Comparison of Nonlinear Identification Techniques on a Benchmark pH Neutralization Process

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***Abstract*—In this paper, several state-of-the-art nonlinear identification methods are compared on a benchmark pH neutralization process. Methods based on orthonormal bases for block-oriented nonlinear models, as well as methods based on support vector regression techniques are considered. The advantages and drawbacks of the different methods, and the properties of the estimated models are analyzed.**

***Index Terms*—Nonlinear identification, Wiener models, NARX models, SVM based identification, Orthonormal Bases based identification, pH neutralization processes.**

I. Introduction

The availability of accurate dynamical models of the processes is of fundamental importance in many areas of science and technology. Models are useful to understand and analyze the system under study, to simulate its behavior, or to design and implement controllers for industrial processes (most of the control design techniques are based on the assumption that a model of the process is available). Most dynamical systems appearing in engineering can be better represented by nonlinear models which are able to describe the global behavior of the system over the whole operating range, rather by linear ones, that are only able to approximate the system around a given operating point.

A variety of techniques has been proposed in the System Identification literature to derive accurate mathematical descriptions of the underlying systems, from input/ouput measurements [1]. A fundamental step in any system identification experiment is the choice of the model structure. Among the different approaches for (nonlinear) model structure selection, the black-box approach is usually preferred, since it provides simple models which are easy to interpret. This is in contrast to the white-box approach, where first principles are used to derived the models, which in general are difficult to parameterized and not always suitable for control design.

One of the most frequently studied classes of nonlinear models are the so-called *block-oriented* models, consisting of the interconnection of Linear Time Invariant (LTI) systems and static nonlinearities, see [2], [3], and an overview of recent advances in [4]. Within this model class, Wiener models, consisting of the cascade connection of an LTI block followed by a static nonlinearity, and Hammerstein models, where the order in the cascade connection is reversed, have been successfully used to represent nonlinear systems in several application areas, such as chemical processes, biological processes, signal processing and control. In addition, it has been proved that Wiener models are capable of representing, with arbitrary accuracy, any nonlinear fading memory system [5].

The objective of this paper is to compare several state-of-the-art identification techniques for Wiener models and nonlinear autoregressive models, on a benchmark pH neutralization process. Methods based on orthonormal bases, as well as methods based on support vector regression techniques are considered. The benchmark process is highly nonlinear, and it has been studied by numerous authors to evaluate identification techniques [6], [3], [7], and nonlinear control performance [8], [9], [10], [11], [12]. The rest of the paper is organized as follows. In Section II, the different model structures are described. In Section III, the identification techniques are presented. The benchmark pH neutralization process is described in Section IV, while the identification results are presented in Section V. Finally, some concluding remarks are given in Section VI.

II. Problem Formulation

A first step in an identification procedure could be to perform a *nonlinearity test*. For instance, step responses around the operating point could be measured, for different amplitudes of the input step signal, to verify if the system behaves linearly (*i.e*, if it verifies the superposition principle). The test could also serve to determine the range of variation around the operating point within which a linear model gives an accurate representation of the system. When large deviations from the operating point are expected to occur, the predictive capability of the linear model will deteriorate, and nonlinear models will be needed to accurately represent the system.

Even when large deviations from the operating point are taking place, it is still useful to estimate the best linear model for these operating conditions, for instance, to be used in a model predictive controller based on a linear model of the plant. The Best Linear Approximation (BLA) of the system is the optimal, in the least squares sense, linear model which minimizes the output prediction errors. In this paper, the BLA model will be computed resorting to subspace identification techniques [13], [14].

A. Nonlinear models

The nonlinear models employed in this paper are briefly described in this section.

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1) Wiener models: Wiener models consist of the cascade connection of an LTI block followed by a static nonlinear block, as depicted in Fig.1, where $u \in \mathbb{R}$ is the scalar input signal, $y \in \mathbb{R}$ is the scalar measured output signal, $\nu \in \mathbb{R}$ is additive noise, $v \in \mathbb{R}$ is the intermediate variable (output of the LTI block), $\tilde{y}$ is the output of the nonlinear block, $\mathcal{N}(\cdot)$ is the nonlinear mapping representing the static nonlinearity, and $G(q^{-1})$ is the transfer function (in the backward-shift operator[1] q^{-1}) of the LTI block.

