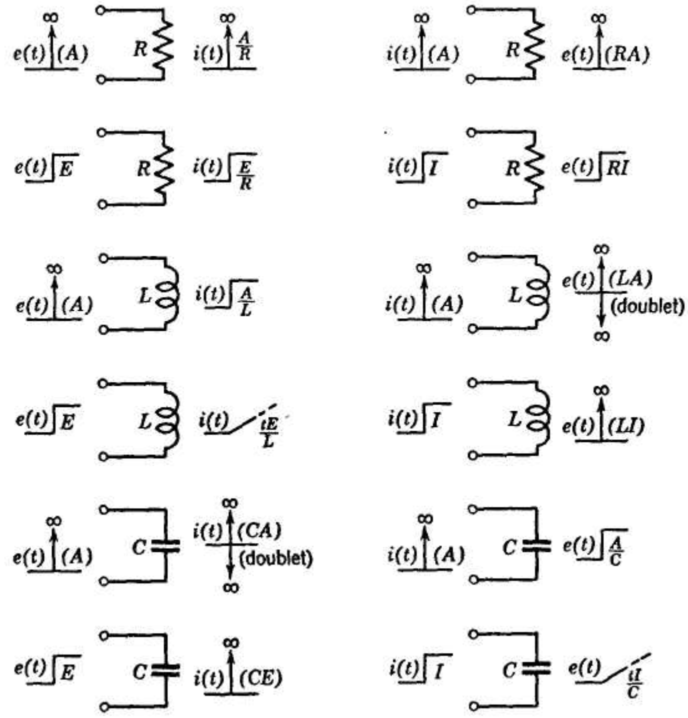
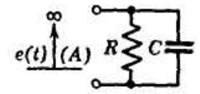
SUMMARY REGARDING THE TRANSIENT RESPONSE OF ONE-, TWO-, AND THREE-ELEMENT COMBINATIONS

A *Single Elements*

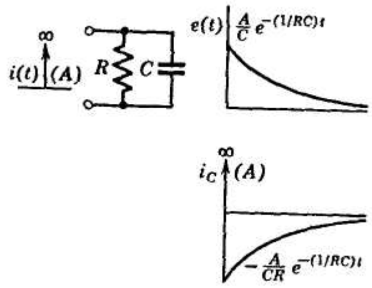
Voltage Source *Current Source*

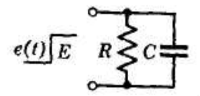


C *Two Elements—R, C*

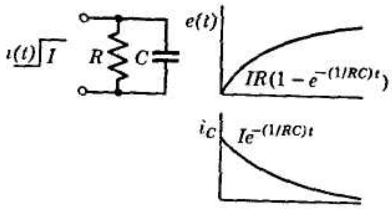


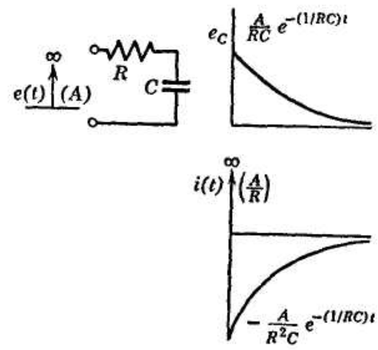
i_R, i_C same as in part (A)

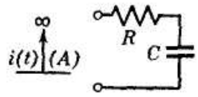




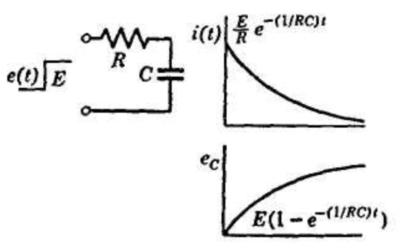
i_R, i_C same as in part (A)

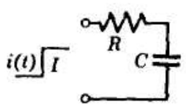






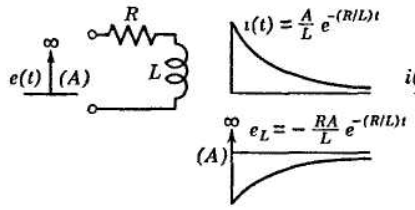
e_R, e_C same as in part (A)

