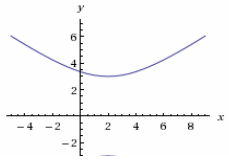
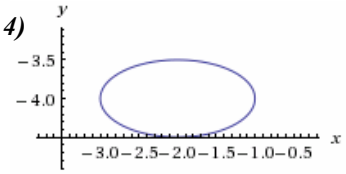
**Respuestas Secciones Cónicas**

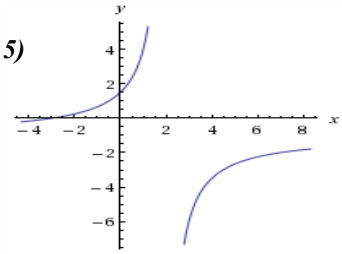
*Colaboración: María Virginia Frontini*
*Santiago Brun*

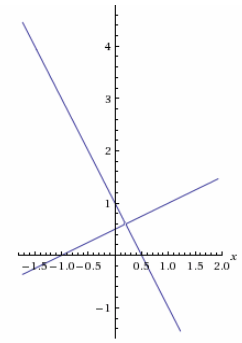
***Actividad 1***: Cuando el plano que corta a la superficie es perpendicular al eje y pasa por el vértice, se obtiene un punto.
Cuando el plano que corta a la superficie contiene a una de las generatrices se obtiene una recta.
Cuando el plano que corta a la superficie contiene al eje se obtienen dos retas que se intersecan en el vértice de la cónica.

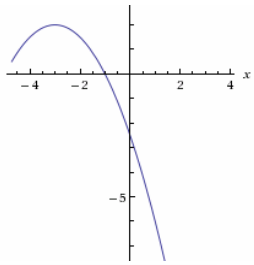
***Actividad Opcional:***

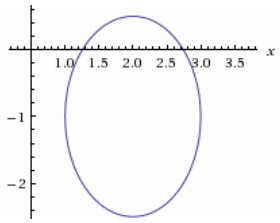
***1)*** ∅ ***2)***  ***3)***

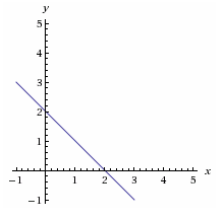
***4)*** 

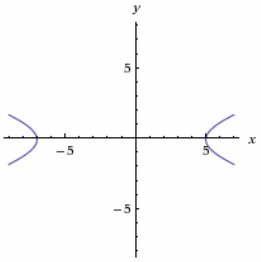
***5)*** 

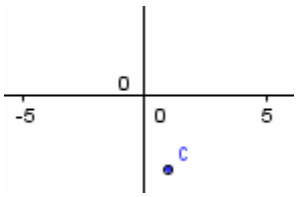
***6)*** 

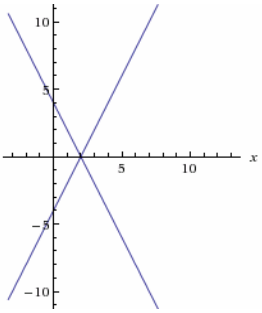
***7)*** 

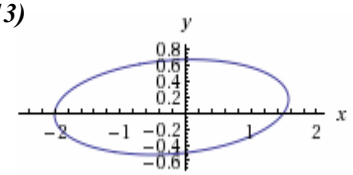
***8)*** 

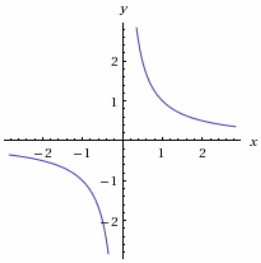
***9)*** 

***10)*** 

***11)*** 

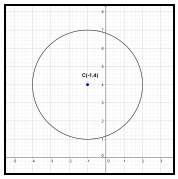

***12)*** 

***13)*** 

***14)*** 

## Respuestas actividades Circunferencia:

***Actividad 2:***



***Actividad 3:***

a) $(x-1)^2+(y+\frac{1}{2})^2=-\frac{19}{2}$ No representan a ningún lugar geométrico b) $(x+1)^2+y^2=0$ Representa al punto (-1,0).

***Actividad 4:***

1) $x^2+(y-3)^2=4$ $2)(x-1)^2+(y-6)^2=25$ $3)(x-1)^2+y^2=16$

4) $(x-1)^2+(y+1)^2=10$ 5) $(x+\frac{11}{14})^2+(y-\frac{55}{14})^2=\frac{845}{98}$

6) Son los puntos que pertenecen a una circunferencia de centro en (-1, 2) y radio $\sqrt{5}$.

