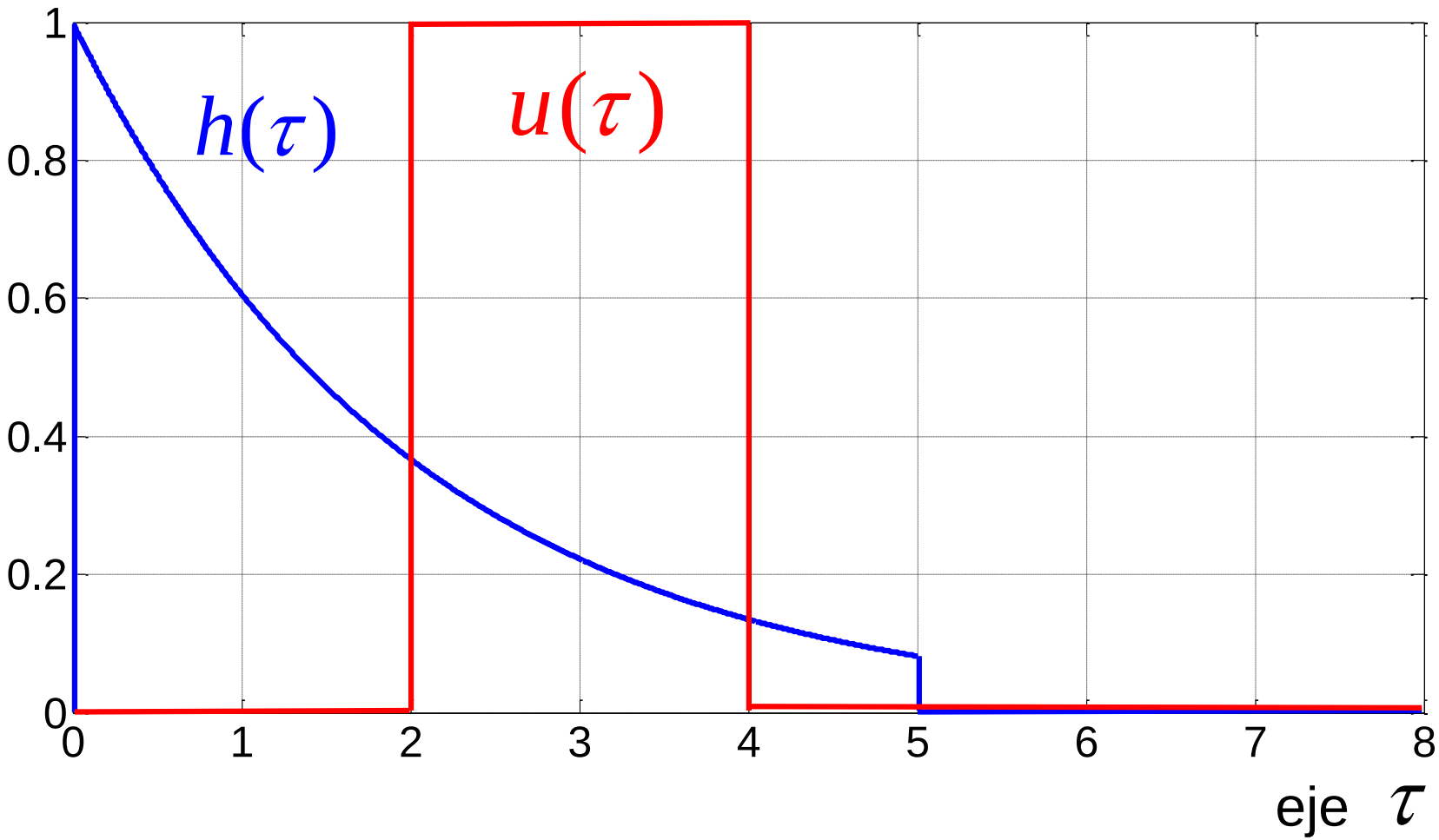
Problema resuelto: Convolución en TC

$$h(t) = e^{-at}\left(\mu(t) - \mu(t-5)\right) \quad , \quad a > 0$$

