Research Seminar

Subspace Identification of Hammerstein and Wiener Models

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Outline

- Introduction: Motivation, New results
- A (**very**) brief review on **S**ubspace **S**tate-**S**pace **S**ystem **ID**entification Methods
- Block-oriented Nonlinear Models
- Subspace Identification of Hammerstein Models
- Subspace Identification of Wiener Models
- Simulation Examples
- Conclusions

Introduction

❑ Motivation for Nonlinear (Subspace) Identification

- Most physical processes have a nonlinear behaviour, except in a limited range where they can be considered linear.
- The performance of controllers designed from a linear approximation is strongly influenced by a change in the operating point of the system.
- Nonlinear models are able to describe more accurately the global behaviour of the system, independently of the operating point.
- Many dynamical systems can be represented by the interconnection of static nonlinearities and LTI systems. These models are called **block-oriented** nonlinear models.

- Subspace Methods have been very successful for the identification of LTI models in many practical applications.

- Although there is a well developed theory for Subspace Identification methods for LTI systems, this is not the case for nonlinear systems. Some recent contributions in this area are: (Verhaegen & Westwick, 1996) in Subspace Identification of Hammersterin and Wiener models, and (Chen & Maciejowski, 2000) and (Favoreel *et al.*, 1999) in Subspace Identification of bilinear systems.

❑ **The new results** (Gomez & Baeyens, 2005)

- New subspace algorithms for the simultaneous identification of the linear and nonlinear parts of **multivariable Hammerstein** and **Wiener** models are presented.
- The proposed algorithms consist basically of two steps:

  Step 1: a standard (linear) subspace algorithm applied to an equivalent linear system whose inputs (outputs) are filtered (by the basis functions describing the static nonlinearities) versions of the original inputs (outputs).

  Step 2: a 2-norm minimization problem which is solved via an SVD.

- Provided the conditions for the consistency of the linear subspace algorithm used in Step 1 are satisfied, **consistency** of the estimates can be guaranteed.

References

1. Gómez, J.C. and Baeyens, E.. Subspace Identification of Multivariable Hammerstein and Wiener Models, *European Journal of Control*, Vol. 11, No. 2, 2005.

2. Gómez, J.C., Jutan, A. and Baeyens, E.. Wiener Model Identification and Predictive Control of a pH Neutralization Process. *IEE Proceedings on Control Theory and Applications*, Vol. 151, No. 3, pp. 329-338, May 2004.

Subspace State-Space System IDentification

4SID Methods

- **Properties**
  - They combine tools of **System Theory**, **Numerical Linear Algebra** and **Geometry** (projections).
  - They have their origin in **Realization Theory** as developed in the 60/70s (Ho & Kalman, 1966).
  - They provide reliable state-space models of **multivariable** LTI systems **directly** from input-output data.
  - They don't require iterative optimization procedures → no problems with local minima, convergence and initialization.

- They don't require a particular (canonical) state-space realization → numerical conditioning improves.
- They require a modest computational load in comparison to traditional identification methods like PEM.
- The algorithms can be (they have been) efficiently implemented in software like **Matlab**.
- Main computational tools are QR and SVD.
- All subspace methods compute at some stage the **subspace** spanned by the columns of the extended observability matrix.
- The various algorithms (*e.g.*, N4SID, MOESP, CVA) differ in the way the extended observability matrix is estimated and also in the way it is used to compute the system matrices.

❑ The system model

$$x_{k+1} = Ax_k + Bu_k + Ke_k$$
$$y_k = Cx_k + Du_k + e_k$$

State-space model in innovation form

❑ The identification problem

To estimate the system matrices *(A, B, C, D)* and *K* , and the model order *n*, from an *(N+α-1)*-point data set of input and output measurements

$$\{u_k , y_k\}_{k=1}^{N+\alpha-1}$$

❑ Realization-based 4SID Methods

For a LTI system, a **minimal** state-space realization *(A, B, C, D)* completely defines the input-output properties of the system through

$$y_k = \sum_{\ell=0}^{\infty} h_\ell u_{k-\ell}$$

convolution sum

where the impulse response coefficients h_ℓ are related to the system matrices by

$$h_\ell = \begin{cases} D & , \quad \ell = 0 \\ CA^{\ell-1}B & , \quad \ell > 0 \end{cases}$$

$$H_{ij} = \begin{bmatrix} h_1 & h_2 & \cdots & h_j \\ h_2 & h_3 & \cdots & h_{j+1} \\ \vdots & \vdots & \ddots & \vdots \\ h_i & h_{i+1} & \cdots & h_{i+j-1} \end{bmatrix} \Rightarrow H_{ij} = \Gamma_i \mathbf{C}_j$$

Impulse Response Hankel Matrix

Extended Observability Matrix ($i>n$)

Extended Controlability Matrix ($j>n$)

An estimate of the extended observability matrix can be computed by a **full rank** factorization of the impulse response Hankel matrix. This factorization is provided by the SVD of matrix H_{ij}.

$$H_{ij} = \begin{bmatrix} U_1 & U_2 \end{bmatrix} \begin{bmatrix} \Sigma_1 & 0 \\ 0 & \Sigma_2 \end{bmatrix} \begin{bmatrix} V_1^T \\ V_2^T \end{bmatrix} \approx U_1 \Sigma_1 V_1^T = \underbrace{\left(U_1 \Sigma_1^{1/2} \right)}_{\hat{\Gamma}_i} \underbrace{\left(\Sigma_1^{1/2} V_1^T \right)}_{\hat{\mathbf{C}}_j}$$

rank reduction

In the absence of noise, H_{ij} will be a rank n matrix, and Σ_1 will contain the n non-zero singular values → **model order is computed.** In the presence of noise, H_{ij} will have full rank and a rank reduction stage will be required for the model order determination.

Problems: it is necessary to measure or to estimate (for example, via correlation analysis) the impulse response of the system → **not good**

❑ Direct 4SID Methods

$$\mathbf{Y}_\alpha = \Gamma_\alpha \mathbf{X} + H_\alpha \mathbf{U}_\alpha + \mathbf{N}_\alpha$$

fundamental equation (1)

$$\mathbf{Y}_\alpha = \begin{bmatrix} y_1 & y_2 & \cdots & y_N \\ y_2 & y_3 & \cdots & y_{N+1} \\ \vdots & \vdots & \vdots & \vdots \\ y_{\alpha-1} & y_\alpha & \cdots & y_{N+\alpha-1} \end{bmatrix}$$

Output block Hankel matrix

(In a similar way are defined the Input block Hankel matrix $\mathbf{U}_\alpha$ and the Noise block Hankel matrix $\mathbf{N}_\alpha$.)

$$\Gamma_\alpha = \begin{bmatrix} C \\ CA \\ \vdots \\ CA^{\alpha-1} \end{bmatrix}$$

Extended ($\alpha > n$)
Observability Matrix

$$\mathbf{X} = [x_1, x_2, \cdots, x_N]$$

State Sequence Matrix

$$H_\alpha = \begin{bmatrix} D & 0 & 0 & \cdots & 0 \\ CB & D & 0 & \cdots & 0 \\ CAB & CB & D & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ CA^{\alpha-2}B & CA^{\alpha-3}B & CA^{\alpha-4}B & \cdots & D \end{bmatrix}$$

Lower triangular block Toeplitz matrix of impulse responses (unknown).

❑ The main idea of Direct 4SID methods

In the absence of noise ($\mathbf{N}_\alpha = 0$), eq. (1) becomes

$$\mathbf{Y}_\alpha = \Gamma_\alpha \mathbf{X} + H_\alpha \mathbf{U}_\alpha \qquad (2)$$

and the part of the output which does not emanate from the state can be removed by multiplying (from the right) both sides of eq. (2) by the **orthogonal projection onto the null space of $\mathbf{U}_\alpha$**, *i.e.* by

$$\Pi^{\perp}_{\mathbf{U}_\alpha^T} \stackrel{\Delta}{=} I - \mathbf{U}_\alpha^T \left(\mathbf{U}_\alpha \mathbf{U}_\alpha^T\right)^{-1} \mathbf{U}_\alpha \stackrel{\Delta}{=} \mathbf{U}_\alpha^{\perp}$$

orthogonal projection

such that $\mathbf{U}_\alpha \mathbf{U}_\alpha^{\perp} = I$

This yields

$$\mathbf{Y}_\alpha \mathbf{U}_\alpha^{\perp} = \Gamma_\alpha \mathbf{X} \mathbf{U}_\alpha^{\perp} \qquad (3)$$

Note that the matrix on the left depends exclusively on the input-output data. Then, a **full rank factorization** of this matrix will provide an estimate $\hat{\Gamma}_\alpha$ of the extended observability matrix. Estimates of the corresponding system matrices can be obtained by resorting to the **shift invariance property** of the extended observability matrix, and by solving a system of linear equations in the least squares sense.