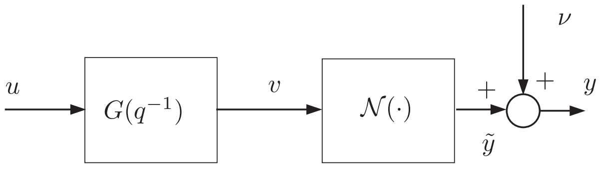

Fig. 1. Wiener Model.

2) NARX models: Nonlinear Auto Regressive with eXogenous inputs (NARX) models are of the form

$$\begin{aligned} y(n) &= \mathcal{F}\left[y(n-1), \cdots, y(n-n_a),\right. \\ &\quad \left. u(n-n_k), \cdots, u(n-n_k-n_b+1)\right] + e(n), \end{aligned} \tag{1}$$

where $u \in \mathbb{R}$ is the scalar input signal, $y \in \mathbb{R}$ is the scalar measured output signal, $e \in \mathbb{R}$ is additive white noise, and where $\mathcal{F}$ is a nonlinear function of past inputs and outputs. Most frequently used functions are polynomials, sigmoid networks, wavelet networks, etc.. Parameters n_a and n_b are the number of past outputs and past inputs, respectively, used to predict the actual output, and n_k is the delay (in samples) from the input to the output.

III. IDENTIFICATION METHODS

A. Wiener model ID based on Orthonormal Bases

The Wiener model identification method based on orthonormal bases introduced in [3] is briefly described in this section.

It is assumed that the transfer function of the linear block in the Wiener structure of Fig. 1 is parameterized using rational orthonormal basis functions[2] on $H_2(\mathbb{T})$, the space of square integrable functions on the unit circle $\mathbb{T}$, which are analytic outside the unit disk, in the form

$$G(q^{-1}) = \sum_{\ell=1}^{p} b_\ell \mathcal{B}_\ell(q^{-1}), \tag{2}$$

where $b_\ell \in \mathbb{R}$ are unknown parameters, and $\left\{\mathcal{B}_\ell(q^{-1})\right\}_{\ell=1}^{\infty}$ are the rational orthonormal bases.

On the other hand, the inverse of the static nonlinearity $\mathcal{N}(\cdot)$ is represented using nonlinear basis functions as follows:

$$\mathcal{N}^{-1}(y(n)) = \sum_{i=1}^{r} d_i f_i(y(n)), \tag{3}$$

where $d_i, i = 1, 2, \cdots, r$, are unknown parameters, and $f_i(\cdot), i = 1, 2, \cdots, r$ are nonlinear basis functions. Without loss of generality it is assumed that $d_1 = 1$.

By equating the values of the intermediate variable $(v(n))$ computed from the input and from the output in Fig. 1, the following linear regressor equation is obtained

$$f_1(y(n)) = \phi^T(n)\theta, \tag{4}$$

where

$$\theta \triangleq [d_2, d_3, \cdots, d_r, b_1, b_2, \cdots, b_p]^T, \tag{5}$$

$$\begin{aligned} \phi(n) \triangleq \; & [-f_2(y(n)), \cdots, -f_r(y(n)), \\ & \mathcal{B}_1(q^{-1})u(n), \cdots, \mathcal{B}_p(q^{-1})u(n)]^T. \end{aligned} \tag{6}$$

Given an N-point data set of measured inputs and outputs $\{u(n), y(n)\}_{n=1}^{N}$, an estimate of the parameter vector θ can be computed by minimizing a quadratic criterion on the prediction errors $e(n) = f_1(y(n)) - \phi^T(n)\theta$. This is the well known least squares estimate, which is given by

$$\widehat{\theta} = (\Phi\Phi^T)^{-1}\Phi\mathbf{f}, \tag{7}$$

provided the indicated inverse exists, and where

$$\Phi \triangleq \left[\phi^T(1); \phi^T(2); \cdots; \phi^T(N)\right]^T, \tag{8}$$

$$\mathbf{f} \triangleq [f_1(y(1)), f_1(y(2)), \cdots, f_1(y(N))]^T. \tag{9}$$

The first $(r-1)$ components of vector $\widehat{\theta}$ correspond to estimates of the parameters $d_i, i = 2, \cdots, r$, used to represent the inverse of the static nonlinearity $\mathcal{N}(\cdot)$, while the last p components correspond to an estimate of vector $\mathbf{b}$ used to represent the LTI block in the Wiener model.

