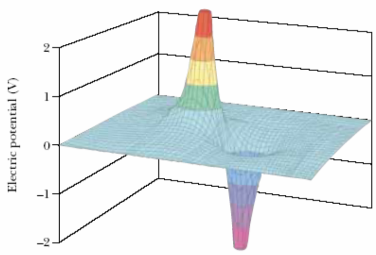


$$V = V_+ + V_- = k\left(\frac{q}{r_+} + \frac{(-q)}{r_-}\right) = kq\,\frac{r_- - r_+}{r_- r_+}$$

$$(r \gg L):\ \ r_- - r_+ \simeq L\cos\theta,\ r_- r_+ \simeq r^2 \Rightarrow\ V(r,\theta) \simeq k\,\frac{qL\cos\theta}{r^2}$$

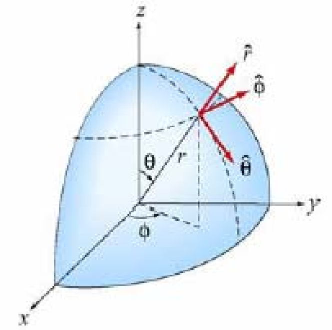
$$p = qL$$

$$\boxed{V(r,\theta) \simeq k\,\frac{p\cos\theta}{r^2}}$$

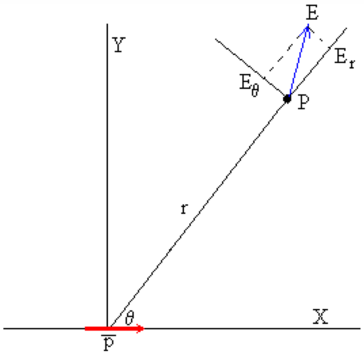


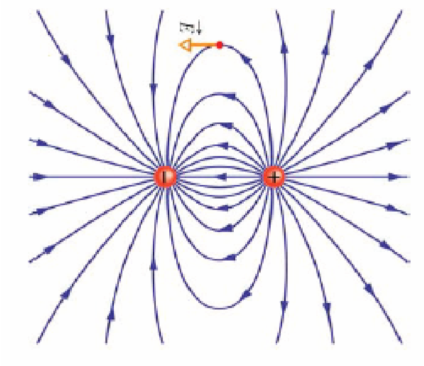
E ?

$$\vec{\nabla} = \frac{\partial}{\partial r}\hat{\mathbf{r}} + \frac{1}{r}\frac{\partial}{\partial \theta}\hat{\boldsymbol{\theta}} + \frac{1}{r\sin\theta}\frac{\partial}{\partial \phi}\hat{\boldsymbol{\varphi}}$$



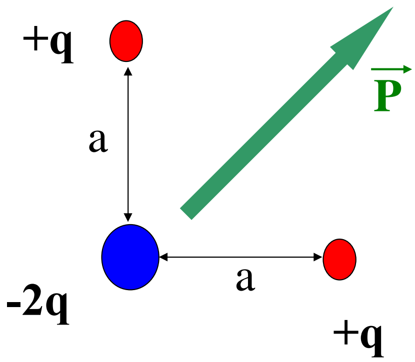
$$\boxed{E_r = -\frac{\partial V}{\partial r} = \frac{p\cos\theta}{2\pi\varepsilon_0 r^3}, \quad E_\theta = -\frac{1}{r}\frac{\partial V}{\partial \theta} = \frac{p\sin\theta}{4\pi\varepsilon_0 r^3}, \quad E_\phi = 0}$$