The factorization is provided by the SVD of the matrix on the left side

$$\mathbf{Y}_\alpha \mathbf{U}_\alpha^\perp = \begin{bmatrix} U_1 & U_2 \end{bmatrix} \begin{bmatrix} \Sigma_1 & 0 \\ 0 & \Sigma_2 \end{bmatrix} \begin{bmatrix} V_1^T \\ V_2^T \end{bmatrix} \approx U_1 \Sigma_1 V_1^T = \underbrace{\left(U_1 \Sigma_1^{1/2} \right)}_{\hat{\Gamma}_\alpha} \left(\Sigma_1^{1/2} V_1^T \right) \quad (4)$$

rank reduction (model order estimation)

(In the absence of noise $\Sigma_2 = 0$)

Weighting Matrices

Row and column weighting matrices can be introduced in (4) before performing the SVD of the matrix in the left hand side. Any choice of positive-definite weighting matrices W_r and W_c will result in consistent estimates of the extended observability matrix.

$$W_r \mathbf{Y}_\alpha \mathbf{U}_\alpha^\perp W_c = \begin{bmatrix} U_1 & U_2 \end{bmatrix} \begin{bmatrix} \Sigma_1 & 0 \\ 0 & \Sigma_2 \end{bmatrix} \begin{bmatrix} V_1^T \\ V_2^T \end{bmatrix} \approx U_1 \Sigma_1 V_1^T = \underbrace{\left(U_1 \Sigma_1^{1/2} \right)}_{\hat{\Gamma}_\alpha} \left(\Sigma_1^{1/2} V_1^T \right)$$

change of coordinates in state-space

Existing algorithms employ the following choices for matrices W_r and W_c,

- **MOESP** (Verhaegen, 1994): $W_r = I, \quad W_c = \left(\frac{1}{N} \Phi \Pi_{U_\alpha^T}^\perp \Phi^T \right)^{-1} \Phi \Pi_{U_\alpha^T}^\perp$
- **CVA** (Larimore, 1990): $W_r = \left(\frac{1}{N} \mathbf{Y}_\alpha \Pi_{U_\alpha^T}^\perp \mathbf{Y}_\alpha{}^T \right)^{-1/2}, \quad W_c = \left(\frac{1}{N} \Phi \Pi_{U_\alpha^T}^\perp \Phi^T \right)^{-1/2}$
- **N4SID** (Van Overschee and de Moor, 1994):

$$W_r = I, \quad W_c = \left(\frac{1}{N} \Phi \Pi_{U_\alpha^T}^\perp \Phi^T \right)^{-1} \Phi$$

Computation of the system matrices

Given an estimate $\hat{\Gamma}_\alpha$ of the extended observability matrix, estimates of the system matrices can be computed as:

- $\hat{C}$: first row block of $\hat{\Gamma}_\alpha$
- $\hat{A}$: solving in the least squares sense

$$\overline{\Gamma_\alpha} = \underline{\Gamma_\alpha}\hat{A}$$

shift-invariance property

- $\hat{B}$ and $\hat{D}$: solving a system of linear equations

❑ Presence of noise

In the presence of noise

$$\mathbf{Y}_\alpha = \Gamma_\alpha \mathbf{X} + H_\alpha \mathbf{U}_\alpha + \mathbf{N}_\alpha$$

and

$$\mathbf{Y}_\alpha \mathbf{U}_\alpha^\perp = \Gamma_\alpha \mathbf{X} + \mathbf{N}_\alpha \mathbf{U}_\alpha^\perp$$

noise term needs to be removed

The noise term can be removed by **correlating it away** with a suitable matrix. This can be interpreted as an **oblique projection.**

Block-oriented Nonlinear Models

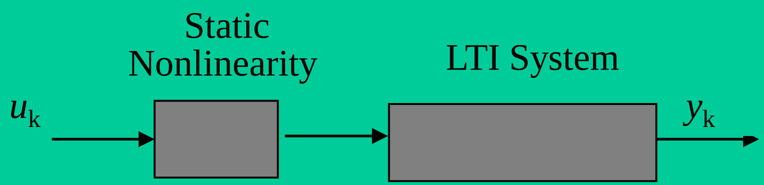


Fig. 1: Hammerstein Model (NL)

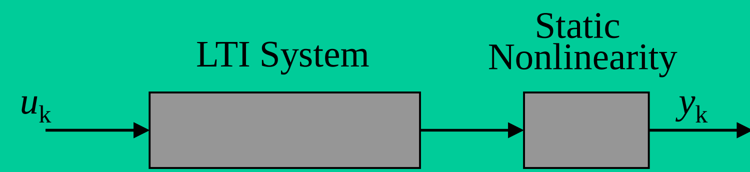


Fig. 2: Wiener Model (LN)

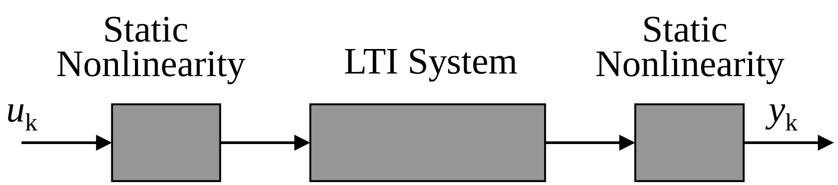


Fig. 3: Hammerstein-Wiener Model (NLN)

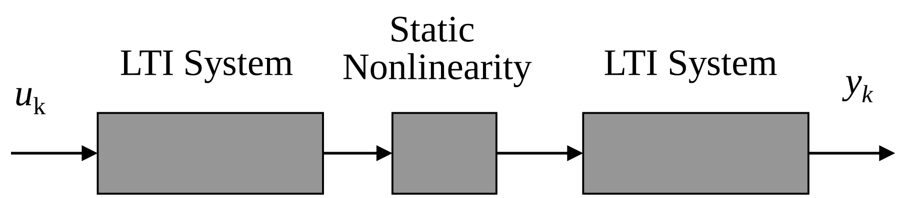


Fig. 4: Hammerstein-Wiener Model (LNL)

Hammerstein Model Identification

Problem Formulation

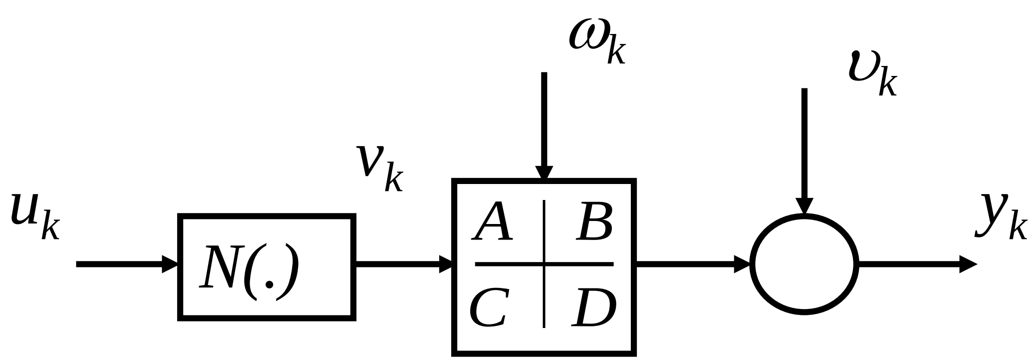


Fig. 5: Hammerstein model

LTI subsystem

$$\begin{cases} x_{k+1} = Ax_k + Bv_k + \omega_k & (1) \\ y_k = Cx_k + Dv_k + \upsilon_k & (2) \end{cases}$$

$y_k \in \Re^m, \; x_k \in \Re^n, \; v_k \in \Re^p$

$\omega_k \in \Re^n, \; \upsilon_k \in \Re^m$

Nonlinear subsystem

$$v_k = N(u_k) = \sum_{i=1}^{r} \alpha_i g_i(u_k) \qquad (3)$$

$g_i(\bullet) : \Re^p \to \Re^p, (i = 1, \cdots, r)$ known basis functions

$\alpha_i \in \Re^{p \times p} \quad (i = 1, \cdots, r)$ unknown matrix parameters

Identification problem: to estimate the unknown parameter matrices $\alpha_i \in \Re^{p \times p}, (i = 1, \cdots, r)$, and A, B, C, and D characterizing the nonlinear and the linear parts, respectively, and the model order n, from an N-point data set $\{u_k, y_k\}_{k=1}^{N}$ of observed input-output measurements.