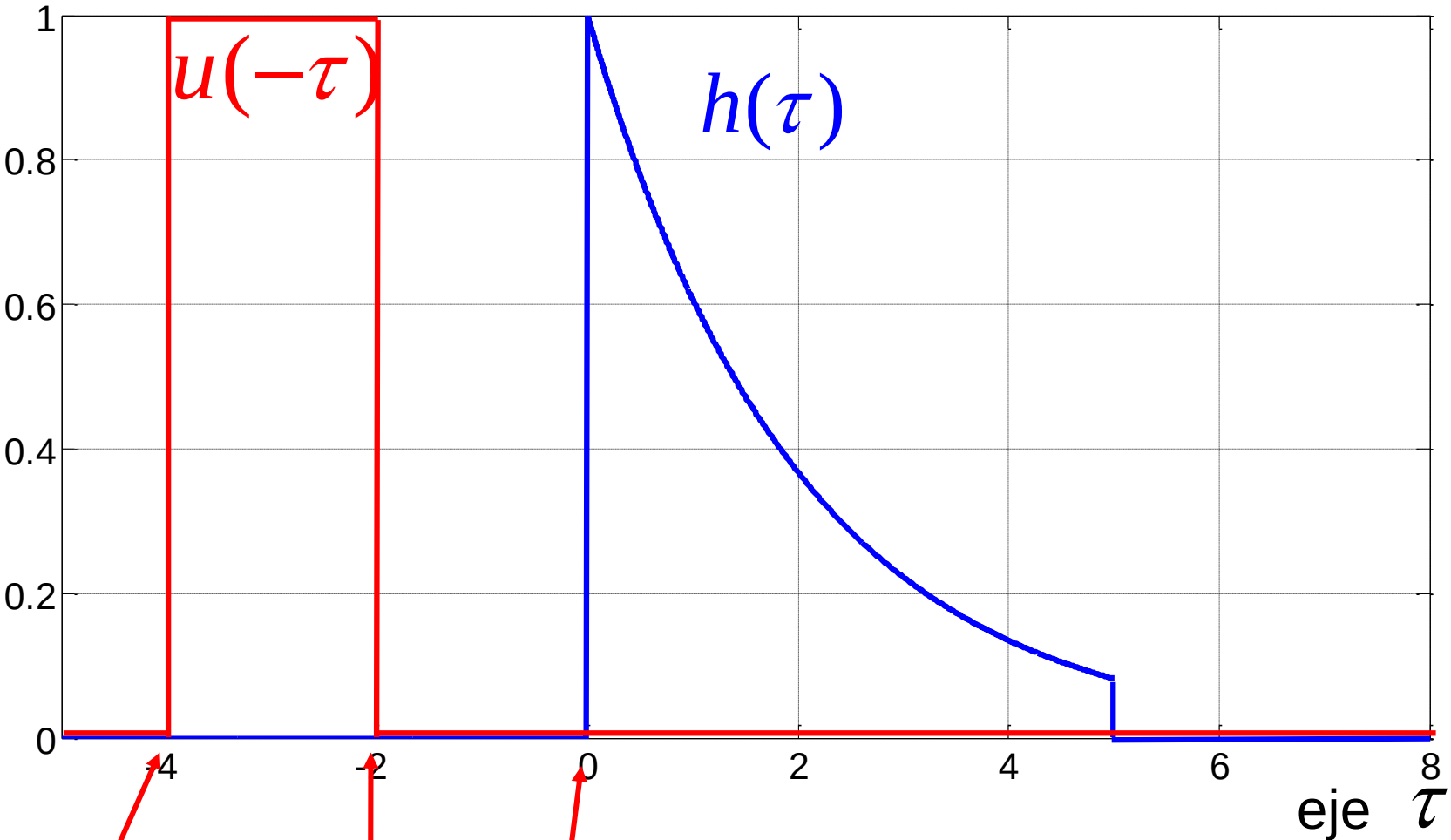
$$u(t) = \mu(t-2) - \mu(t-4)$$

Calcular

$$y(t) = h(t) * u(t)$$


$h(\tau)$
$u(\tau)$
1
0.8
0.6
0.4
0.2
0
0
1