The Wiener model estimated with the described method will hereafter be referred to as *Wiener-OB* model.

B. Wiener model ID based on BLA

In this case, the BLA approximation of the system is used as the estimated linear block in the Wiener structure. The nonlinear block is parameterized using basis functions in the form

$$\mathcal{N}(v(n)) = \sum_{i=1}^{r} a_i g_i(v(n)), \tag{10}$$

where $a_i, i = 1, 2, \cdots, r$, are unknown parameters, and $g_i(\cdot), i = 1, 2, \cdots, r$ are nonlinear basis functions, such as polynomials, piecewise-linear functions, radial basis functions, etc.. Given the input estimation data, the input to the static nonlinearity can be straightforwardly computed, by filtering the input with the BLA transfer function. Then, the parameters in (10) can be estimated by least squares fitting.

The Wiener model estimated with the described method will hereafter be referred to as *Wiener-BLA* model.

C. Wiener model ID based on SV-Regression

The Wiener model identification method based on SV-regression introduced in [15] is summarized in this section.

It is assumed that the transfer function of the linear block in the Wiener structure of Fig. 1 is parameterized using rational orthonormal basis functions as in (2). On the other hand, the

[1]The backward-shift operator q^{-1} is defined as: $q^{-1}x(n) \triangleq x(n-1)$.

[2]The bases are orthonormal in the sense that $\langle \mathcal{B}_\ell, \mathcal{B}_k \rangle = \delta_{\ell k}$, where $\delta_{\ell k}$ is the Kronecker delta, and $\langle \cdot, \cdot \rangle$ is the standard inner product in $L_2(\mathbb{T})$.

nonlinear block is parameterized using basis functions as in (10).

With these parameterizations, the Wiener model can be represented as in Fig. 2. Note that all the unknowns are now concentrated in the Multiple-Input Single-Output (MISO) static block $\tilde{\mathcal{N}}(\cdot)$. The inputs $x_1, x_2, \cdots, x_p$ of this block are computed by filtering the actual input u with the basis functions $\mathcal{B}_1(q^{-1}), \mathcal{B}_2(q^{-1}), \cdots, \mathcal{B}_p(q^{-1})$ used to represent the LTI block.

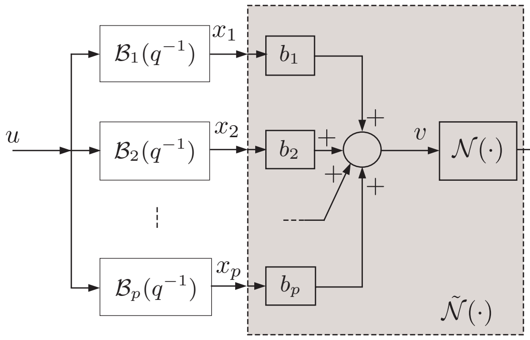

Fig. 2. Parameterized Wiener Model.

Defining now

$$\mathbf{x}(n) \triangleq \left[x_1(n), x_2(n), \cdots, x_p(n)\right]^T \in \mathbb{R}^p, \quad (11)$$

$$\mathbf{a} \triangleq \left[a_1, a_2, \cdots, a_r\right]^T \in \mathbb{R}^r, \quad (12)$$

$$\mathbf{b} \triangleq \left[b_1, b_2, \cdots, b_p\right]^T \in \mathbb{R}^p, \quad (13)$$

$$\mathbf{g}(\cdot) \triangleq \left[g_1(\cdot), g_2(\cdot), \cdots, g_r(\cdot)\right]^T : \mathbb{R} \to \mathbb{R}^r, \quad (14)$$

$$\tilde{\mathbf{g}}(\mathbf{x}(n)) \triangleq \mathbf{g}(\mathbf{b}^T\mathbf{x}(n)) : \mathbb{R}^p \to \mathbb{R}^r, \quad (15)$$

the output of the system can be written as

$$y(n) = \mathbf{a}^T\tilde{\mathbf{g}}(\mathbf{x}(n)) + \nu(n). \quad (16)$$

Equation (16) is the starting point for the formulation of the estimation problem within the framework of Support Vector regression [16]. The estimation problem, in the so-called *primal weight space*, can be formulated as follows: Given a data set of measured inputs and outputs $\{u(n), y(n)\}_{n=1}^N$, the goal is to estimate a model of the form

$$y(n) = \mathbf{a}^T\tilde{\mathbf{g}}(\mathbf{x}(n)) + c + \nu(n), \quad (17)$$

where c is a *bias* term, and $\{\nu(n)\}$ is an i.i.d. random process with zero mean and finite variance. The unknowns in the model are $\mathbf{a} \in \mathbb{R}^r$, $c \in \mathbb{R}$, and the order r.