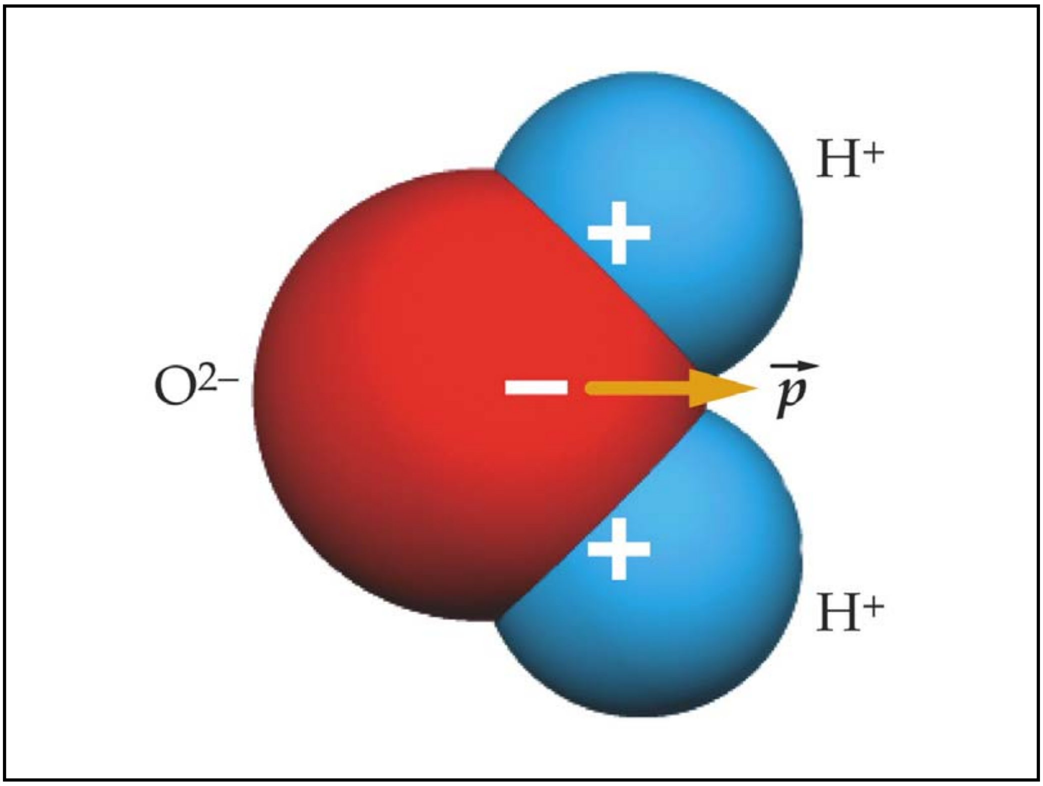




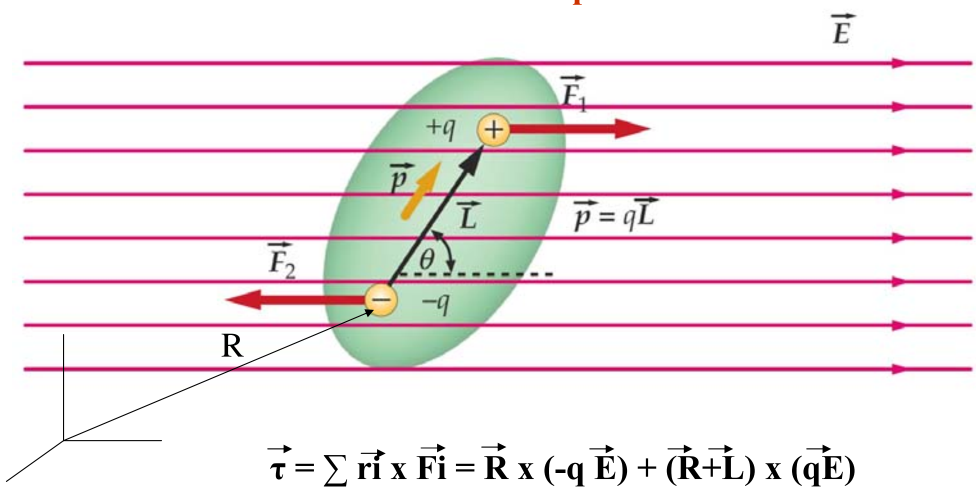
Generalización del concepto de dipolo

$$\mathbf{Q = \sum q_j = 0 \qquad y \qquad \vec{P} = \sum q_j\,\vec{r}_j}$$





Efecto de E sobre un dipolo

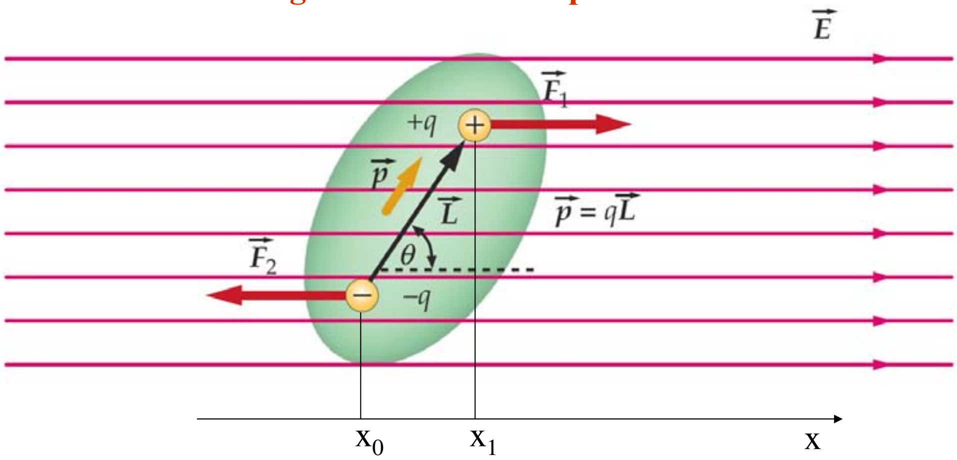


$$\vec{\tau} = \sum \vec{r}i \times \vec{F}i = \vec{R} \times (-q\,\vec{E}) + (\vec{R}+\vec{L}) \times (q\vec{E})$$