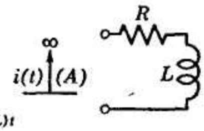




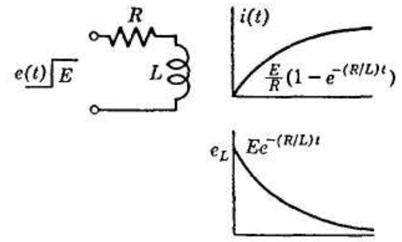
e_R, e_C same as in part (A)

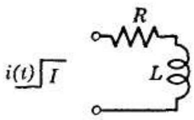
B *Two Elements—R, L*



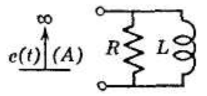


e_L, e_R same as in part (A)

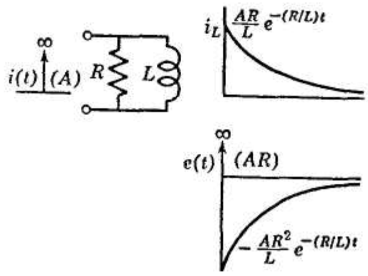


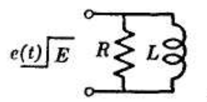


e_L, e_R same as in part (A)

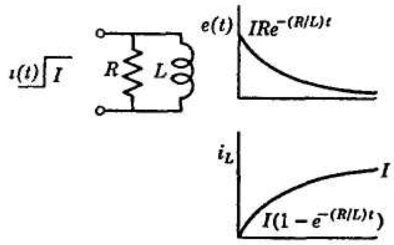


i_R, i_L same as in part (A)

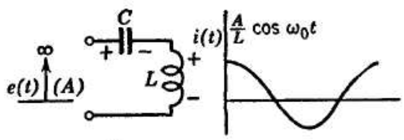


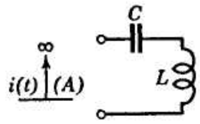


i_R, i_L same as in part (A)



D *Two Elements—L, C* $\omega_0 = 1/\sqrt{LC}$

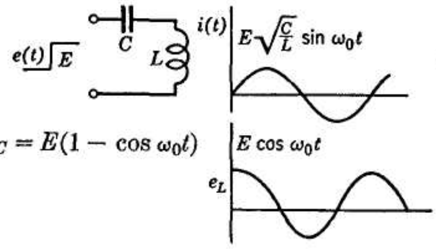




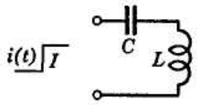
$$e_L = L\frac{di}{dt} = Au_0(t) - A\omega_0 \sin \omega_0 t$$

$$e_C = \frac{1}{C}\int i\,dt = A\omega_0 \sin \omega_0 t$$

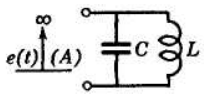
e_C, e_L same as in part (A)



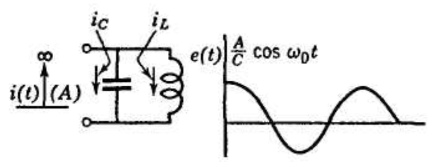
$e_C = E(1 - \cos \omega_0 t)$



e_C, e_L same as in part (A)

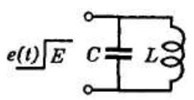


i_C, i_L same as in part (A)

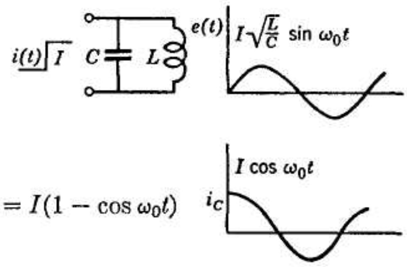


$$i_C = C\frac{de}{dt} = Au_0(t) - A\omega_0 \sin \omega_0 t$$

$$i_L = \frac{1}{L}\int e\,dt = A\omega_0 \sin \omega_0 t$$



i_C, i_L same as in part (A)