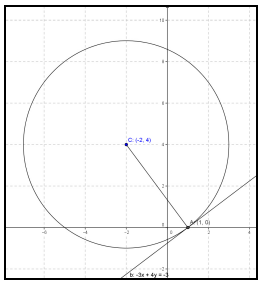
7) El conjunto solución es vacío.

8) Existen dos soluciones: $(x+5)^2+(y+5)^2=25$ y $(x+13)^2+(y+13)^2=169$.

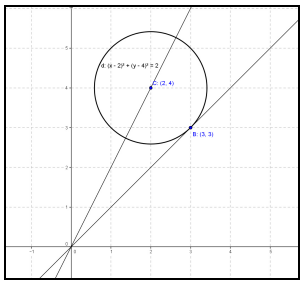
9) $x^2+(y-b)^2=1+b^2$, centro *(0,b)* y radio $1+b^2$.

***Actividad 5:***

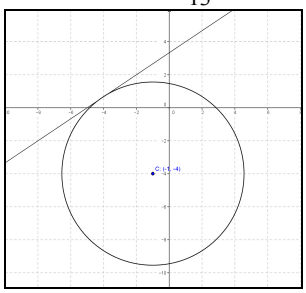
1) *3 x – 4 y -3 = 0*



2) $(x-2)^2+(y-4)^2=2$

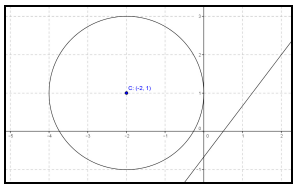


3) $(x+1)^2+(y+4)^2=\frac{400}{13}$



***Actividad 6:***

1) No hay intersección



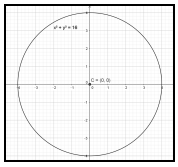
2) Secante $|k|<5$, tangente k =5, exterior $|k|>5$.

3) $r<\sqrt{8}$

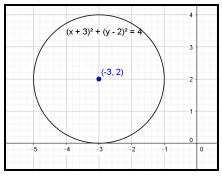
***Actividad 7:***

1) $\begin{cases} x=1+4\cos t \\ y=-2+4\,sent \end{cases}$ $0\le t<2\pi$

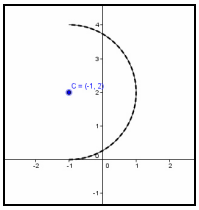
2) *a)* $x^2+y^2=16$



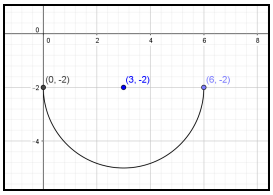
*b)* $(x+3)^2+(y-2)^2=4$



3) *a)* $(x+1)^2+(y-2)^2=4$



*b)* $(x-3)^2+(y+2)^2=9$



***Actividad complementaria:***

1) Dos soluciones: $x^2+(y-2)^2=16$; y $(x^2-48)^2+(y+34)^2=16$

2) a) tangente: $a=\pm\sqrt{40}$ b) secante: $|a|<\sqrt{40}$ c) exterior: $|a|>\sqrt{40}$

3) $(x-2)^2+(y+1)^2=17$

4) El área de la región cubierta por el césped es 62493 π cm2

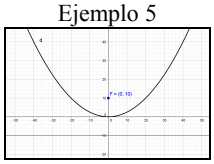
5) Área de la corona circular : 4 π cm2

6) Área círculo: 625 π cm2

## Respuestas actividades Parábola:

***Actividad 9:***

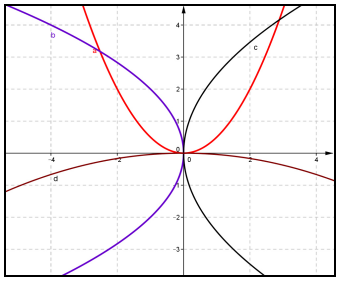
Ejemplo 5



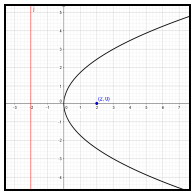
Ejemplo 6

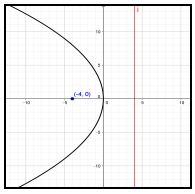
***Actividad 10:***

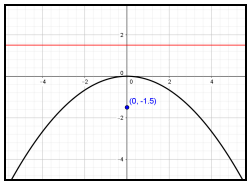
1) a) *F(0, ½) d) y= - ½* b) *F(-1,0) d) x= 1* c) *F(3/2, 0) d) y=-3/2* d) *F(0,-6) d) y=6 ½*