It is well known that the unknowns $\mathbf{a}$ and c can be determined by solving the following constrained optimization problem [17]

$$\min_{\mathbf{a},c,\nu} \quad \frac{1}{2}\mathbf{a}^T\mathbf{a} + \gamma\sum_{n=1}^{N} L_\epsilon(\nu(n)) \quad (18)$$

$$\text{subject to} \quad y(n) - \mathbf{a}^T\tilde{\mathbf{g}}(\mathbf{x}(n)) - c - \nu(n) = 0, \quad n = 1, \cdots, N$$

where $\gamma > 0$ is a regularization constant providing a tradeoff between model complexity (penalized by the first term in (18)) and fitting accuracy to the experimental data (penalized by the second term in (18)), and $L_\epsilon(\nu(n))$ is Vapnik's ϵ-insensitivity loss function, defined as

$$L_\epsilon(\nu(n)) = \begin{cases} |\nu(n)| - \epsilon & \text{if } |\nu(n)| \geq \epsilon \\ 0 & \text{otherwise} \end{cases} \quad (19)$$

The optimization problem (18) can be solved more easily in its dual formulation using Lagrange multipliers, [18]. Introducing the positive definite kernels [17]

$$K(\mathbf{x}(n), \mathbf{x}(k)) \triangleq \tilde{\mathbf{g}}^T(\mathbf{x}(n))\tilde{\mathbf{g}}(\mathbf{x}(k)) \quad (20)$$

associated with the functions $\tilde{\mathbf{g}}(\mathbf{x}(n))$, the *dual* problem in the Lagrange multipliers (α_n and α_n^*) can be formulated as follows:

$$\max_{\alpha_n,\alpha_n^*} \quad -\frac{1}{2}\sum_{n,k=1}^{N}(\alpha_n - \alpha_n^*)(\alpha_k - \alpha_k^*)K(\mathbf{x}(n), \mathbf{x}(k)) - \epsilon\sum_{n=1}^{N}(\alpha_n + \alpha_n^*) + \sum_{n=1}^{N} y(n)(\alpha_n - \alpha_n^*)$$

$$\text{subject to} \quad \sum_{n=1}^{N}(\alpha_n - \alpha_n^*) = 0, \quad \alpha_n, \alpha_n^* \in [0, \gamma], \quad n = 1, \cdots, N$$

which is a quadratic programming (QP) problem with box constraints, [19]. The *dual* model representation is given by

$$y(n) = \sum_{k=1}^{N}(\alpha_k - \alpha_k^*)K(\mathbf{x}(n), \mathbf{x}(k)) + c \quad (21)$$

Although the number of terms in the representation (21) equals the number of data points N, only a reduced number of terms, corresponding to a small number of vectors $\mathbf{x}(k)$, will have non vanishing coefficients $(\alpha_k - \alpha_k^*)$. These vectors are the so-called *support vectors*. The number of support vectors will depend on the chosen values for ϵ and γ, and on the chosen kernel function. Commonly used kernels are Gaussian Radial Basis Functions (RBF), polynomial kernels, and MultiLayer Perceptrons (MLP).

The Wiener model estimated with the described method will hereafter be referred to as *Wiener-SVM* model.

D. *Identification using* nlhw *and* nlarx

Several functions for the estimation of nonlinear models are available in the System Identification Toolbox for use with Matlab [20]. Two of these functions, namely nlhw and nlarx will be considered in this paper.