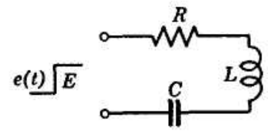


$i_L = I(1 - \cos \omega_0 t)$

E *Three Elements—R, L, C*

$(\omega_0 > \alpha)$

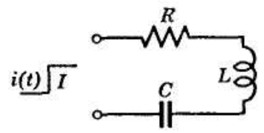
$$\omega_0 = \frac{1}{\sqrt{LC}} \qquad \alpha = \frac{R}{2L} \qquad \omega_d = \sqrt{{\omega_0}^2 - \alpha^2}$$



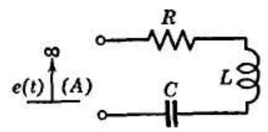
$$i(t) = E\sqrt{\frac{C}{L}}\frac{\omega_0}{\omega_d}e^{-\alpha t}\sin\omega_d t$$

when $\alpha \ll \omega_0$

$$i(t) \approx E\sqrt{\frac{C}{L}}e^{-\alpha t}\sin\omega_0 t$$

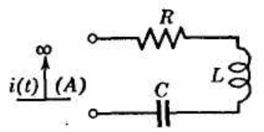


e_L, e_R, e_C as shown in part (A)

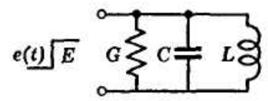


$$i(t) = \frac{A}{L}\frac{\omega_0}{\omega_d}e^{-\alpha t}\cos\left(\omega_d t + \cos^{-1}\frac{\omega_d}{\omega_0}\right)$$

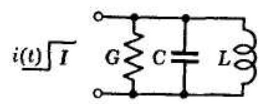
when $\alpha \ll \omega_0$; $i(t) \approx \dfrac{A}{L}e^{-\alpha t}\cos\omega_0 t$



e_L, e_R, e_C as shown in part (A)



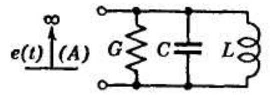
i_G, i_L, i_C as shown in part (A)



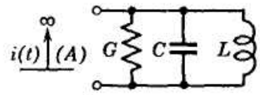
$$e(t) = I\sqrt{\frac{L}{C}}\frac{\omega_0}{\omega_d}e^{-\alpha t}\sin\omega_d t$$

when $\alpha \ll \omega_0$

$$e(t) \approx I\sqrt{\frac{L}{C}}e^{-\alpha t}\sin\omega_0 t$$



i_G, i_L, i_C as shown in part (A)



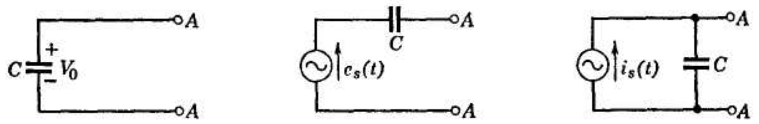
$$e(t) = \frac{A}{C}\frac{\omega_0}{\omega_d} e^{-\alpha t} \cos\left(\omega_d t + \cos^{-1}\frac{\omega_d}{\omega_0}\right)$$

when $\alpha \ll \omega_0$; $e(t) \approx \dfrac{A}{C} e^{-\alpha t} \cos \omega_0 t$

Note: When $\alpha \ll \omega_0$, it is necessary only to insert $e^{-\alpha t}$ as a multiplier to the results of part (D).

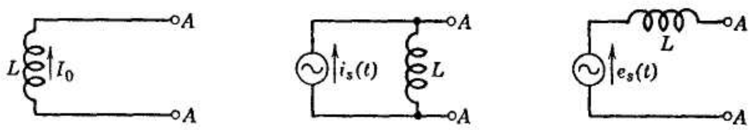
F *Initial Conditions*

1 *Initial voltages on condensers*



Charged condenser $e_s(t) = V_0 u_{-1}(t)$ in series $i_s(t) = CV_0 u_0(t)$ in parallel