2
3
4
5
6
7
8
eje τ

$t<2 \quad (t^*<0) \quad \Rightarrow y(t)=0$

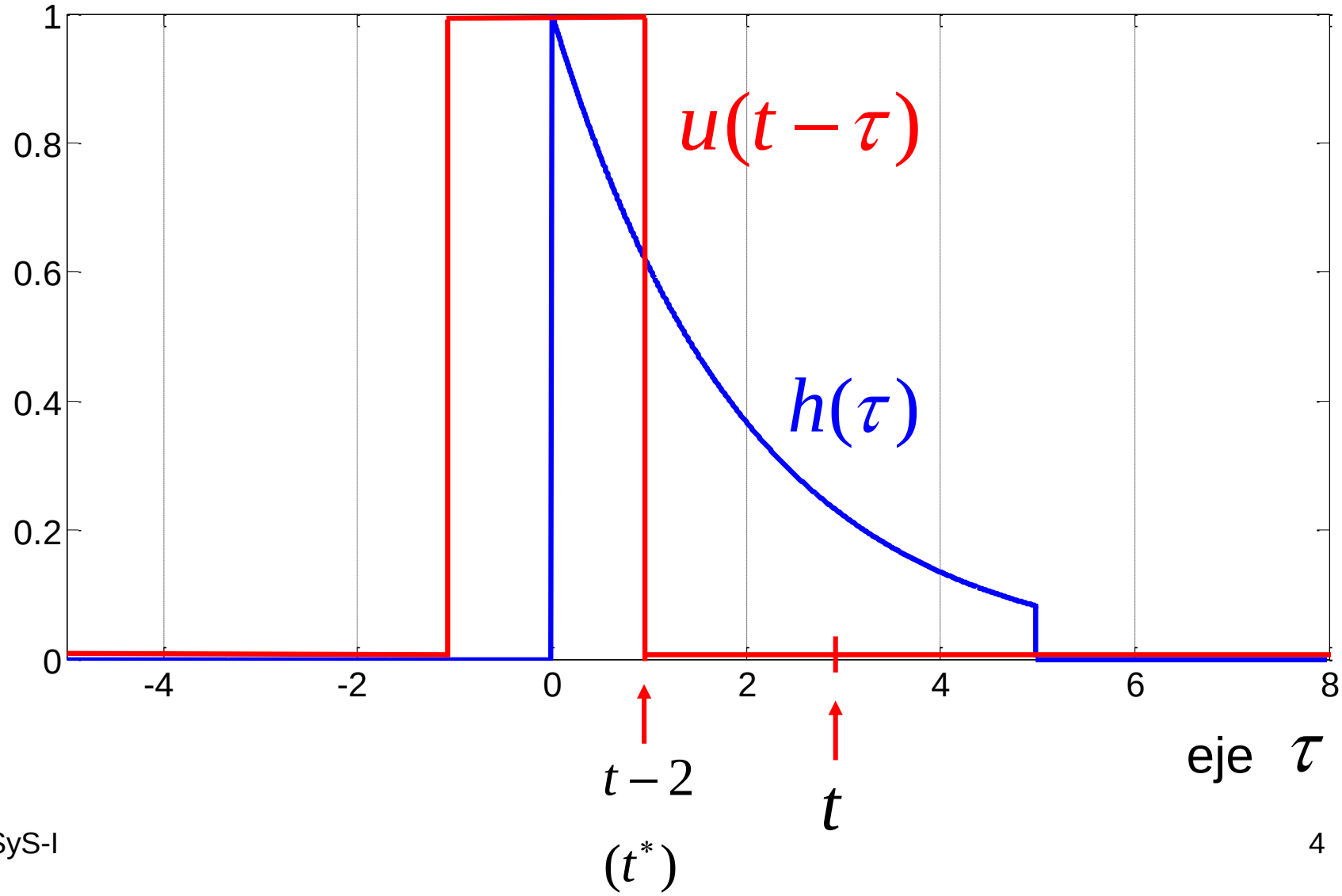


$t-4$ $\quad t-2$ $\quad t(=0)$