2) a) $y^2 = 8x$ b) $y^2 = -16x$ c) $x^2 = -6y$



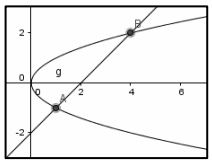




3) $x^2 = -6y$

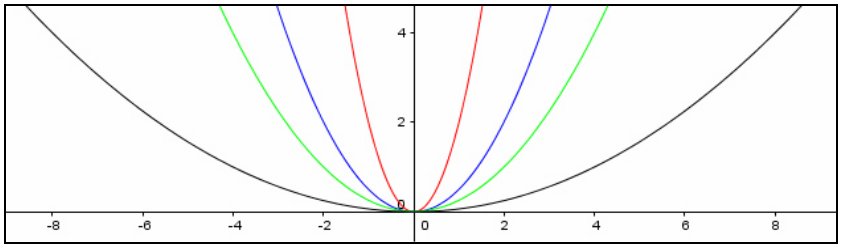
4) $(3, \pm 2\sqrt{3})$

5) A (1,-1) B (4,2)



6) Ecuación de parábola con foco en $F_1(0,1/8)$: $x^2 = \frac{1}{2}y$ ; Ecuación de parábola con foco en $F_2(0,1/2)$: $x^2 = 2y$

Ecuación de parábola con foco en $F_3(0,1)$: $x^2 = 4y$ ; Ecuación de parábola con foco en $F_4(0,4)$: $x^2 = 16y$



Cuanto más cerca esté el foco del vértice, más se contraen las ramas de la parábola.

7) Dos soluciones $x^2 = y$ e $y^2 = 8x$

***Actividad 11:***

Foco*: F(h+p,k)* Diretriz*: r) x=h-p* Por definición de parábola:

$$d(P,F) = d(P,r) \Leftrightarrow \sqrt{(x-(h-p))^2 + (y-k)^2} = |x-(h-p)|$$

Elevando ambos miembros al cuadrado, se obtiene:

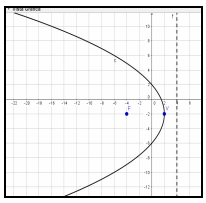
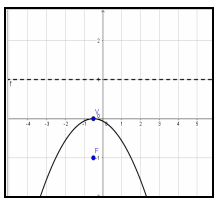
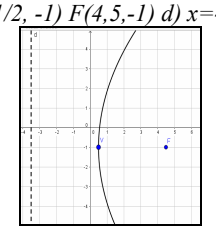
$$(x-(h-p))^2 + (y-k)^2 = (x-(h-p))^2$$

Desarrollando:

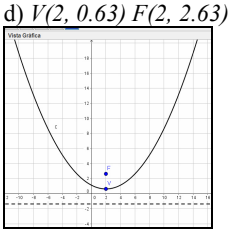
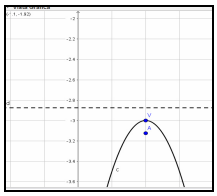
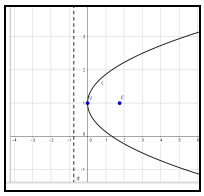
$$x^2 - 2x(h+p) + (h+p)^2 = x^2 - 2x(h-p) + (h-p)^2$$
$$(y-k)^2 = 2xp - 2hp + 2xp - 2hp$$
$$(y-h)^2 = 4p(x-h)$$

***Actividad 12:***

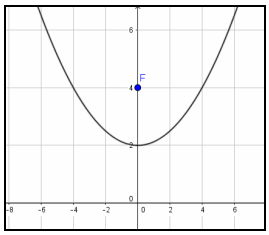
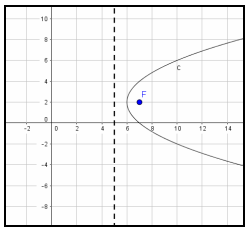
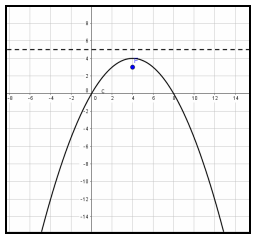
1) a) *V(2,-2) F(4,-2) d) x=4* b) *V(-1/2, 0) F(-1/2,-1) d) y=1* c) *V(1/2, -1) F(4,5,-1) d) x=-3*

