Se presentan a continuación algunas resoluciones y respuestas de los ejercicios propuestos en el apunte de Integrales. Le **corresponde al estudiante "justificar"** cada una de las respuestas de acuerdo a los conocimientos desarrollados.

Integrales

1) a) $\int x^6 dx = \frac{x^7}{7} + k \quad (k \quad cte)$

b) $\int (6x^3 + 8x^2 - 3)dx = \int 6x^3 dx + \int 8x^2 dx + \int -3\,dx = 6\int x^3 dx + 8\int x^2 dx - 3\int dx =$

$= 6\frac{x^4}{4} + 8\frac{x^3}{3} - 3x + k$

c) $\int -3\sqrt{2x}\; dx = -3\int (2x)^{1/2} dx = -3\int \sqrt{2}\sqrt{x}\; dx = -3\sqrt{2}\int x^{1/2} dx =$

$= -3\sqrt{2}\,\frac{x^{3/2}}{3/2} + k = -2\sqrt{2}x^{3/2} + k$

d) $\int \frac{1}{\sqrt[5]{x}} dx = \int x^{-1/5} dx = \frac{x^{4/5}}{4/5} + k = \frac{5}{4}x^{4/5} + k$

e) $\int (5x^2 + 2x - 3)\,dx = 5\frac{x^3}{3} + x^2 - 3x + k$

f) $\int (3e^x - 4)\,dx = 3e^x - 4x + k$

g) $\int (sen\,x + \sqrt{x} + e^x)\,dx = -\cos x + \frac{2}{3}x^{3/2} + e^x + k$

h) $\int (x-1)(x^2 - 3x)dx = \int (x^3 - 4x^2 + 3x)dx = \frac{x^4}{4} - 4\frac{x^3}{3} + 3\frac{x^2}{2} + k$

i) $\int \left(\frac{1}{2}x + \frac{1}{\sqrt{x}} - 3\right)dx = \frac{x^2}{4} + 2\sqrt{x} - 3x + k$

j) $\int \left(\cos x + \frac{sen\,x}{2}\right)dx = senx - \frac{\cos x}{2} + k$

2) a) $e - 1$ **b)** 0 **c)** $-\frac{4}{3}$ **d)** -9 **e)** $\frac{2}{5}$ **f)** 2

3) $A = 36$ **4)** $A = \frac{4}{3}$ **5)** $A = \frac{125}{6}$ **6)** $A = \frac{9}{8}$ **7)** $A = \frac{23}{3}$

8) a) $\int (x-5)^4 dx = \int t^4 dt = \frac{t^5}{5} + k = \frac{(x-5)^5}{5} + k$ $\qquad x - 5 = t$, $dx = dt$

b) $\int e^{2x+1} dx = \int e^t \frac{1}{2} dt = \frac{1}{2}\int e^t dt = \frac{1}{2}e^t + k = \frac{1}{2}e^{2x+1} + k$ $\qquad 2x + 1 = t$, $2dx = dt \quad dx = \frac{1}{2}dt$

c) $\int 6\cos(2x-1)\,dx = 6\int \cos t\,\frac{dt}{2} = 6\frac{1}{2}\int \cos t dt = 3\,sent + k = 3\,sen(2x-1)+k$

$2x-1=t$

$2dx = dt \quad dx = \frac{1}{2}dt$

d) $\int \sqrt{(x+3)^5}\,dx = \int t^{5/2}dt = \frac{t^{7/2}}{7/2} + k = \frac{2}{7}(x+3)^{7/2} + k$ $\qquad x+3=t$, $dx = dt$

e) $\int sen(x+5)\,dx = \int sen t\; dt = -\cos t + k = -\cos(x+5)+k$ $\qquad x+5=t$, $dx=dt$

f) $\int senx\cos x\,dx = \int t\;dt = \frac{t^2}{2} + k = \frac{(senx)^2}{2} + k$ $\qquad senx = t$, $\cos x\,dx = dt$

g) $\int \sqrt{5x+2}\,dx = \int t^{1/2}\frac{1}{5}dx = \frac{1}{5}\frac{t^{3/2}}{3/2} + k = 2\frac{t^{3/2}}{15} + k = \frac{2}{15}(5x+2)^{3/2} + k$

$5x+2=t$

$5\,dx = dt \qquad dx = \frac{dt}{5}$

h) $\int x\cos x^2\,dx = \int \cos t\frac{dt}{2} = \frac{1}{2}\int \cos t\,dt = \frac{1}{2}sent + k = \frac{1}{2}senx^2 + k$