Subspace Identification Algorithm

$$(3) \rightarrow (1), (2) \;\Rightarrow\; \begin{cases} x_{k+1} = Ax_k + \sum_{i=1}^{r} B\alpha_i g_i(u_k) + \omega_k \\ y_k = Cx_k + \sum_{i=1}^{r} D\alpha_i g_i(u_k) + \upsilon_k \end{cases}$$

Identifiability problem

Normalization $\|\alpha_i\|_2 = 1$

Defining $\tilde{B} \overset{\Delta}{=} [B\alpha_1, \cdots, B\alpha_r]$, $\tilde{D} \overset{\Delta}{=} [D\alpha_1, \cdots, D\alpha_r]$, $U_k \overset{\Delta}{=} \left[g_1^T(u_k), \cdots, g_r^T(u_k)\right]^T$

$$\begin{cases} x_{k+1} = Ax_k + \tilde{B}U_k + \omega_k \\ y_k = Cx_k + \tilde{D}U_k + \upsilon_k \end{cases}$$

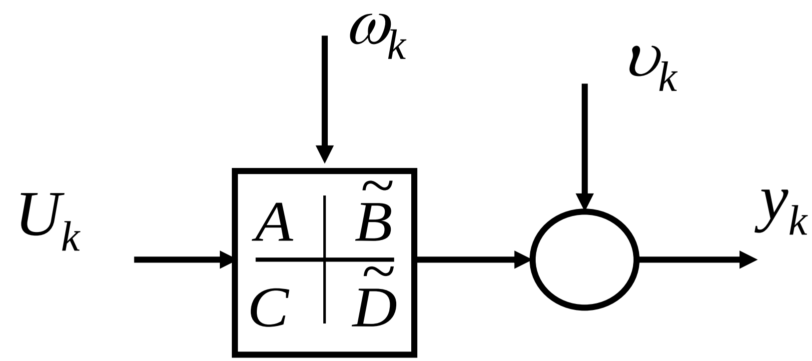


Fig. 6: Equivalent LTI system with input U_k

Linear Subspace Algorithms

(N4SID, MOESP,CVA)

Estimates $\hat{A}, \hat{\tilde{B}}, \hat{C}, \hat{\tilde{D}}$, model order n

Defining $\alpha = [\alpha_1, \cdots, \alpha_r]^T$, then $\tilde{B} = B\alpha^T$, and $\tilde{D} = D\alpha^T$, so that

$$\Theta_{BD} \overset{\Delta}{=} \begin{bmatrix} \tilde{B} \\ \tilde{D} \end{bmatrix} = \begin{bmatrix} B \\ D \end{bmatrix} \alpha^T$$

The **problem** then is how to compute estimates of matrices B, D, and α from the estimate of the matrices $\tilde{B}$, and $\tilde{D}$ (i.e., from an estimate of Θ_{BD})

It is clear that the closest, in the 2-norm sense, estimates $\hat{B}$, $\hat{D}$, and $\hat{\alpha}$ are such that

$$\left(\hat{B}, \hat{D}, \hat{\alpha}\right) = \underset{B,D,\alpha}{\operatorname{argmin}} \left\{ \left\| \hat{\Theta}_{BD} - \begin{bmatrix} B \\ D \end{bmatrix} \alpha^T \right\|_2^2 \right\}$$

The **solution** to this optimization problem is provided by the SVD of $\hat{\Theta}_{BD}$.

Result 1

Let $\hat{\Theta}_{BD} \in \Re^{(n+m)\times rp}$ have rank *s*>*p*, and let its economy size SVD be partitioned as

$$\hat{\Theta}_{BD} = U\Sigma V^T = \sum_{i=1}^{s} \sigma_i u_i v_i^T = \begin{bmatrix} U_1 & U_2 \end{bmatrix} \begin{bmatrix} \Sigma_1 & 0 \\ 0 & \Sigma_2 \end{bmatrix} \begin{bmatrix} V_1^T \\ V_2^T \end{bmatrix} \qquad (4)$$

with $U_1 \in \Re^{(n+m)\times p}$, $V_1 \in \Re^{rp\times p}$, and $\Sigma_1 = \mathrm{diag}(\sigma_1, \sigma_2, \cdots, \sigma_p)$.

Then

$$\left(\begin{bmatrix} \hat{B} \\ \hat{D} \end{bmatrix}, \hat{\alpha} \right) = \underset{B,D,\alpha}{\mathrm{argmin}} \left\| \hat{\Theta}_{BD} - \begin{bmatrix} B \\ D \end{bmatrix} \alpha^T \right\|_2^2 = (U_1\Sigma_1, V_1),$$

and the approximation error is given by

$$\left\| \hat{\Theta}_{BD} - \begin{bmatrix} \hat{B} \\ \hat{D} \end{bmatrix} \alpha^T \right\|_2^2 = \sigma_{p+1}^2.$$

Normalization in α provided by the SVD

Identification Algorithm

The subspace algorithm can be summarized as follows.

Step 1: Compute estimates of the system matrices $\left(A, \tilde{B}, C, \tilde{D}\right)$, and the model order *n*, using any available (linear) subspace algorithm, such as N4SID, MOESP, CVA.

Step 2: Based on the estimates $\hat{\tilde{B}}$ and $\hat{\tilde{D}}$ compute an estimate $\hat{\Theta}_{BD}$ of matrix Θ_{BD} .

Step 3: Compute the SVD of $\hat{\Theta}_{BD}$ and its partition as in (4).

Step 4: Compute the estimates of the parameter matrices *B*, *D*, and α as

$$\begin{bmatrix} \hat{B} \\ \hat{D} \end{bmatrix} = U_1 \Sigma_1$$

$$\hat{\alpha} = V_1$$

respectively.

Result 2: Consistency Analysis

Under some assumptions on **persistency of excitation** of the inputs, which depend on the particular subspace method used in **Step 1** of the algorithm, the estimates $\left(\hat{A}, \hat{\tilde{B}}, \hat{C}, \hat{\tilde{D}}\right)$ are **consistent** in the sense that they converge to the true values when the number of data points $N \to \infty$.

The consistency of $\hat{\tilde{B}}$ and $\hat{\tilde{D}}$, implies that of B, D, and α.

Wiener Model Identification

Problem Formulation

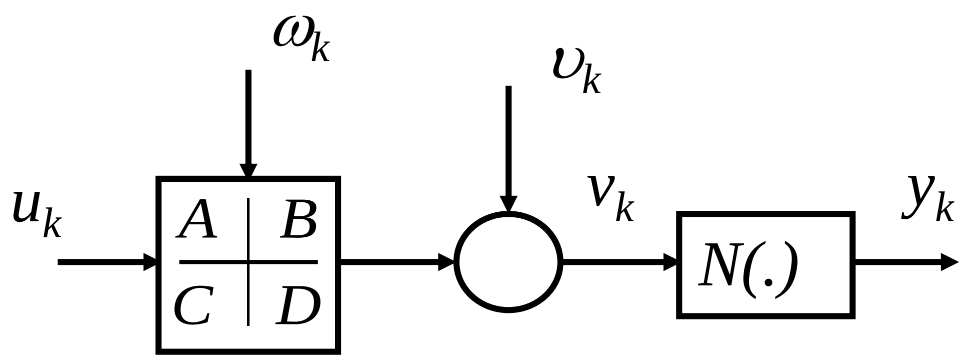


Fig. 7: Wiener model

LTI subsystem

$$\begin{cases} x_{k+1} = Ax_k + Bu_k + \omega_k & (5) \\ v_k = Cx_k + Du_k + \upsilon_k & (6) \end{cases}$$

$$u_k \in \Re^p,\ x_k \in \Re^n,\ v_k \in \Re^m$$

$$\omega_k \in \Re^n,\ \upsilon_k \in \Re^m$$

Nonlinear subsystem

$$v_k = N^{-1}(y_k) = \sum_{i=1}^{r} \alpha_i g_i(y_k) \quad (7)$$

$g_i(\bullet): \Re^m \to \Re^m, (i = 1, \cdots, r)$ known basis functions

$\alpha_i \in \Re^{m\times m} \quad (i = 1, \cdots, r)$ unknown matrix parameters

Identification problem: to estimate the unknown parameter matrices $\alpha_i \in \Re^{m \times m}, (i = 1, \cdots, r)$, and A, B, C, and D characterizing the nonlinear and the linear parts, respectively, and the model order n, from an N-point data set $\{u_k, y_k\}_{k=1}^{N}$ of observed input-output measurements.