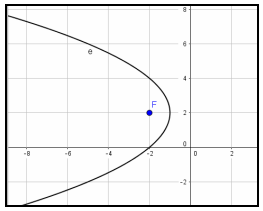
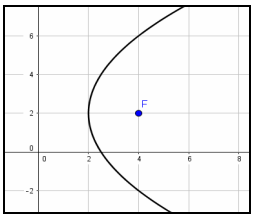
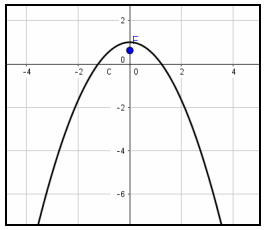
d) *V(2, 0.63) F(2, 2.63) d) y=-1.37* e) *V(1,-3) F(1-3.13) d) y=-2.88* f) *V(0,1) F(3/16,1) d) x=-3/4*

2) a) $x^2 = 8(y-2)$ b) $(y-2)^2 = 4(x-6)$ c) $(x-4)^2 = -4(y-4)$

d) $(y-2)^2 = -4(x+1)$ e) $(y-2)^2 = 8(x-2)$ f) $x^2 = -\frac{3}{2}(y-1)$

3) $(y+2)^2 = -(x-4)$

***Actividad 13:***

1) a) dos rectas paralelas b) una recta c) conjunto vacío d) parábola

2) $Ax^2 + Dx + Ey + F = 0$

Sacando factor común $A$: $x^2 + \frac{D}{A}x + \frac{E}{A}y + \frac{F}{A} = 0$

Completando cuadrados en la variable $x$: $\left(x + \frac{D}{2A}\right)^2 - \frac{D^2}{4A^2} + \frac{E}{A}y + \frac{F}{A} = 0;\ \left(x + \frac{D}{2A}\right)^2 = \frac{D^2}{4A^2} - \frac{E}{A}y - \frac{F}{A}$

,

Sacando factor común $-\frac{E}{A}$: $\left(x + \frac{D}{2A}\right)^2 = \frac{-E}{A\left(y + \left(\frac{F}{E} - \frac{D^2}{2EA}\right)\right)}$

$$\left(x + \frac{D}{2A}\right)^2 = \frac{-E}{A\left(y + \left(\frac{4AF - D^2}{4EAE}\right)\right)}$$

Vértice: $\left(\frac{-D}{2A}, \frac{D^2 - 4AF}{4AE}\right)$, Eje de simetría: $x = -D/2A$

***Actividad Complementaria***

1) 2,5 m del vértice.

2) La profundidad deberá ser $\frac{\sqrt{2}}{4}$ m.

3) La distancia que más se acercaran será de 25 millones de millas.

4) Tirante 1: 19,2 m, Tirante 2: $\frac{278}{15}$ m, Tirante 3: 18 m, Tirante 4: 17,6 m, Tirante 5: $\frac{52}{3}$ m

Tirante 6: 17,2 m, Tirante 7: 17,2 m, Tirante 8: $\frac{52}{3}$ m, Tirante 9: 17,6 m, Tirante 10: 18 m

Tirante 11: $\frac{278}{15}$ m, Tirante 12: 19,2 m, Tirante 13: 20 m, Tirante 14: $\frac{314}{15}$ m

5) La longitud de las péndolas, de izquierda a derecha, cada 50 pies son:

Péndola 1: 67,6 pies, Péndola 2: 42,4 pies, Péndola 3: 24,4 pies, Péndola 4: 13,6 pies

Péndola 5: 10 pies, Péndola 6: 13,6 pies, Péndola 7: 24,4 pies, Péndola 8: 42,4 pies, Péndola 9: 67,6 pies

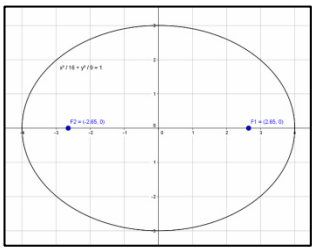
6) $\begin{cases} x = h + t \\ y = k + \frac{1}{4p}t^2 \end{cases} \quad t \in \Re$

7) $\begin{cases} x = \frac{1}{4}t^2 \\ y = k + t \end{cases} \quad t \in [0,6]$

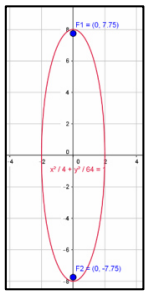
## Respuestas actividades Elipse:

***Actividad 18:***

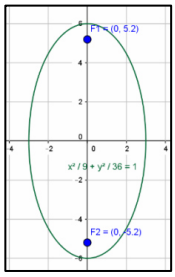
1) a) vértices: $(\pm 4,0)$
focos: $(\pm\sqrt{7},0)$
excentricidad: $\frac{\sqrt{7}}{4}$,
eje mayor: 8, semieje menor: 3