$x^2 = t$

$2x\,dx = dt \qquad xdx = \frac{dt}{2}$

i) $\int 3x\sqrt{2-x^2}\,dx = 3\int \sqrt{t}\,\frac{dt}{-2} = \frac{-3}{2}\int t^{1/2}dt = \frac{-3}{2}\frac{t^{3/2}}{3/2} + k = -t^{3/2} + k = -(2-x^2)^{3/2} + k$

$2-x^2 = t$

$-2x\,dx = dt \qquad xdx = \frac{dt}{-2}$

j) $\int 4\,sen(2x+3)\,dx = 2\int sen t\;dt = -2\cos t + K = -2\cos(2x+3)+K$

$2x+3=t$

$2x\,dx = dt$

9) a) $\int 2x\,e^x dx = 2xe^x - \int 2e^x dx = 2xe^x - 2e^x + k$ $\qquad u = 2x \quad du = 2dx$, $dv = e^x dx \quad v = e^x$

b) $\int \ln x\,dx = x\ln x - \int x\frac{1}{x}dx = x\ln x - x + k$ $\qquad u = \ln x \quad du = \frac{1}{x}dx$, $dv = dx \quad v = x$

c) $\int x \ln x \, dx = \ln x \frac{x^2}{2} - \int \frac{x^2}{2} \frac{1}{x} dx = \frac{x^2}{2} \ln x - \frac{x^2}{4} + k$

$u = \ln x \qquad du = \frac{1}{x} dx$

$dv = x dx \qquad v = \frac{x^2}{2}$

d) $\int x \, senx \, dx = -x\cos x - \int -\cos x dx = -x\cos x + \int \cos x dx = -x\cos x + senx + k$

$u = x \qquad du = dx$

$dv = senx dx \qquad v = -\cos x$

e) $\int x \cos x \, dx = xsenx - \int senx dx = xsenx - (-\cos x) + k = xsenx + \cos x + k$

$u = x \qquad du = dx$

$dv = \cos x dx \qquad v = senx$

f) $\int x^3 \ln x \, dx = \ln x \ \frac{x^4}{4} - \int \frac{x^4}{4} \frac{1}{x} dx = \frac{x^4}{4} \ln x - \frac{1}{4} \int x^3 dx = \ln x \ \frac{x^4}{4} - \frac{x^4}{16} + k$

$u = \ln x \qquad du = \frac{1}{x} dx \qquad dv = x^3 dx \qquad v = \frac{x^4}{4}$

g) $\int x \cos(3x) \, dx = x \frac{sen3x}{3} - \int \left(\frac{sen3x}{3} \right) dx = x \frac{sen3x}{3} - \frac{1}{3} \int sen3x dx =$

$u = x \qquad du = dx$

$dv = \cos 3x dx \qquad v = \frac{sen3x}{3}$

$= x \frac{sen3x}{3} - \frac{1}{3} \frac{(-\cos 3x)}{3} + k = x \frac{sen3x}{3} + \frac{\cos 3x}{9} + k$

h) $\int (x-2) e^x \, dx = (x-2)e^x - \int e^x dx = (x-2)e^x - e^x + k$

$u = x - 2 \qquad du = dx$

$dv = e^x dx \qquad v = e^x$

i) $\int \sqrt{x} \ln x \, dx = \ln x \ \frac{2}{3} x^{3/2} - \int \frac{2}{3} x^{3/2} \frac{1}{x} dx = \ln x \frac{2}{3} x^{3/2} - \frac{2}{3} \int x^{1/2} dx =$

$u = \ln x \qquad du = \frac{1}{x} dx$

$= \ln x \frac{2}{3} x^{3/2} - \frac{4}{9} x^{3/2} + k$

$dv = \sqrt{x} \, dx \qquad v = \frac{x^{3/2}}{3/2} = \frac{2}{3} x^{3/2}$

j) $\int x^2 \ln x dx = \ln x \ \frac{x^3}{3} - \int \frac{x^3}{3} \frac{1}{x} dx = \frac{x^3}{3} \ln x - \frac{1}{3} \int x^2 = \ln x \ \frac{x^3}{3} - \frac{x^3}{9} + k$