Subspace Identification Algorithm

$$(7) \rightarrow (6) \;\Rightarrow \begin{cases} x_{k+1} = Ax_k + Bu_k + \omega_k \\ \alpha Y_k \overset{\Delta}{=} \sum_{i=1}^{r} \alpha_i g_i(y_k) = Cx_k + Du_k + \upsilon_k \end{cases} \quad \Rightarrow \begin{cases} x_{k+1} = Ax_k + Bu_k + \omega_k \\ Y_k = \tilde{C}x_k + \tilde{D}u_k + \tilde{\upsilon}_k \end{cases}$$

$$\alpha = [\alpha_1, \cdots, \alpha_r], \quad Y_k = [g_1^T(y_k), \cdots, g_r^T(y_k)]^T$$

$$\tilde{C} \overset{\Delta}{=} \alpha^+ C, \quad \tilde{D} \overset{\Delta}{=} \alpha^+ D$$

Identifiability problem

Normalization $\left\| \alpha^+ \right\|_2 = 1$

$$\begin{cases} x_{k+1} = Ax_k + Bu_k + \omega_k \\ Y_k = \tilde{C}x_k + \tilde{D}u_k + \tilde{\upsilon}_k \end{cases}$$

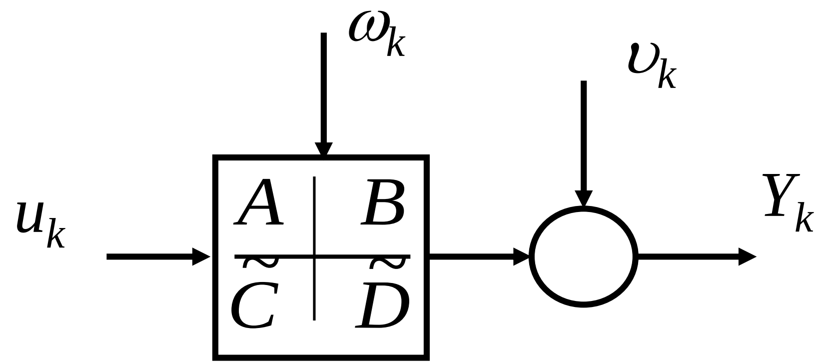


Fig. 8: Equivalent LTI model with output Y_k

Linear Subspace Algorithms

(N4SID, MOESP,CVA)

Estimates $\hat{A}, \hat{B}, \hat{\tilde{C}}, \hat{\tilde{D}}$, model order n

The **problem** is how to compute estimates of matrices C, D, and α^+ from the estimates of the matrices $\tilde{C}$, and $\tilde{D}$

Similarly to what was done for the Hammerstein model the closest, in the 2-norm sense, estimates $\hat{C}$, $\hat{D}$, and $\hat{\alpha}^+$ are such that

$$\left(\hat{C}, \hat{D}, \hat{\alpha}^+\right) = \underset{C,D,\alpha^+}{\operatorname{argmin}} \left\{ \left\| \begin{bmatrix} \hat{\tilde{C}} & \hat{\tilde{D}} \end{bmatrix} - \alpha^+ \begin{bmatrix} C & D \end{bmatrix} \right\|_2^2 \right\}$$

The **solution** to this optimization problem is provided by the SVD of the matrix $\begin{bmatrix} \hat{\tilde{C}} & \hat{\tilde{D}} \end{bmatrix}$

Result 3

Let $\begin{bmatrix}\hat{\tilde{C}} & \hat{\tilde{D}}\end{bmatrix} \in \Re^{mr\times(n+p)}$ have rank $s>m$, and let its economy size SVD be partitioned as

$$\begin{bmatrix}\hat{\tilde{C}} & \hat{\tilde{D}}\end{bmatrix} = U\Sigma V^T = \sum_{i=1}^{s}\sigma_i u_i v_i^T = \begin{bmatrix}U_1 & U_2\end{bmatrix}\begin{bmatrix}\Sigma_1 & 0 \\ 0 & \Sigma_2\end{bmatrix}\begin{bmatrix}V_1^T \\ V_2^T\end{bmatrix} \qquad (8)$$

with $U_1 \in \Re^{mr\times m}$, $V_1 \in \Re^{(n+p)\times m}$, and $\Sigma_1 = \mathrm{diag}(\sigma_1, \sigma_2, \cdots, \sigma_m)$.

Then

$$\left(\hat{\alpha}^+, \begin{bmatrix}\hat{C} & \hat{D}\end{bmatrix}\right) = \operatorname*{argmin}_{C,D,\alpha^+} \left\| \begin{bmatrix}\hat{\tilde{C}} & \hat{\tilde{D}}\end{bmatrix} - \alpha^+\begin{bmatrix}C & D\end{bmatrix} \right\|_2^2 = \left(U_1, \Sigma_1 V_1^T\right),$$

Normalization in α^+ provided by the SVD

and the approximation error is given by

$$\left\| \begin{bmatrix}\hat{\tilde{C}} & \hat{\tilde{D}}\end{bmatrix} - \hat{\alpha}^+\begin{bmatrix}\hat{C} & \hat{D}\end{bmatrix} \right\|_2^2 = \sigma_{m+1}^2.$$

Identification Algorithm

The subspace algorithm can be summarized as follows.

Step 1: Compute estimates of the system matrices $\left(A, B, \tilde{C}, \tilde{D}\right)$, and the model order n, using any available (linear) subspace algorithm, such as N4SID, MOESP, CVA.

Step 2: Compute the SVD of $\begin{bmatrix} \hat{\tilde{C}} & \hat{\tilde{D}} \end{bmatrix}$ and its partition as in (8).

Step 3: Compute the estimates of the parameter matrices C, D, and α^{+} as

$$\begin{bmatrix} \hat{C} & \hat{D} \end{bmatrix} = \Sigma_1 V_1^T$$

$$\hat{\alpha} = U_1^{+}$$

respectively.

Simulation Examples

Example 1: Hammerstein Model ID ("academic")

- **The True System**

$$G(z) = \frac{z^2 + 0.7z - 1.5}{z^3 + 0.9z^2 + 0.15z + 0.002}$$

linear subsystem

$$N(u_k) = 0.8589u_k + 0.0149u_k^2 - 0.5113u_k^3 - 0.0263u_k^4$$

nonlinear subsystem

- **The input and noise**

$$u_k = \sin(0.0005\pi k) + 0.5\sin(0.0015\pi k) + 0.3\sin(0.0025\pi k) + 0.1\sin(0.0035\pi k) + \gamma_k$$

input
(γ_k white noise with variance 10^{-6})

$$\Phi_v(\omega) = \frac{0.64 \times 10^{-8}}{1.2 - 0.4\cos(\omega)}$$

Spectrum of the zero mean coloured noise

❑ The Estimated Nonlinear Subsystem

$$\hat{N}(u_k) = 0.8589\, u_k + 0.0142\, u_k^2 - 0.5113\, u_k^3 - 0.0260\, u_k^4$$

Estimated nonlinear subsystem

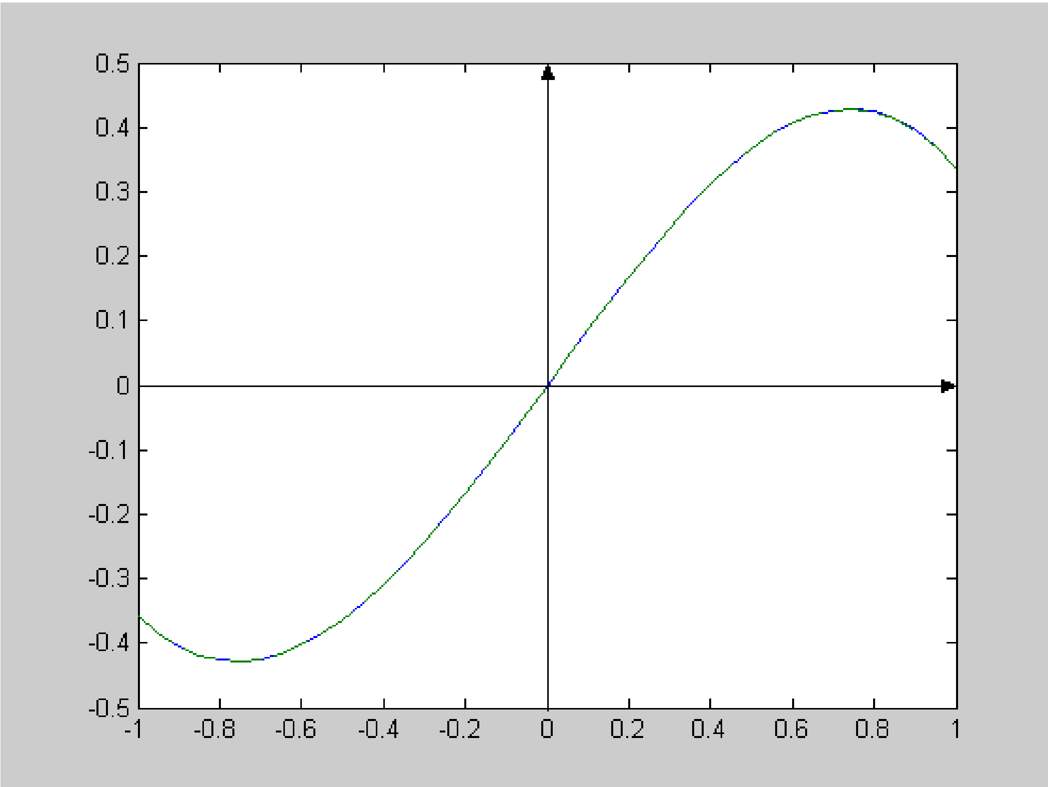


Fig.9: True (blue) and Estimated (green) nonlinear characteristic.