t^*-2 $\quad t^*$ $\quad t^*+2$

Reflejamos $u(\tau)$ para obtener $u(-\tau)$

$2 \le t < 4 \quad (0 \le t^* < 2)$



Para este caso resulta

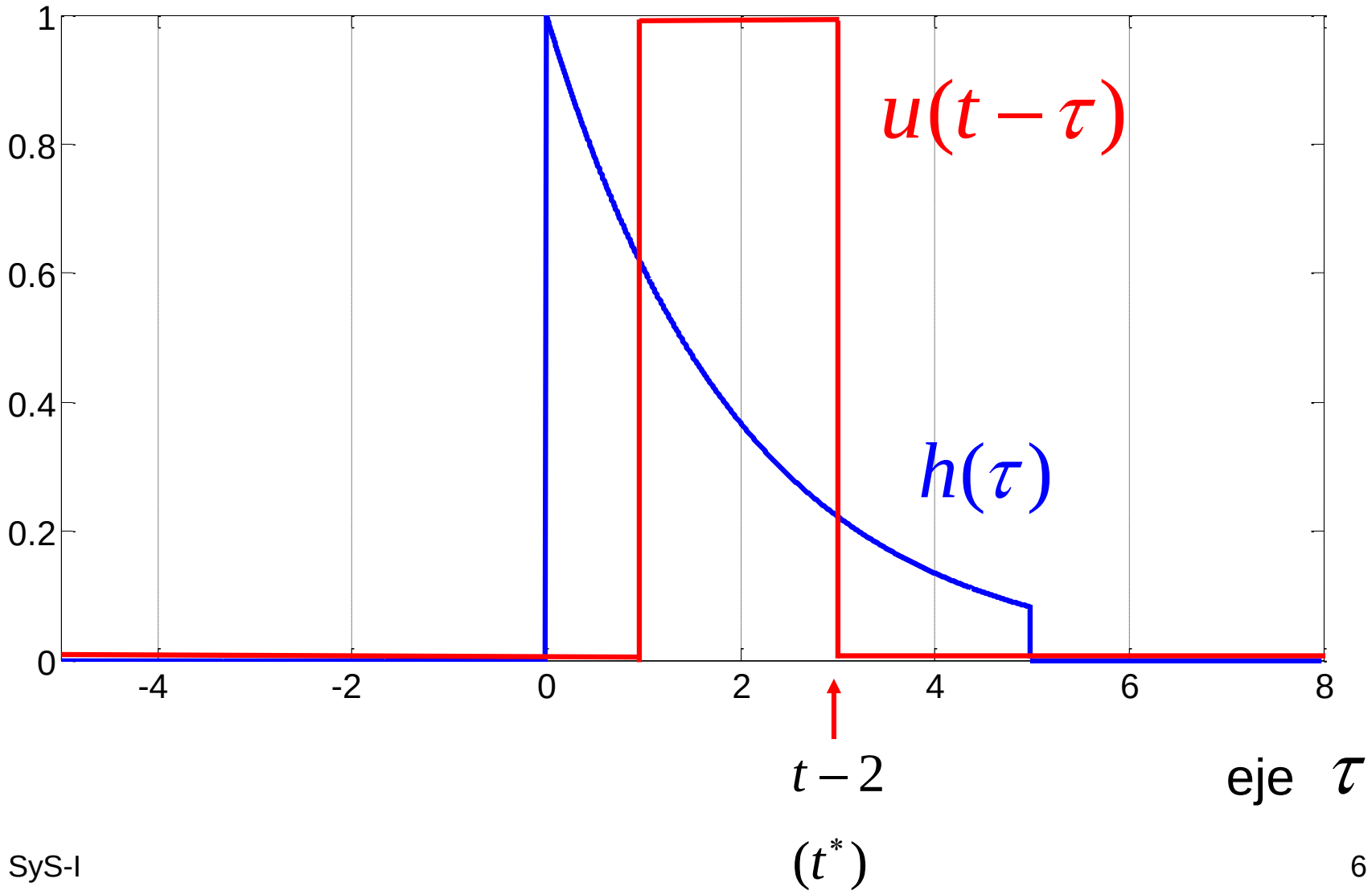
$$y(t) = \int_0^{t-2} e^{-a\tau} d\tau = -\frac{1}{a} e^{-a\tau} \Big|_0^{t-2} = \frac{1}{a}\left(1 - e^{-a(t-2)}\right)$$