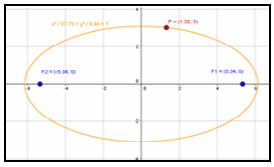
b) vértices: $(0,\pm 8)$
focos: $(0,\pm\sqrt{60})$
excentricidad: $\frac{\sqrt{15}}{4}$
eje mayor: 16, semieje menor: 2



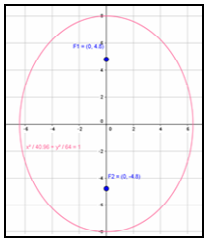
c) vértices: $(0,\pm 6)$
focos: $(0,\pm 3\sqrt{3})$
excentricidad: $\frac{\sqrt{3}}{2}$
eje mayor: 12, semieje menor: 3



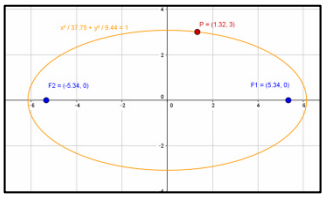
2) a) $\frac{x^2}{151/4} + \frac{y^2}{151/16} = 1$



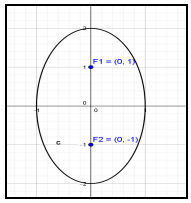
b) $\frac{x^2}{1024/254} + \frac{y^2}{64} = 1$



c) $\frac{x^2}{66/18} + \frac{y^2}{22} = 1$



d) $x^2 + \frac{y^2}{4} = 1$

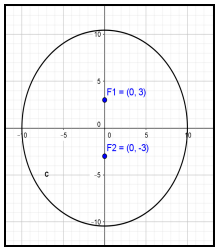


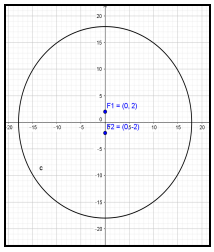
e) $\frac{x^2}{100} + \frac{y^2}{109} = 1$

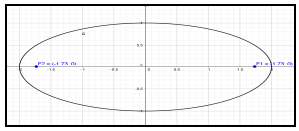
f) $\frac{x^2}{320} + \frac{y^2}{324} = 1$

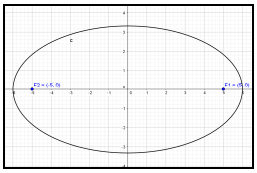
h) $\frac{x^2}{4} + y^2 = 1$

g) $\frac{x^2}{36} + \frac{y^2}{11} = 1$

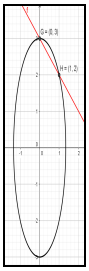


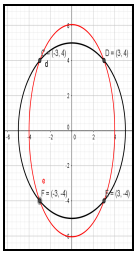



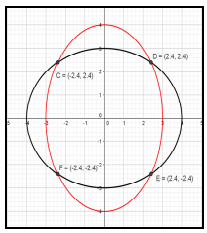





3) a) (0,3) y (1,2)  b) (-3,4), (3,4), (3,-4) y (-3,-4)  c) (-2,4, 2,4), (2,4, 2,4),(2,4, -2,4),(-2,4,- 2,4)

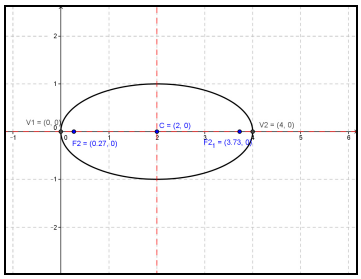




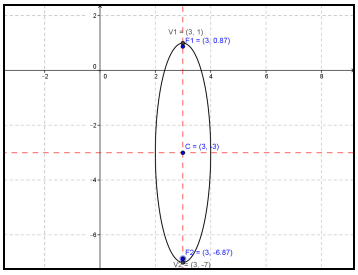


***Actividad 20:***

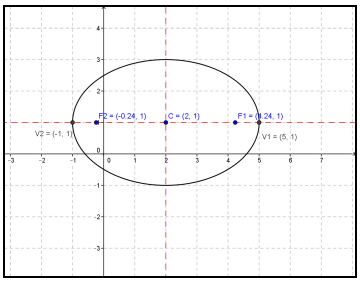
1) a) Centro (2,0), Vértices $(2 \pm 2,0)$,
Focos $\left(2 \pm \sqrt{3},0\right)$, excentricidad $\frac{\sqrt{3}}{2}$