Function nlhw allows for the estimation of block oriented models of the Hammerstein-Wiener type, that is models composed by a static input nonlinearity followed by LTI block, followed by a static output nonlinearity in a cascade connection. Different input and output nonlinearity estimators, such as piecewise linear, polynomial, saturation, deadzone, etc., can be specified. The user must also specify the orders of the numerator and denominator polynomials of the transfer function of the LTI block, as well as the input-output delay of that block. The model can be iteratively refined to avoid local minima. The Wiener model estimated with this method will hereafter be referred to as *Wiener-nlhw* model.

Function nlhw allows for the estimation of Nonlinear ARX models of the form (1). The user must specify the model orders and delay, as well as the type on nonlinearity (the nonlinear function $\mathcal{F}$ in (1)).

IV. pH NEUTRALIZATION PROCESS

The pH neutralization process considered in this paper consists of an acid stream (HNO_3), a base stream ($NaOH$), and a buffer stream ($NaHCO_3$), that are mixed in a constant volume (V) stirring tank. The process has been studied by numerous authors, and constitutes a benchmark for nonlinear process identification due to its highly nonlinear characteristics [8], [21], [3], [7], and also for nonlinear model-based control design, see for instance [10], [11], and [12].

The inputs of the system are the base flow rate (u_1), which is usually the manipulated variable, the buffer flow rate (u_2), and the acid flow rate (u_3), while the output (y) is the pH of the effluent solution. A computational model, based on first principles, was presented in [8]. The authors introduce the concept of reaction invariants related to charge balance and carbonate ion balance, associated with each inlet/outlet stream. The dynamic model for the reaction invariants of the effluent solution (W_a and W_b), in state-space form, is given by

$$\dot{x} = f(x) + g(x)u_1 + p(x)u_2, \quad (22)$$
$$h(x, y) = 0, \quad (23)$$

where $x = [x_1, x_2]^T = [W_a, W_b]^T$, and

$$f(x) = \left[\frac{u_3}{V}(W_{a3} - x_1), \frac{u_3}{V}(W_{b3} - x_2)\right]^T, \quad (24)$$
$$g(x) = \left[\frac{1}{V}(W_{a1} - x_1), \frac{1}{V}(W_{b1} - x_2)\right]^T, \quad (25)$$
$$p(x) = \left[\frac{1}{V}(W_{a2} - x_1), \frac{1}{V}(W_{b2} - x_2)\right]^T, \quad (26)$$

$$h(x, y) = x_1 + 10^{y-14} - 10^{-y} + x_2 \frac{1 + 2 \times 10^{y-pK_2}}{1 + 10^{pK_1-y} + 10^{y-pK_2}} \quad (27)$$

where pK_1 and pK_2 are the first and second disassociation constants of the weak acid (H_2CO_3).

The nominal operating conditions are given in Table I.

TABLE I
NOMINAL OPERATING CONDITIONS OF THE pH NEUTRALIZATION PROCESS.

$\bar{u}_3 = 16.60 ml/s$	$\bar{u}_2 = 0.55 ml/s$
$\bar{u}_3 = 15.55 ml/s$	$V = 2900 ml$
$W_{a1} = -3.05 \times 10^{-3} M$	$W_{b1} = 5 \times 10^{-5} M$
$W_{a2} = -3 \times 10^{-2} M$	$W_{b2} = 3 \times 10^{-2} M$
$W_{a3} = 3 \times 10^{-3} M$	$W_{b3} = 0 M$
$W_a = -4.32 \times 10^{-4} M$	$W_b = 5.28 \times 10^{-4} M$
$pK_1 = 6.35$	$pK_2 = 10.25$
$\bar{y} = 7.0$	

V. EXPERIMENTAL RESULTS

For the purposes of identification the computational model of the pH-neutralization process described in Section IV was excited with band-limited white noise around the nominal value of the base flow rate u_1, keeping the buffer flow rate and the acid flow rate constant in their nominal values, corresponding to the operating point (pH = 7). In order to simulate the more realistic situation of having measurement noise, the output of the system was corrupted with additive band limited white noise. The input-output data are shown in Fig.3. A set of 1600 samples were collected with a sample period of 1 second. From this set, the first 1000 samples were used for the estimation of the different nonlinear models, while the remaining 600 were used for the purposes of validation.