Si lo calculamos en función de t^* resulta

$$y(t^*) = \int_0^{t^*} e^{-a\tau} d\tau = -\frac{1}{a} e^{-a\tau} \Big|_0^{t^*} = \frac{1}{a}\left(1 - e^{-at^*}\right)$$

que coincide con la expresión anterior ya que $t^* = t - 2$

$4 \le t < 7 \quad (2 \le t^* < 5)$



Para este caso resulta

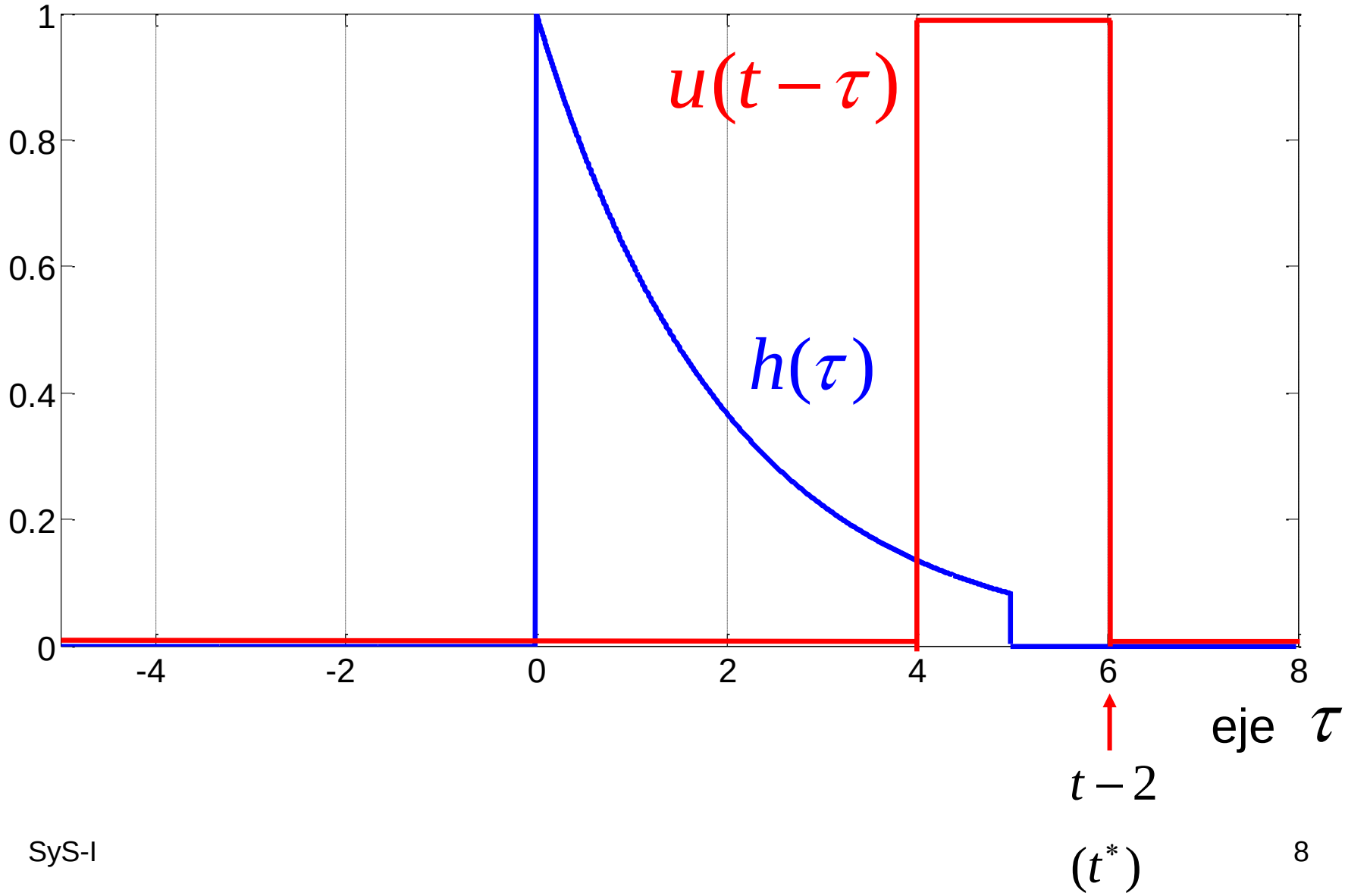
$$y(t) = \int_{t-4}^{t-2} e^{-a\tau} d\tau = -\frac{1}{a} e^{-a\tau} \Big|_{t-4}^{t-2} = \frac{1}{a}\left(e^{-a(t-4)} - e^{-a(t-2)}\right)$$

