**Serie 8 – Parte I - Problema 3**: Calcule y represente los espectros en magnitud y fase de las siguientes señales ( $a>0$ )

a. $$x(t)=\begin{cases} Ae^{-at} & t\geq 0 \\ 0 & t<0 \end{cases}$$

Se verifica

$$X(F)=\int_{-\infty}^{\infty} x(t)e^{-j2\pi Ft}dt$$

$$=A\int_{0}^{\infty} e^{-at}e^{-j2\pi Ft}dt = A\int_{0}^{\infty} e^{-(a+j2\pi F)t}dt$$

$$=-\frac{A}{(a+j2\pi F)}e^{-(a+j2\pi F)t}\Big|_{0}^{\infty}$$

$$=\frac{A}{(a+j2\pi F)}$$

Luego

$$|X(F)|=\frac{A}{\sqrt{a^2+4\pi^2F^2}}$$

$$\angle X(F)=-\arctan\left(\frac{2\pi F}{a}\right)$$

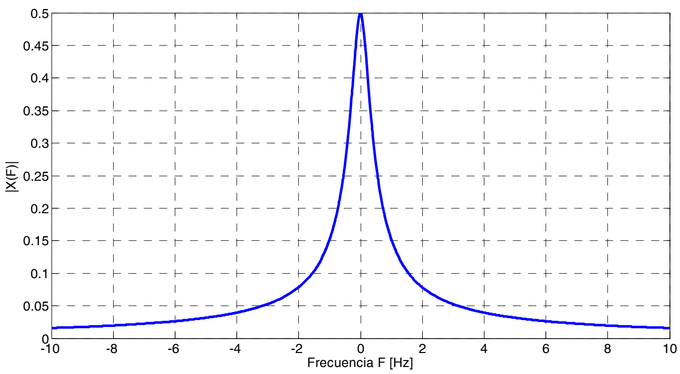


Figura 1: $|X(F)|$ vs. $F$, para $A = 1$ y $a=2$

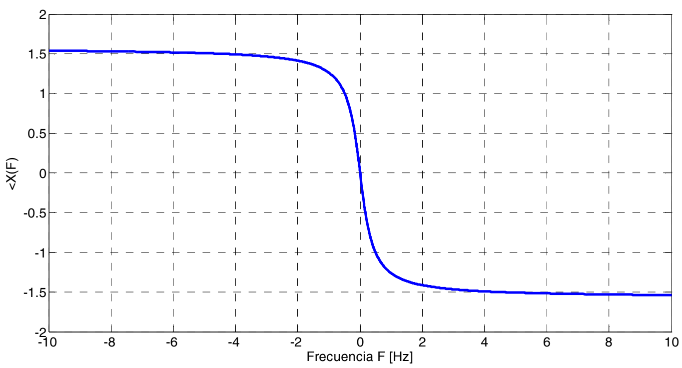


Figura 2: $\angle X(F)$ vs. $F$, para $A = 1$ y $a=2$.

**Serie 8 – Parte I - Problema 4:**
Considere la señal

$$x(t)=\begin{cases}1-\dfrac{|t|}{\tau} & |t|\le\tau\\ 0 & c.o.c.\end{cases}$$

a. Determine y dibuje sus espectros en magnitud y fase, $|X(F)|$ , $\angle X(F)$ respectivamente.

b. Cree una señal periódica $x_p(t)$ con período fundamental $T_p\ge 2\tau$ , de modo que $x(t)=x_p(t)\quad \text{si}\quad -\dfrac{T_p}{2}\le t\le\dfrac{T_p}{2}$. ¿Cuáles son los coeficientes de Fourier $c_k$ para la señal $x_p(t)$ ?

c. Usando los resultados de los apartados a y b, demuestre que $c_k$ es igual a $\dfrac{1}{T_p}X\left(\dfrac{k}{T_p}\right)$

a. Se verifica

$$\begin{aligned}X(F)&=\int_{-\infty}^{\infty}x(t)e^{-j2\pi Ft}dt\\&=\int_{-\tau}^{0}\left(1+\frac{t}{\tau}\right)e^{-j2\pi Ft}dt+\int_{0}^{\tau}\left(1-\frac{t}{\tau}\right)e^{-j2\pi Ft}dt\\&=\int_{-\tau}^{\tau}e^{-j2\pi Ft}dt+\frac{1}{\tau}\int_{-\tau}^{0}te^{-j2\pi Ft}dt-\frac{1}{\tau}\int_{0}^{\tau}te^{-j2\pi Ft}dt\end{aligned}$$