2 *Initial currents in coils*

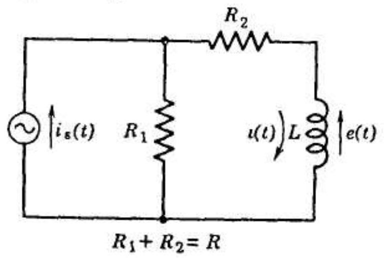


Initial current $i_s(t) = I_0 u_{-1}(t)$ in parallel $e_s(t) = LI_0 u_0(t)$ in series

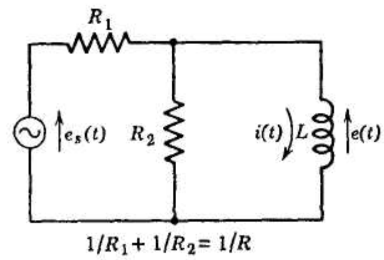
If other sources are present in the circuit solve by superposition.

PROBLEMS

1. For the circuit shown below, determine $e(t)$ and $i(t)$ (a) for $i_s(t) = u_0(t)$, (b) for $i_s(t) = u_{-1}(t)$.



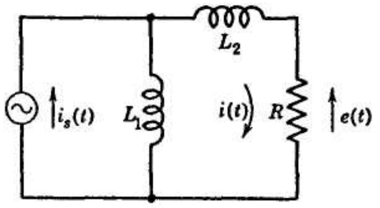
PROB. 1.



PROB. 2.

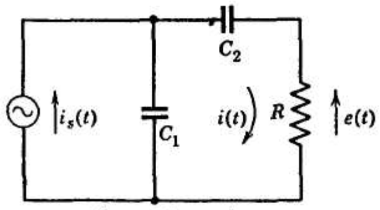
2. For the circuit shown above, determine $e(t)$ and $i(t)$ (a) for $e_s(t) = u_0(t)$, (b) for $e_s(t) = u_{-1}(t)$.

3. Given $i_s(t) = u_{-1}(t)$. Determine $e(t)$ and $i(t)$.



$L_1 + L_2 = L$

PROB. 3.

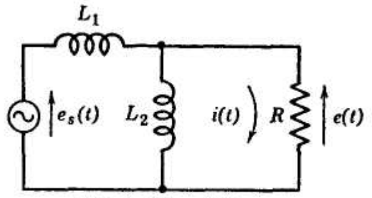


$1/C_1 + 1/C_2 = 1/C$

PROB. 4.

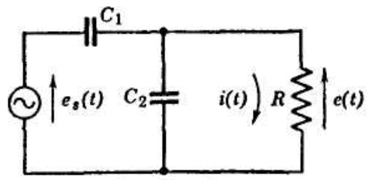
4. Given $i_s(t) = u_0(t)$. Determine $e(t)$ and $i(t)$.

5. Given $e_s(t) = u_0(t)$. Determine $e(t)$ and $i(t)$.



$1/L_1 + 1/L_2 = 1/L$

PROB. 5.

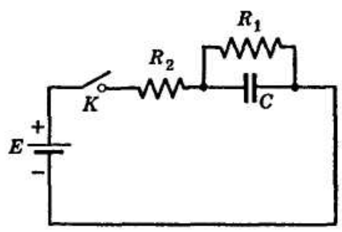


$C_1 + C_2 = C$

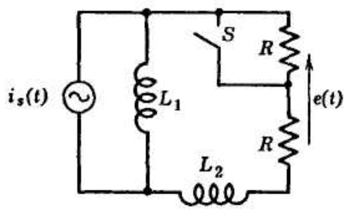
PROB. 6.

6. Given $e_s(t) = u_{-1}(t)$. Determine $e(t)$ and $i(t)$.

7. For the circuit shown below, R_1 represents the leakage resistance of the condenser of capacitance C. What is the equation for the charge on the condenser as a function of time after the switch K is closed?



PROB. 7.



$L = L_1 + L_2$

$i_s(t) = u_{-1}(t)$

PROB. 9.

8. Which pairs of the circuits in Probs. 1 to 6 inclusive are potential duals? Using this information, check your answers to these problems.

9. Switch S is closed for $0 < t < L/R$ and is open during the interval $L/R < t < \infty$. Find analytic expressions for the voltage $e(t)$ valid for $0 < t < L/R$ and again for $L/R < t < \infty$.