Si lo calculamos en función de t^* resulta

$$y(t^*) = \int_{t^*-2}^{t^*} e^{-a\tau} d\tau = -\frac{1}{a} e^{-a\tau} \Big|_{t^*-2}^{t^*} = \frac{1}{a}\left(e^{-a(t^*-2)} - e^{-at^*}\right)$$

que coincide con la expresión anterior ya que $t^* = t - 2$

$7 \le t < 9 \quad (5 \le t^* < 7)$



Para este caso resulta

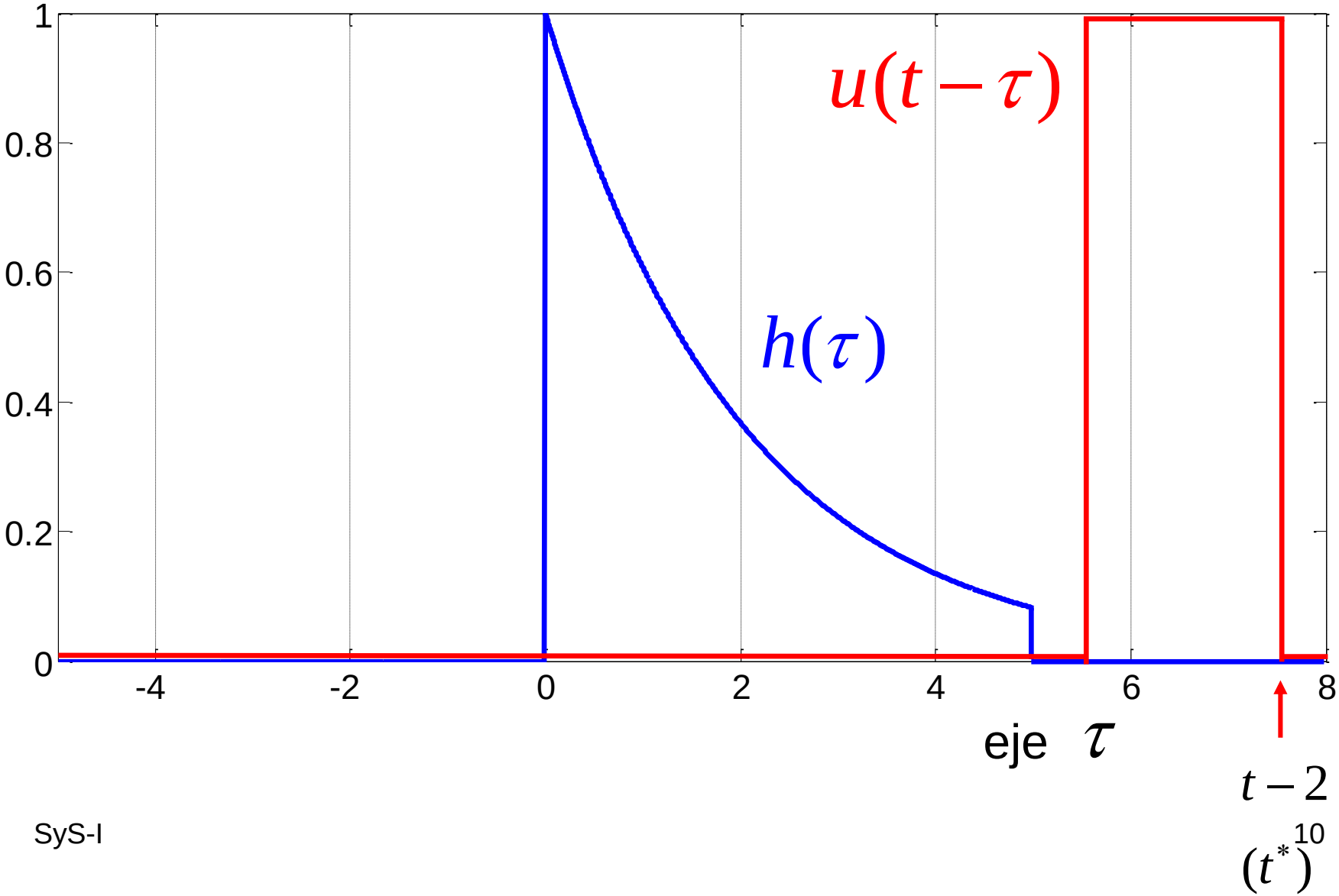
$$y(t) = \int_{t-4}^{5} e^{-a\tau} d\tau = -\frac{1}{a} e^{-a\tau} \Big|_{t-4}^{5} = \frac{1}{a}\left(e^{-a(t-4)} - e^{-a5}\right)$$