❑ The Estimated Linear Subsystem

$$\hat{G}(z) = \frac{0.9986z^2 + 0.6997z - 1.4984}{z^3 + 0.9002z^2 + 0.1495z + 0.0014}$$

Estimated linear subsystem

❑ Validation results

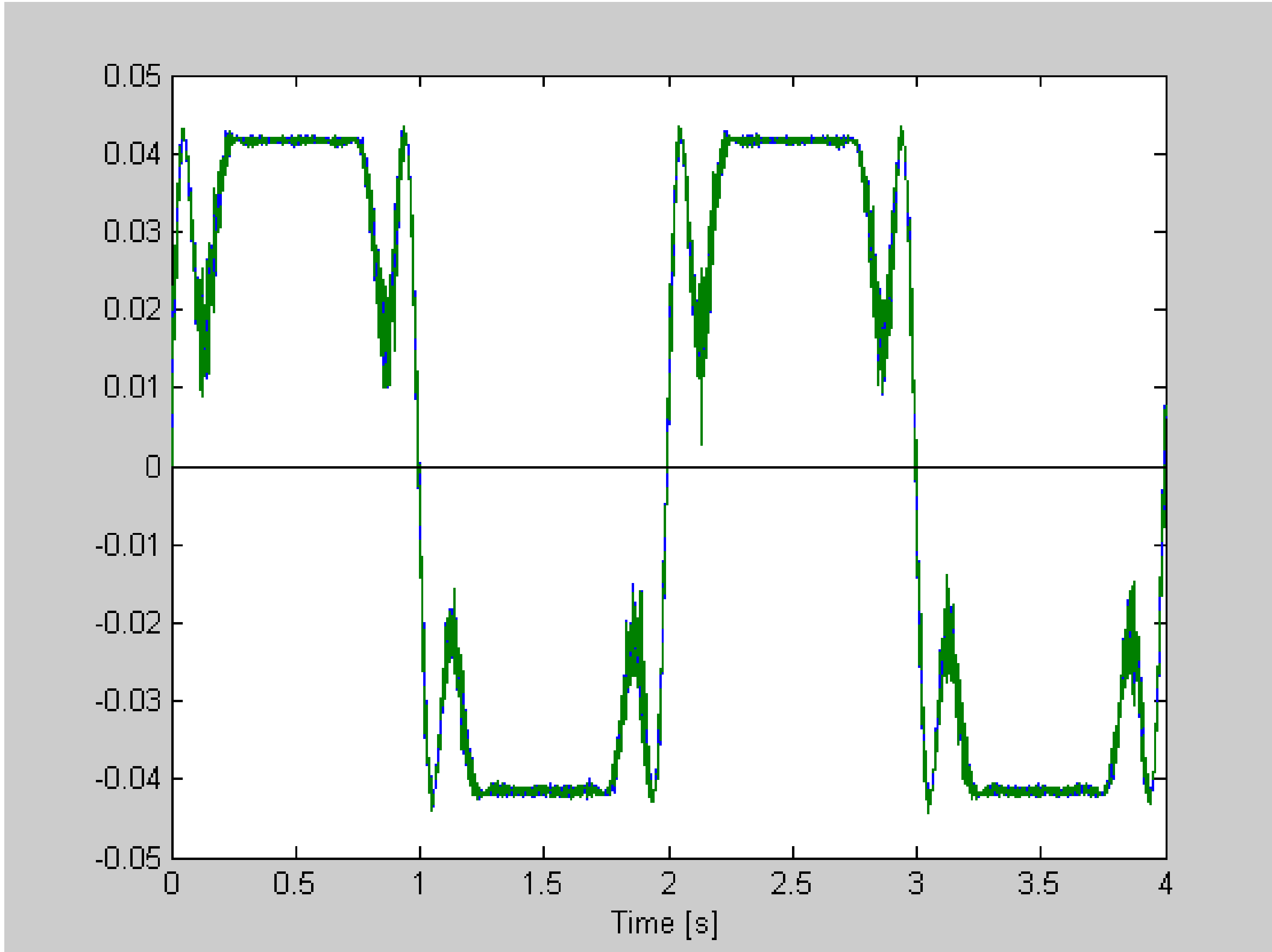


Fig. 10: True (green) and Estimated (blue) Output.

Example 2: Hammerstein Model ID (Binary Distillation Column)

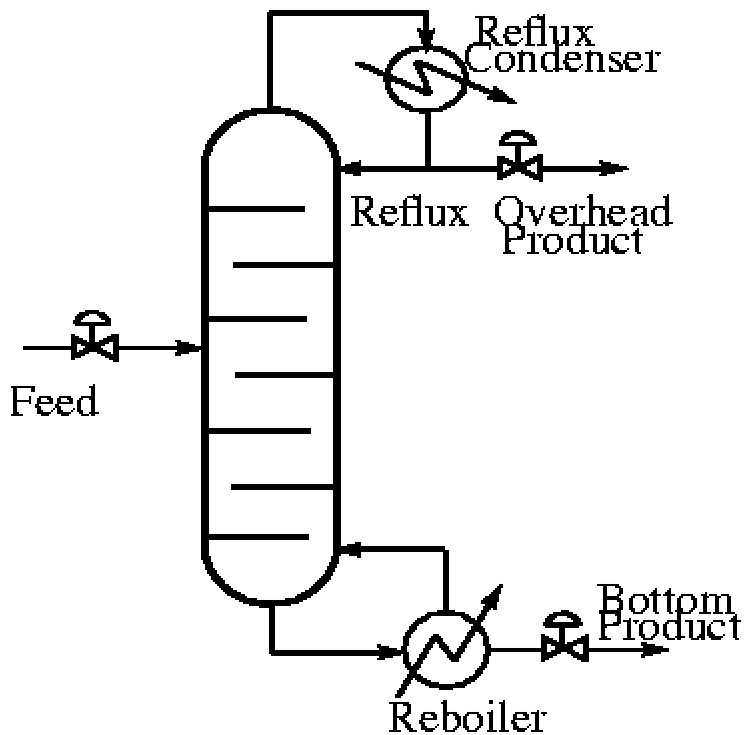


Fig. 11: Schematic representation of the distillation column

(Weischedel & McAvoy, 1980)

Input: reflux ratio (u)

Outputs: overhead flow rate (y_1)

overhead methanol concentration (y_2)

bottom flow rate (y_3)

bottom methanol concentration (y_4)