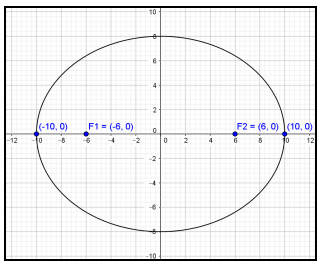

b) Centro (3,-3), Vértices $(3,-3 \pm 4)$,
Focos $\left(3,-3 \pm \sqrt{15}\right)$, excentricidad $\frac{\sqrt{15}}{4}$



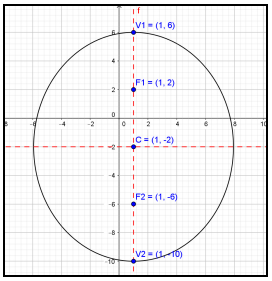

c) Centro (2,1), Vértices $(2 \pm 3,1)$,
Focos $\left(2 \pm \sqrt{5},1\right)$, excentricidad $\frac{\sqrt{5}}{3}$



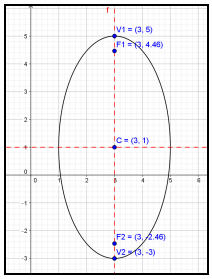

2) a) $\frac{x^2}{100}+\frac{y^2}{64}=1$



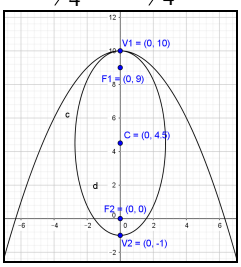

b) $\frac{(x-1)^2}{16}+\frac{(y+2)^2}{12}=1$



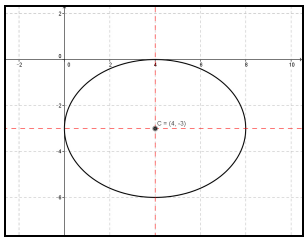

c) $\frac{(x-3)^2}{4}+\frac{(y-1)^2}{16}=1$




d) $\frac{x^2}{30/4}+\frac{(y-9/2)^2}{121/4}=1$



3) $\frac{(x-4)^2}{16}+\frac{(y+3)^2}{9}=1$



***Actividad 21:***

1) a) punto de coordenadas (1, -2) b) conjunto vacío c) elipse de ecuación $2(x+\frac{3}{2})^2+(y+1)^2=\frac{91}{2}$

d) $x^2+\frac{(y-1)^2}{2}=1$ e) conjunto vacío e) punto de coordenadas (1,0)

2) a) F < 17 b) F = 17 c) F < 17

***Actividad Complementaria***

1) longitud del hilo $\approx \pi\, 2{,}5$ m. distancia entre tachuelas $\approx$ 3.29 m

2) $e \approx 0.00167$ longitud eje mayor $\approx 299.25$ longitud eje menor $\approx$ 299.21

3) $e \approx 0.067$ longitud del eje mayor $\approx 763.11$ longitud eje menor $\approx$ 761.45

4) $\frac{x^2}{25^2}+\frac{(y-20)^2}{10^2}=1$

5) $\frac{x^2}{1942^2}+\frac{y^2}{1939^2}=1$

6) Reemplazando $\frac{x}{a}=\cos t;\ \frac{y}{b}=sent$ elevando cada término al cuadrado y sumando se llega a $\frac{x^2}{a^2}+\frac{y^2}{b^2}=\cos^2 t+sen^2 t=1$

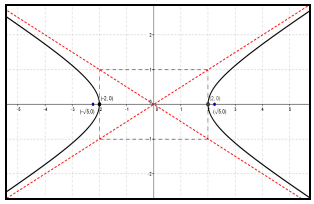
7) a) Focos en $\left(0,\pm\frac{4}{3}\sqrt{2}\right)$ b) Focos en $\left(\pm\sqrt{1},0\right)$

## Respuestas actividades Hipérbola:

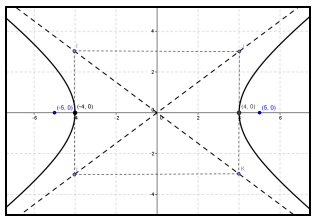
***Actividad 24***

1)

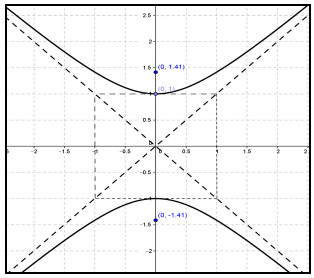
a) Vértices $(\pm 2,0)$, Focos $\left(\pm\sqrt{5},0\right)$, asíntotas $y=\pm\frac{1}{2}x$, excentricidad $\frac{\sqrt{5}}{2}$



b) Vértices $(\pm 4,0)$, Focos $(\pm 5,0)$, asíntotas $y=\pm\frac{3}{4}x$, excentricidad $\frac{5}{4}$