$u = \ln x \qquad du = \frac{1}{x} dx$

$dv = x^2 dx \qquad v = \frac{x^3}{3}$

k) $\int \frac{1}{2} x\, e^{x-1}\, dx = \frac{1}{2}\left[x e^{x-1} - \int e^{x-1} dx\right] = \frac{1}{2}\left[x e^{x-1} - e^{x-1} + k\right] = \frac{1}{2} x e^{x-1} - \frac{1}{2} e^{x-1} + k$

$u = x \qquad du = dx$

$dv = e^{x-1} dx \qquad v = e^{x-1}$

PRÁCTICA COMPLEMENTARIA

1) 9 **2)** $\frac{13}{3}$ **3)** $\frac{16}{3}$ **4)** $\frac{4}{3}$

5) a) $\int_{-1}^{2}\left(x^3 - \frac{3}{2}x^2\right) dx = \ldots = \frac{-3}{4}$ **b)** $\int_{1}^{3}\left(e^x + x + 3\right) dx = \ldots = e^3 + 10 - e$

c) $\int_{0}^{1}\left(\sqrt{x} + 5x^4\right) dx = \ldots = \frac{5}{3}$ **d)** $\int_{1}^{4}\left(x + \sqrt{x}\right)\, dx = \ldots = \frac{73}{6}$

6) a) $\int 4x\left(2x^2 - 1\right)^2 dx = \frac{\left(2x^2-1\right)^3}{3} + K$ **b)** $\int (2x+1)\left(x^2 + x - 1\right)^2 dx = \frac{\left(x^2+x-1\right)^3}{3} + K$

c) $\int 3(6x-2)^2 dx = \frac{(6x-2)^3}{6} + K$ **d)** $\int 3sen(3x+2) dx = -\cos(3x+2) + K$

e) $\int \frac{1}{2}\cos(x-3) dx = \frac{sen(x-3)}{2} + K$ **f)** $\int x\, sen\left(2x^2\right) dx = \frac{-\cos\left(2x^2\right)}{4} + K$

g) $\int \frac{1}{3} e^{3x+1} dx = \frac{e^{(3x+1)}}{9} + K$ **h)** $\int (x+2)^{3/2} dx = \frac{2}{5}(x+2)^{5/2} + K$

i) $\int 2x\sqrt{3x^2 - 1}\, dx = \frac{2}{9}\left(3x^2-1\right)^{3/2} + K$ **j)** $\int \cos x\, senx\, senx\, dx = \frac{(senx)^3}{3} + K$

k) $\int \frac{x}{\left(x^2-1\right)^3} dx = \frac{-1}{4}\frac{1}{\left(x^2-1\right)^2} + K$ **l)** $\int x^4 e^{x^5} dx = \frac{e^{x^5}}{5} + K$

7) a) $\int \frac{x}{\sqrt[3]{x}} dx = \frac{3}{5} x^{5/3} + K$ **b)** $\int \frac{\ln x}{x^3} dx = \frac{-\ln x}{2x^2} - \frac{1}{4x^2} + K$

c) $\int x^2 \ln x\, dx = \frac{x^3}{3}\ln x - \frac{x^3}{9} + K$ **d)** $\int 2x\cos x\, dx = 2x senx + 2\cos x + K$

e) $\int x^2 \cos x\, dx = x^2 senx + 2x\cos x - 2senx + K$ **f)** $\int \frac{x}{e^x} dx = \frac{-x}{e^x} - e^{-x} + K$

g) $\int \left(x^3 - 2x\right)(x-3)\, dx = \frac{x^5}{5} - \frac{3x^4}{4} - \frac{2x^3}{3} + \frac{6x^2}{2} + K$

h) $\int 6sen(3x-2) dx = -2\cos(3x-2) + K$

i) $\int_0^1 \left(\sqrt{x}+5x^4\right) dx = \frac{5}{3}$

j) $\int \left(e^x + x + 3\right) dx = e^x + \frac{x^2}{2} + 3x + K$

k) $\int 3x\sqrt{3x^2+7}\,dx = \frac{1}{3}\left(3x^2+7\right)^{3/2} + K$

l) $\int 2x\,e^{3x^2+1}\,dx = \frac{1}{3}e^{3x^2+1} + K$

m) $\int \frac{2x}{\left(x^2-3\right)^4}\,dx = \frac{-1}{3\left(x^2-3\right)^3} + K$

n) $\int x\sqrt{2x^2-3}\,dx = \frac{1}{6}\left(2x^2-3\right)^{3/2} + K$

ñ) $\int_0^1 \left(\sqrt{x}+2e^x+x^3\right) dx = \frac{-13}{12} + 2e + K$