$$= \vec{L} \times q\,\vec{E} = q\vec{L} \times \vec{E}$$

$$\boxed{\vec{\tau} = \vec{P} \times \vec{E}}$$

Energía Potencial del dipolo



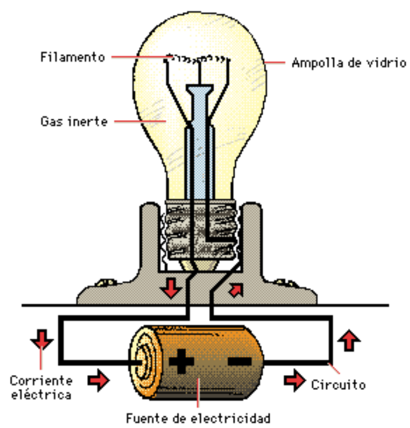
$V(x) = - E\, x$

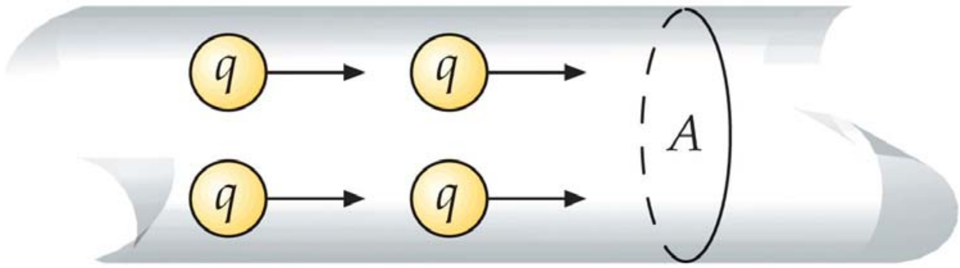
$U = \sum q_i V_i = (-q)\, V(x_0) + q\, V(x_1) = q\, E\, x_0 - q\, E\, x_1 = -q\, E\, (x_1 - x_0)$

$U = - q\ \ a \cos\theta = - p\, E \cos\theta$

$$\boxed{U = - \vec{p} \cdot \vec{E}}$$

Cargas en Movimento
Corriente Eléctrica





$$I_m = \frac{\Delta q}{\Delta t}$$

Corriente promedio: carga que pasa por A por unidad de tiempo

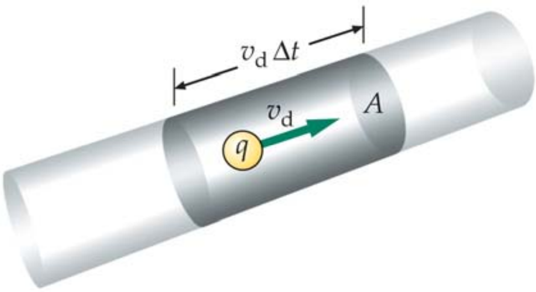
$$I = \frac{dq}{dt}$$

Corriente Instantánea

[I] = C/s = A (Ampere)

Se le asigna un sentido coincidente con $q\,\vec{v}$

Densidad de corriente



n: densidad de carga movil

N: número de portadores que atraviesan A en Δt

N = n A v_d Δt

$$I = \frac{\Delta Q}{\Delta t} = nqv_d A$$

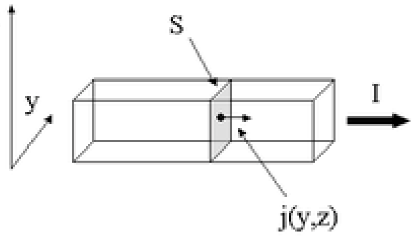
$$J = \frac{I}{A} = nqv_d$$

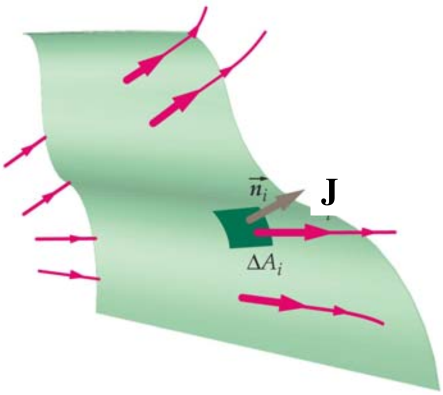
Densidad de corriente (carga por unidad de A y t)

J es un vector !!!! ⇨ $\boxed{\vec{J} = q\ n\ \vec{v}_d}$

En general:

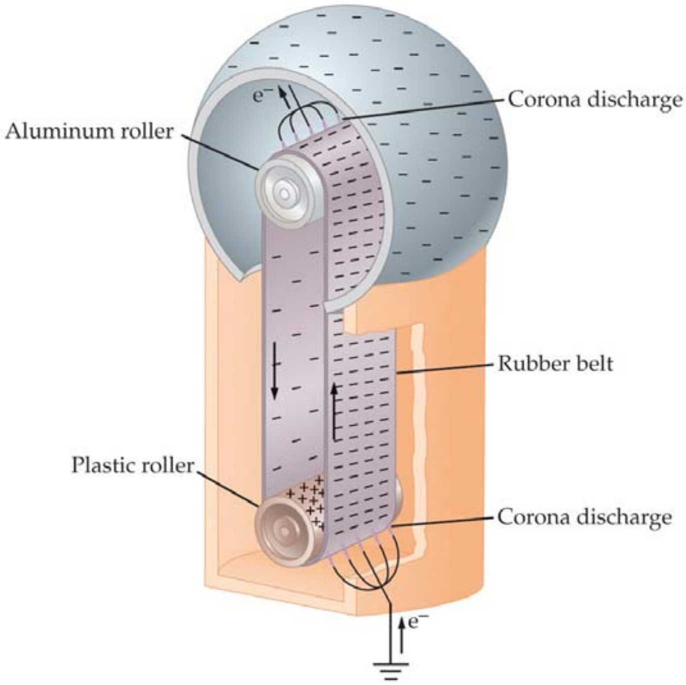
$$I = \int_S \mathbf{j} \cdot d\mathbf{S}$$

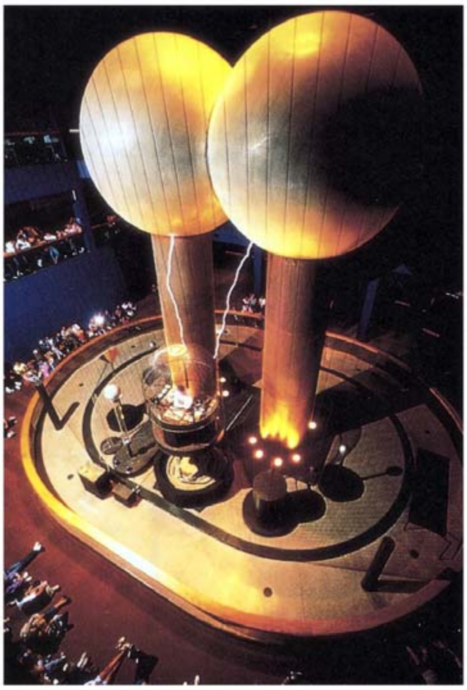


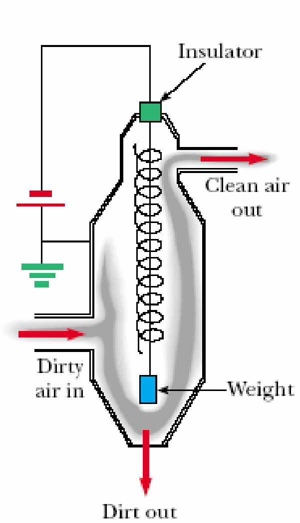


La corriente eléctrica (I) es el flujo del vector densidad de corriente (J) a traves de una superficie.

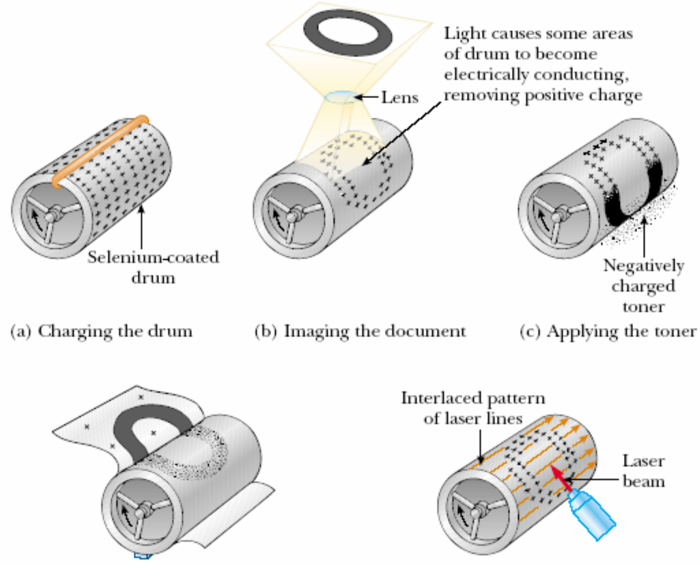
Aplicaciones


Corona discharge
Aluminum roller
Rubber belt
Plastic roller
Corona discharge
e⁻
e⁻





Rei O'Hara/Black Star/PNI



(a) Charging the drum (b) Imaging the document (c) Applying the toner

(d) Transferring the toner to the paper (e) Laser printer drum