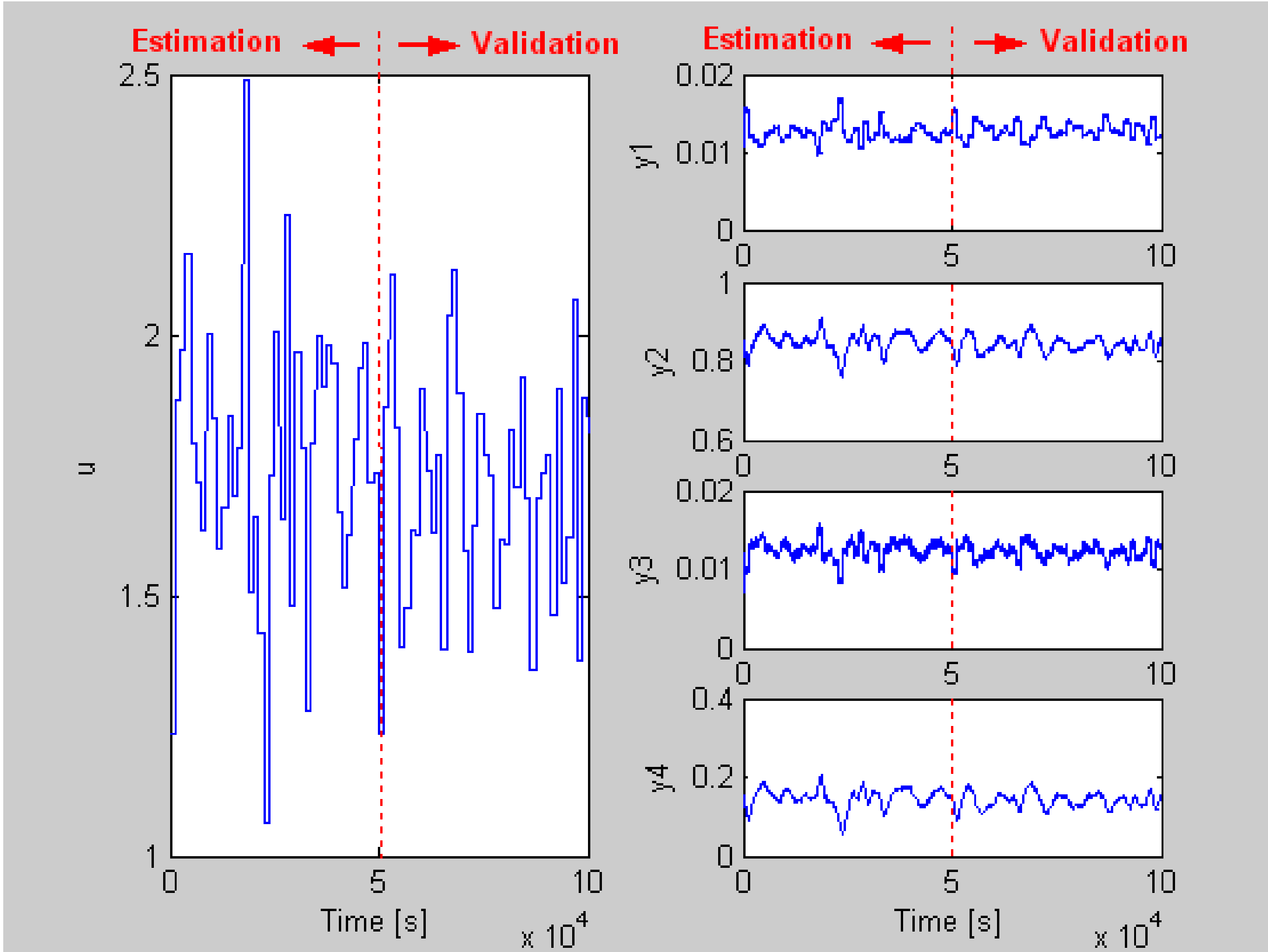


Fig. 12: Left Plot: Estimation (first 1000 points), and validation (remaining 1000 points) Input Data. Right Plot: Estimation (first 1000 points) and Validation (remaining 1000 points) Output Data.

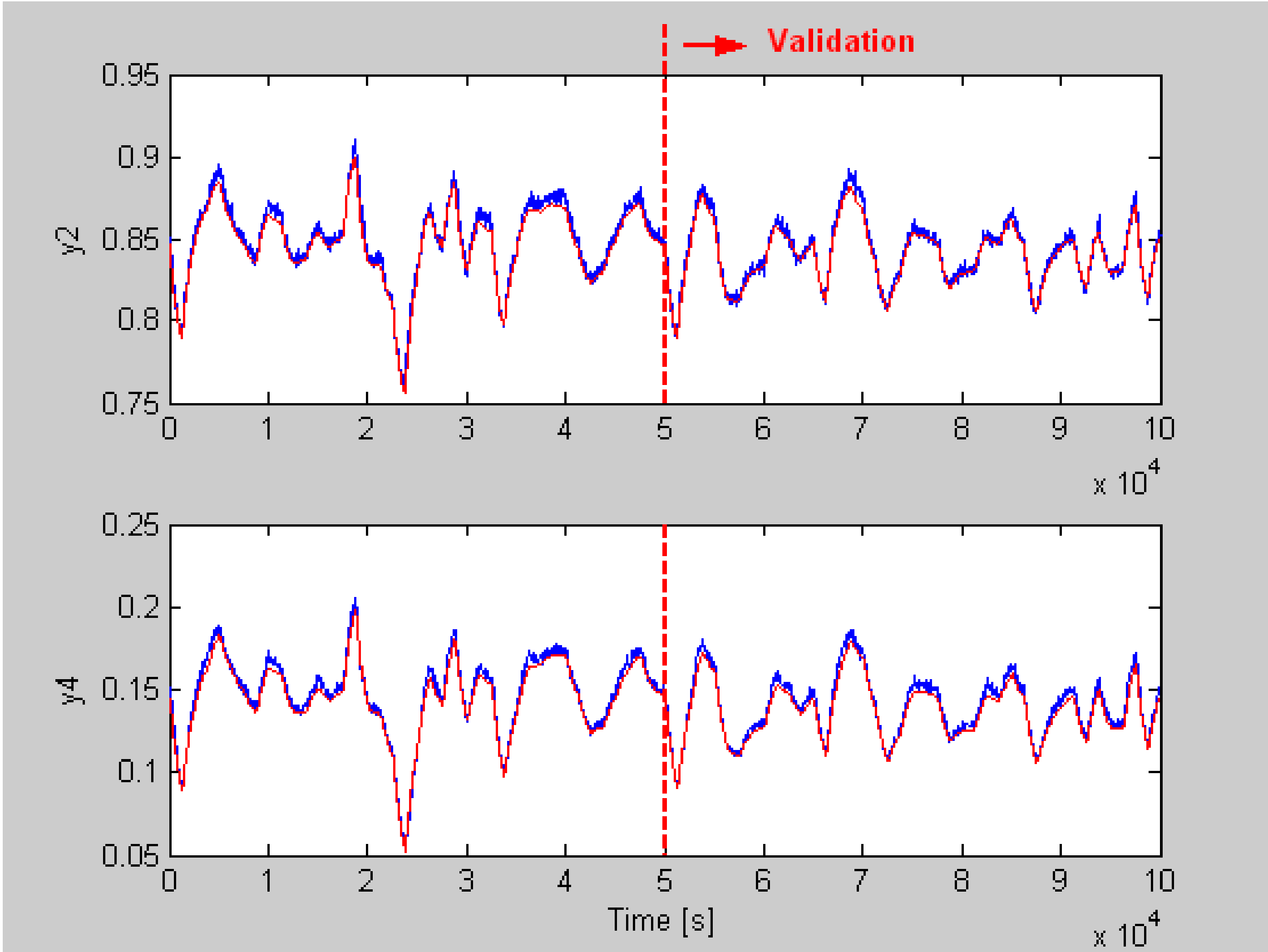
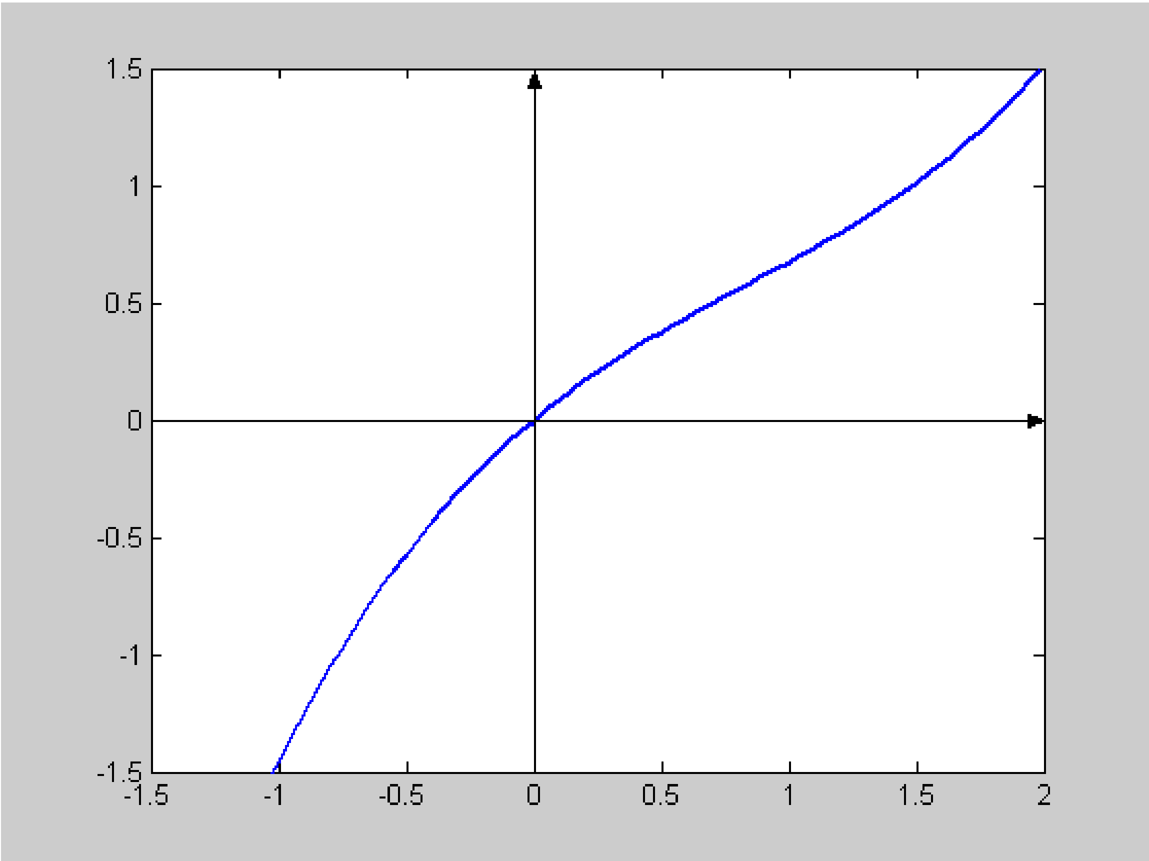