Si lo calculamos en función de t^* resulta

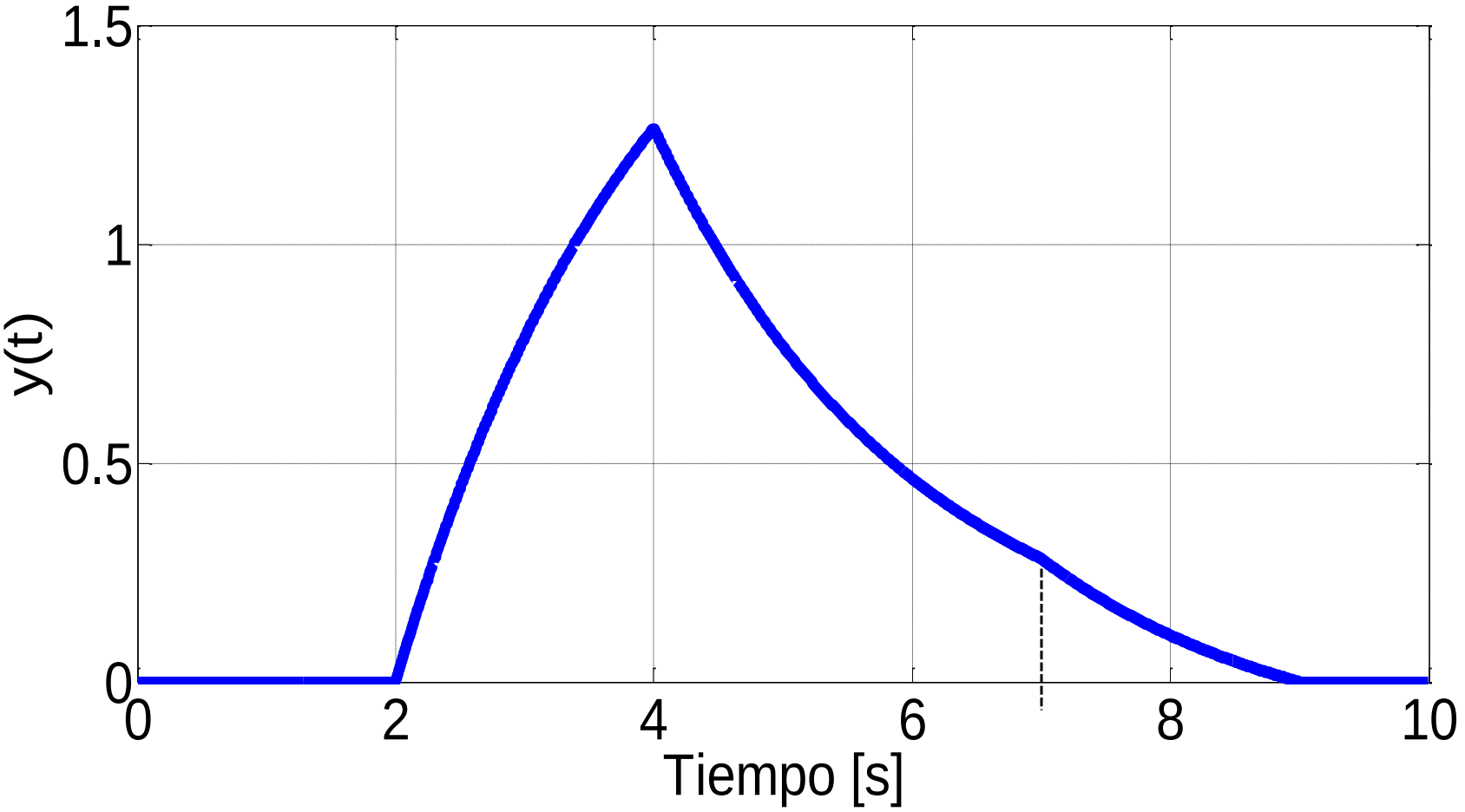
$$y(t^*) = \int_{t^*-2}^{5} e^{-a\tau} d\tau = -\frac{1}{a} e^{-a\tau} \Big|_{t^*-2}^{5} = \frac{1}{a}\left(e^{-a(t^*-2)} - e^{-a5}\right)$$

que coincide con la expresión anterior ya que $t^* = t - 2$

$$9 \le t \quad (7 \le t^*) \quad \Rightarrow y(t) = 0$$



$$y(t)=\begin{cases}0 & t<2\\ \frac{1}{a}\left(1-e^{-a(t-2)}\right) & 2\le t<4\\ \frac{1}{a}\left(e^{-a(t-4)}-e^{-a(t-2)}\right) & 4\le t<7\\ \frac{1}{a}\left(e^{-a(t-4)}-e^{-a5}\right) & 7\le t<9\\ 0 & 9\le t\end{cases}$$


1.5
1
0.5
0
y(t)
0
2
4
6
8
10
Tiempo [s]