Considerando que $\int te^{at}dt=\dfrac{e^{at}}{a^2}(at-1)$, resulta

$$\begin{aligned}X(F)&=\frac{1}{\pi F}\frac{e^{j2\pi F\tau}-e^{-j2\pi F\tau}}{2j}\\&\quad-\frac{1}{\tau}\frac{e^{-j2\pi Ft}}{4\pi^2F^2}(-j2\pi Ft-1)\Big|_{-\tau}^{0}+\frac{1}{\tau}\frac{e^{-j2\pi Ft}}{4\pi^2F^2}(-j2\pi Ft-1)\Big|_{0}^{\tau}\\&=\frac{\sin(2\pi F\tau)}{\pi F}+\frac{1}{\tau}\frac{1}{4\pi^2F^2}-\frac{1}{\tau}\frac{e^{j2\pi F\tau}}{4\pi^2F^2}+\frac{1}{\tau}\frac{e^{j2\pi F\tau}}{4\pi^2F^2}j2\pi F\tau-\frac{1}{\tau}\frac{e^{-j2\pi F\tau}}{4\pi^2F^2}+\frac{1}{\tau}\frac{1}{4\pi^2F^2}-\frac{1}{\tau}\frac{e^{-j2\pi F\tau}}{4\pi^2F^2}j2\pi F\tau\\&=\frac{\sin(2\pi F\tau)}{\pi F}+\frac{1}{\tau}\frac{1}{2\pi^2F^2}-\frac{1}{\tau}\frac{1}{2\pi^2F^2}\cos(2\pi F\tau)-\frac{\sin(2\pi F\tau)}{\pi F}\\&=\frac{1}{\tau}\frac{1}{2\pi^2F^2}-\frac{1}{\tau}\frac{1}{2\pi^2F^2}\cos(2\pi F\tau)=\frac{1}{\tau}\frac{1}{2\pi^2F^2}\left(1-\cos(2\pi F\tau)\right)\\&=\frac{1}{\tau}\frac{1}{\pi^2F^2}\sin^2(\pi F\tau)\end{aligned}$$

Luego vemos que *X(F)* es real y positiva $\Rightarrow \angle X(F)=0$

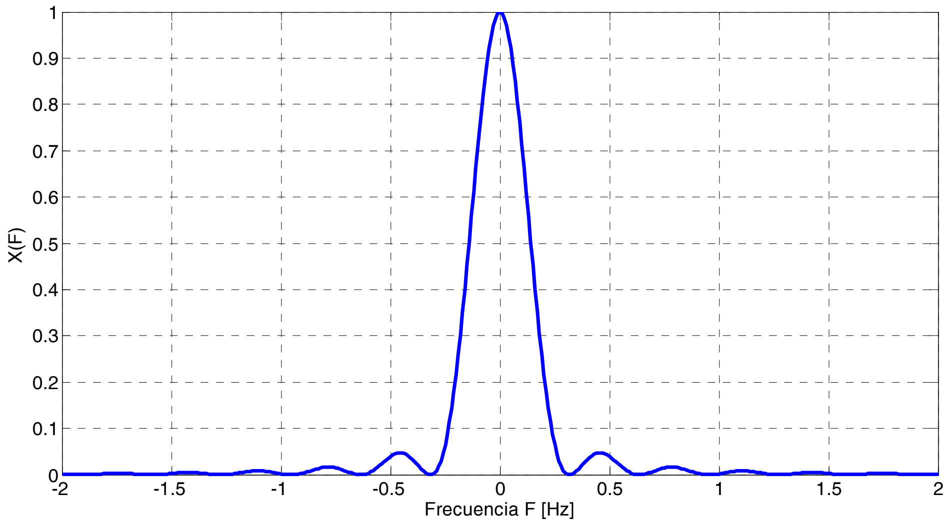


Figura 3: $X(F)$ vs. $F$ , para $\tau = 1$