Fig. 13: True (blue) and Estimated (red) Outputs (validation data)

- **The Estimated Linear Subsystem**

Third order model with eigenvalues at $\{0.4916, 0.9557, 0.9726\}$

- **The Estimated Nonlinear Subsystem**



Third order polynomial

Fig. 14: Estimated Nonlinear Characteristic

Example 3: Wiener Model ID (pH Neutralization Process)

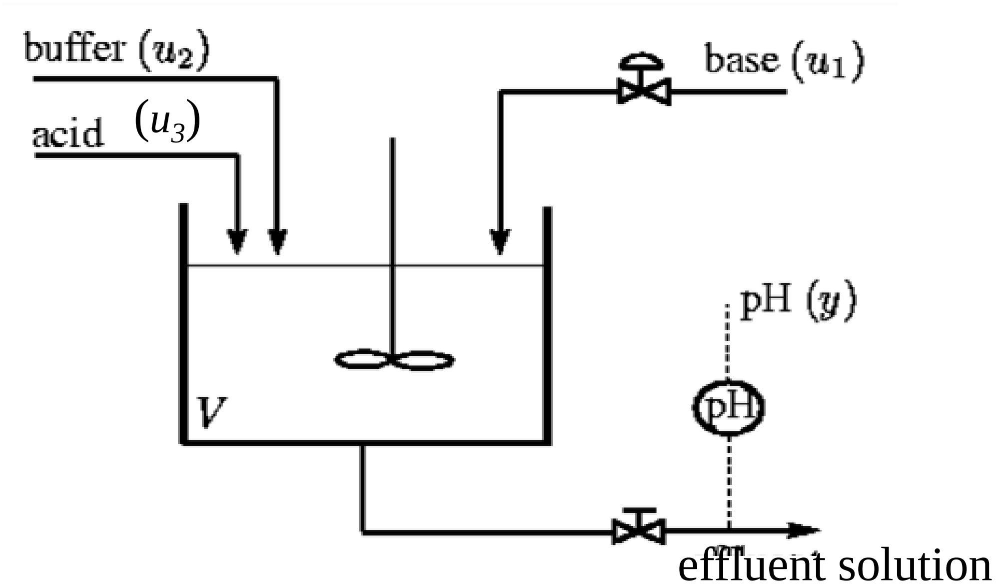


Fig. 15: Schematic representation of the pH Neutralization Process

(Henson & Seborg, 92, 94, 97)

- **base:** NaOH **acid:** HNO_3

  buffer: $NaHCO_3$
- **Manipulated variable:** base flow rate (u_1)
- **Disturbances:** buffer flow rate (u_2) and acid flow rate (u_3)
- **Output:** pH of the effluent solution (y)

- **Simulation Model** based on **first principles** (introducing two reaction invariants for each inlet stream)

$$\dot{x} = f(x) + g(x)u_1 + p(x)u_2$$
$$h(x, y) = 0$$

where

$$x \stackrel{\Delta}{=} [x_1, x_2]^T = [W_a, W_b]^T$$

$$f(x) = \left[\frac{u_3}{V}(W_{a3} - x_1), \frac{u_3}{V}(W_{b3} - x_2) \right]^T$$

$$g(x) = \left[\frac{1}{V}(W_{a1} - x_1), \frac{1}{V}(W_{b1} - x_2) \right]^T$$

$$p(x) = \left[\frac{1}{V}(W_{a2} - x_1), \frac{1}{V}(W_{b2} - x_2) \right]^T$$

$$h(x, y) = x_1 + 10^{y-14} - 10^{-y} + x_2 \frac{1 + 2 \times 10^{y - pK_2}}{1 + 10^{pK_1 - y} + 10^{y - pK_2}}$$

❑ Estimation and Validation data

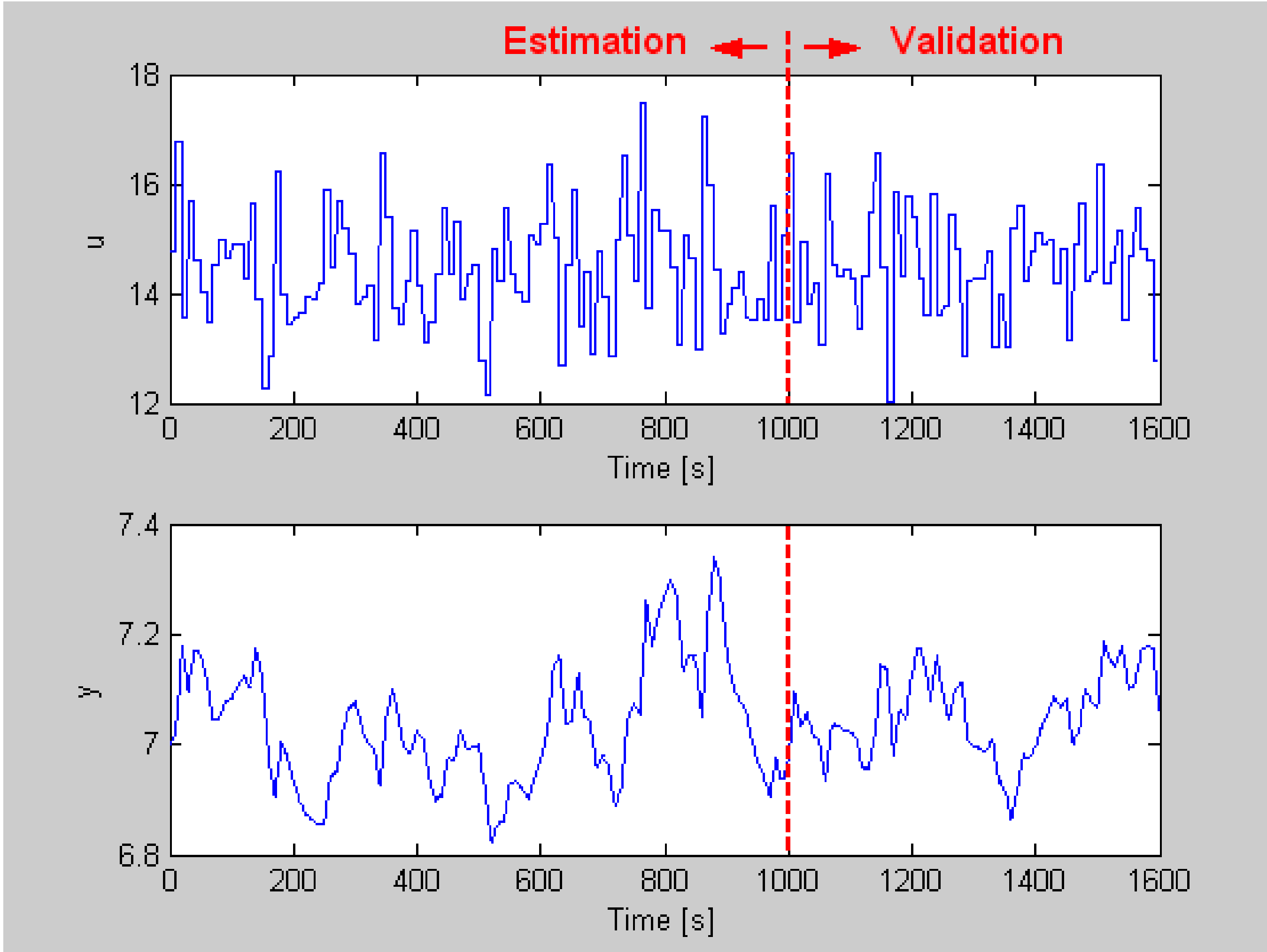


Fig. 16: Estimation (first 1000 points) and validation (remaining 600 points) input-output data.