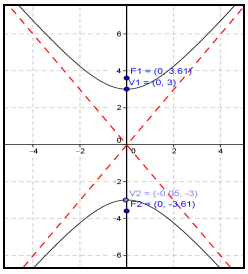


c) Vértices $(0,\pm 1)$, Focos $\left(0,\pm\sqrt{2}\right)$, asíntotas $y=\pm x$, excentricidad $\sqrt{2}$

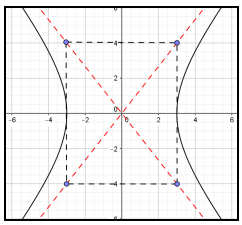
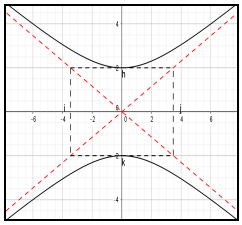
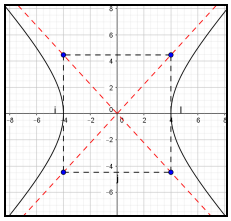
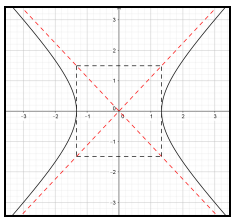


d) Vértices $(0,\pm 3)$, Focos $\left(0,\pm\sqrt{13}\right)$,

asíntotas $y=\pm\frac{3}{2}x$, excentricidad $\sqrt{13}/2$

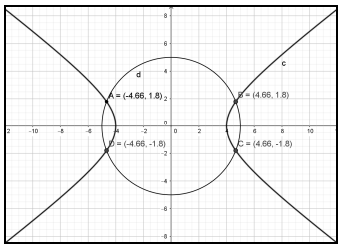



2) a) $\frac{x^2}{9}-\frac{y^2}{16}=1$ b) $\frac{y^2}{4}-\frac{x^2}{12}=1$ c) $\frac{x^2}{16}-\frac{y^2}{36}=1$ d) $\frac{x^2}{16/9}-\frac{y^2}{20/9}=1$

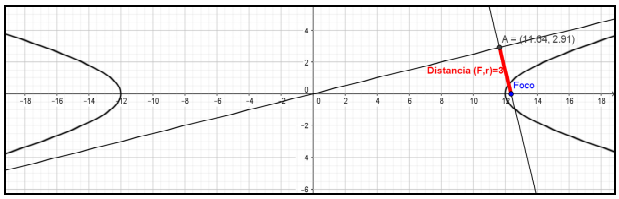





3) No hay puntos de intersección, la hipérbola no se cruza con la recta.

4)

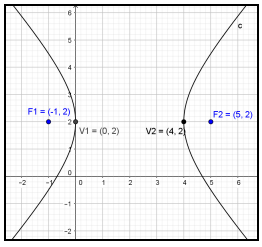


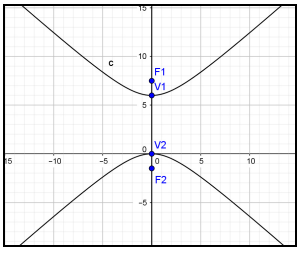

5 ) F $\left(3\sqrt{17},0\right)$, distancia=3

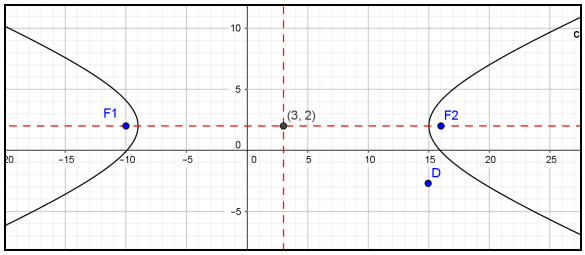



***Actividad 27***

1) a) $\frac{(x-2^2}{4}-\frac{(y-2)^2}{5}=1$ b) $\frac{(y-3)^2}{9}-\frac{x^2}{45/4}=1$ c) $\frac{(x-3)^2}{144}-\frac{(y-2)^2}{25}=1$



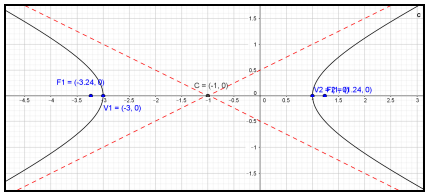





3) a) centro(-1,0), vertices ( -1±2,0),

focos $\left(-1\pm\sqrt{5},0\right)$, excentricidad $\frac{\sqrt{5}}{2}$,

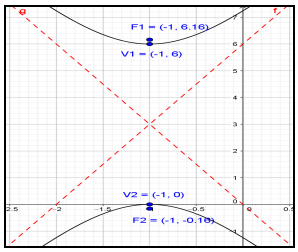
asíntotas $y=\pm\frac{1}{2}(x+1)$



b) centro(-1,3), vertices ( -1,3±3),

focos $\left(-1{,}3 \pm \sqrt{10}\right)$, excentricidad $\dfrac{\sqrt{10}}{3}$,

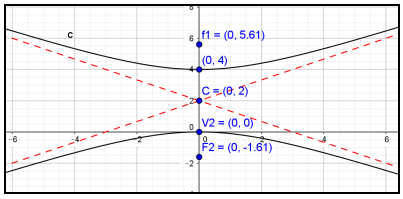
asíntotas $y = \pm 3(x+1)+3$




c) centro(0,2), vertices ( 0,2±2),

focos $\left(0{,}2 \pm \sqrt{13}\right)$, excentricidad $\dfrac{\sqrt{13}}{2}$,

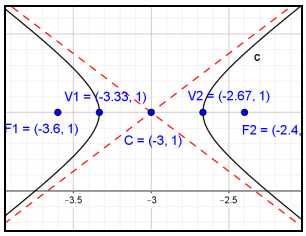
asíntotas $y = \pm\dfrac{2}{3}x + 2$




d) centro(-3,1), vertices ( $-3\pm\dfrac{1}{3}$,1),

focos $\left(-3 \pm \dfrac{\sqrt{13}}{6}, 1\right)$, excentricidad $\dfrac{\sqrt{13}}{2}$,

asíntotas $y = \pm\dfrac{3}{2}(x+3)+1$




***Actividad 28***

1) a) Par de rectas secantes que se cortan en el punto (3, 0), de ecuaciones $y = \dfrac{1}{3}x - 1,\ y = -\dfrac{1}{3}x + 1$.

b) $8(y-1)^2 - (x+2)^2 = 1$, hipérbola con centro en (-2,1), vértices $(-2, \frac{3}{2}), (-2, \frac{1}{2})$; focos $(-2, \frac{2 \pm \sqrt{5}}{2})$,

asíntotas: $y = \dfrac{1}{2}x + 2,\ y = -\dfrac{1}{2}x - 1$.

c) No representa a ningún punto.

d) Par de rectas secantes que se cortan en el punto (1, 3), de ecuaciones $y = -3x + 6; y = 3x$.

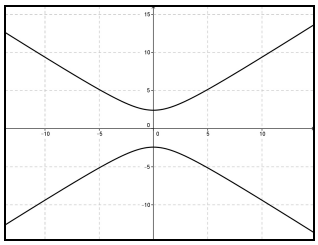
e) No representa a ningún punto.

f) $y^2 - x^2 = 1$, hipérbola con centro en (0,0), vértices en (0, $\pm$ 1), focos (0, $\pm\sqrt{2}$); asíntotas $y = \pm x$.

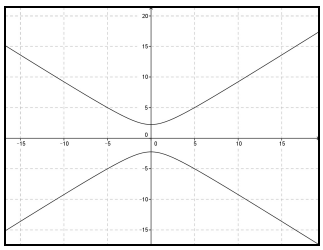
2) a) F<0 b) F>0 c) F=0.

***Actividad 29***

a) $\dfrac{(y+4)^2}{29/5} - \dfrac{x^2}{29/4} = 1$



b) $\dfrac{y^2}{5} - \dfrac{x^2}{25/4} = 1$



***Actividad Complementaria***

1) $\dfrac{(x-9)^2}{a^2} - \dfrac{y^2}{36-a^2}$ o $\dfrac{(x+9)^2}{a^2} - \dfrac{y^2}{36-a^2}$ ; $x = 40\,y^2$

2) $\dfrac{x^2}{40} - \dfrac{y^2}{(40/3)^2} = 1$

4) $\begin{cases} x = 2\sec(t) \\ y = tg(t) \end{cases}$ $t \in [0,2\pi] - \left\{\dfrac{\pi}{2}, \dfrac{3}{2}\pi\right\}$