❑ The Estimated Linear Subsystem

Third order model

$$\hat{G}(z) = \frac{0.0062z^2 - 0.0122z + 0.006}{z^3 - 2.9466z^2 + 2.8940z - 0.9474}$$

❑ The Estimated Nonlinear Subsystem

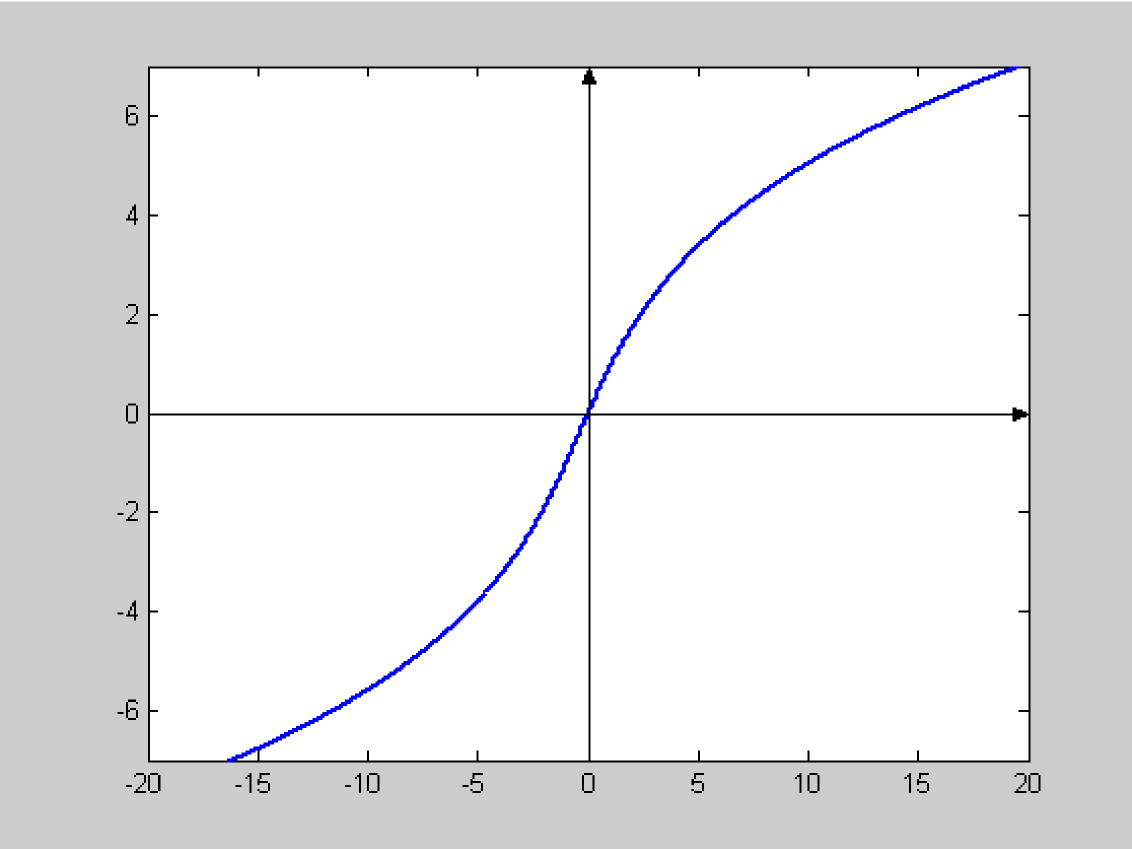


Fig. 17: Estimated Nonlinear Characteristic.

Third order polynomial

$$\hat{N}^{-1}(y_k) = 0.0319\,y_k^3 + 0.0358\,y_k^2 + 0.9989\,y_k$$

❑ Validation results

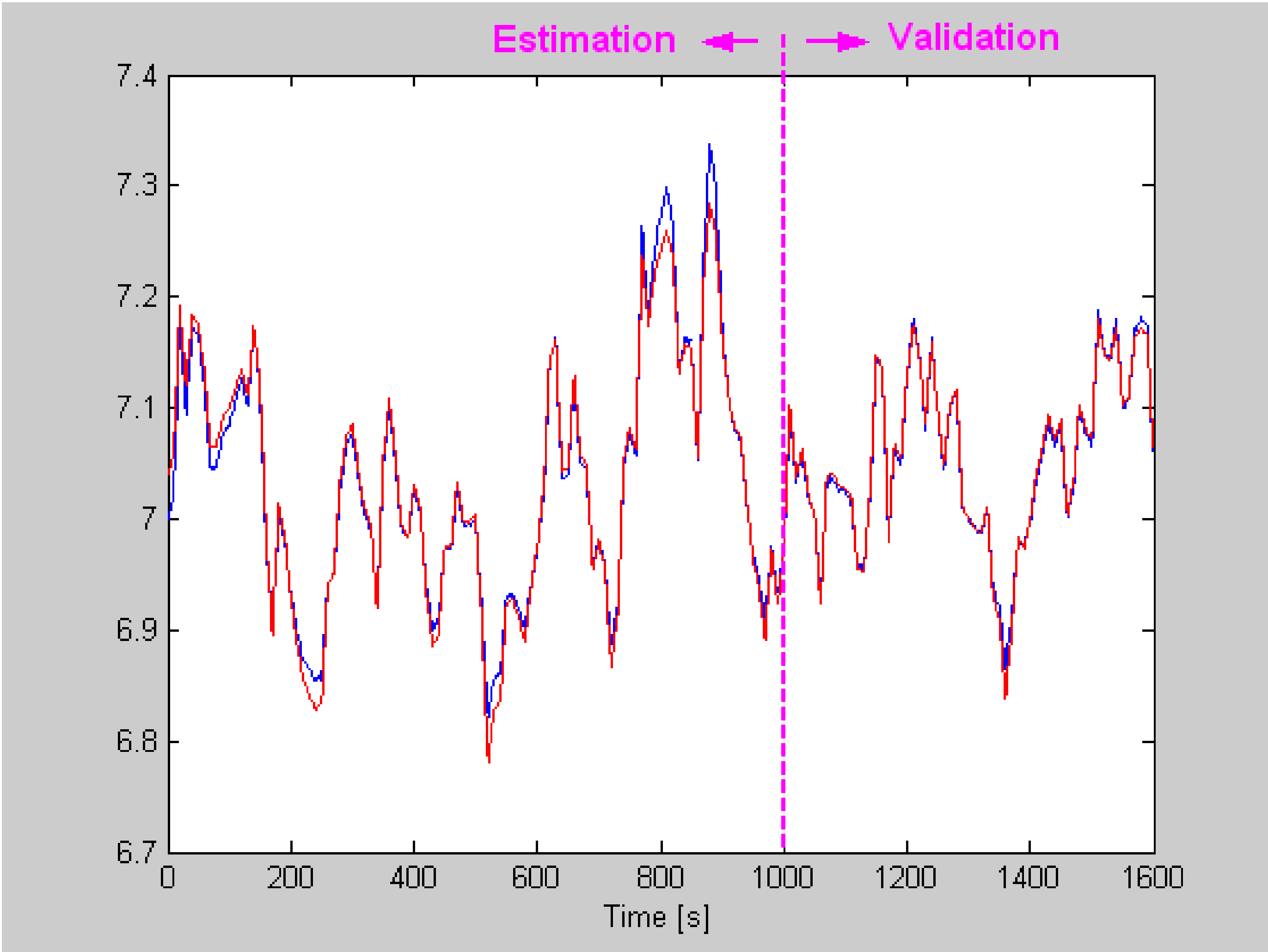


Fig. 18: True (blue) and estimated (red) Output (Estimation/Validation data).

Conclusions

- New subspace methods for the simultaneous identification of the linear and nonlinear parts of **multivariable Hammerstein and Wiener models** have been presented.
- The proposed methods make use of a standard (linear) subspace method followed by a 2-norm minimization problem which is solved via an SVD.
- The proposed methods **generalize** all the families of linear subspace methods to this class of nonlinear models.
- The method provides **consistent estimates** under the same conditions on persistency of excitation required by the (linear) subspace method used as the first step of the algorithm.
- The estimated models are in a format which is suitable for their use in standard (linear) Model Predictive Control schemes.

Research Seminar

Subspace Identification of Hammerstein and Wiener Models

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