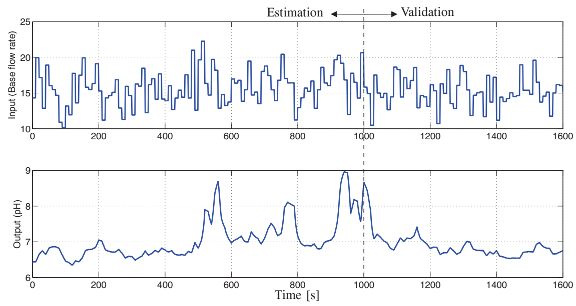

Fig. 3. Estimation and validation input (top) - output (bottom) data.

The data were used to estimate the following models: BLA, Wiener-OB, Wiener-BLA, Wiener-nlhw, Wiener-SVM, and NARX, presented in Section II-A, using the identification techniques described in Section III.

Details of the different estimated models follow:

- *BLA model:* State-space model, estimated with the n4sid routine of the System Identification Toolbox [20]. The 'best' option for the 'model order' argument was employed. Selected order: $n = 3$. The option 'prediction' was selected for the 'focus' argument.
- *Wiener-OB model:* A 4th. order linear block was estimated with poles of the bases at $\{0.98, 0.95, 0.97, 0.60\}$, while a 7th. order polynomial was estimated for the static nonlinearity. The orthonormal bases with fixed poles (OBFP) studied in [22], that have the more common FIR, Laguerre [23], and Kautz bases as special cases, were considered.
- *Wiener-BLA model:* The estimated BLA model was used as the linear block, while a 7th. order polynomial was estimated for the static nonlinearity.
- *Wiener-SVM model:* The meta-parameters of the SV-regression algorithm were set to: $\gamma = 2000$, $\epsilon = 0.001$. Gaussian Radial Basis Functions (RBF) were used as kernel functions, with a kernel bandwidth $\sigma^2 = 10$. On the other hand, OBFP with poles at $\{0.9849, 09849, 0.8305, 08305, 0.8305, 0.8305\}$, were employed for the linear block estimation.
- *Wiener-nlhw model:* The nlhw routine of the System Identification Toolbox [20] was used to estimate a Wiener model. A 3rd. order model was selected for the linear

block, while piecewise-linear functions where selected to represent the nonlinear block.

- *NARX model:* The nlarx routine of the System Identification Toolbox [20] was used to estimate a NARX model. The following parameters were chosen for the estimation: $n_a = 3$, $n_b = 3$, $n_k = 1$. A sigmoid network was chosen as the nonlinear function $\mathcal{F}$.

To quantify the prediction accuracy of the estimated models, the Best FIT, defined as:

$$\text{Best FIT} = \left(1 - \frac{\|\mathbf{y} - \mathbf{y}_v\|}{\|\mathbf{y}_v - y_{mean}\|}\right) \times 100, \qquad (28)$$

was used as the error criterion. Here $\mathbf{y}$ is a vector with the output of the model when excited with the validation input data, $\mathbf{y}_v$ is a vector with the validation output data, y_{mean} is the mean value of the validation output, and $\|\cdot\|$ stands for the Euclidean vector norm. The resulting Best FITs for the different estimated models are included in Table II. To give an idea of the complexity of the estimated models, the number of estimated parameters for each model is also included in the fourth column of Table II.

TABLE II
PREDICTION ACCURACY AND NUMBER OF PARAMETERS FOR THE DIFFERENT ESTIMATED MODELS.

Model	Method	Best FIT [%]	# parameters
Wiener	OB	80.3912	**12**
Wiener	BLA	83.8640	23
Wiener	nlhw	87.4905	25
Wiener	SVM	84.3179	#SV: 891
NARX	nlarx	**89.1412**	15
BLA	n4sid	60.3541	15

From the results in Table II, it can be concluded that the estimated linear model (BLA model) is not capable of accurately describing the system when large deviations from the operating point are taking place. Regarding the estimated Wiener models, the one with the best accuracy is the Wiener-nlhw model. However if one trades off fitting accuracy and model complexity, a good choice would be the Wiener-OB model. The overall best fitting accuracy was obtained by the estimated NARX model, with a reasonable model complexity. The Wiener-SVM also resulted with a very good fitting accuracy. Note however that only a kernel representation of the system has been estimated, instead of the parameters of the linear and nonlinear blocks in the Wiener structure.

The validation output and the outputs predicted by the estimated Wiener-nlhw and NARX-nlarx models (the two models with the best accuracy) are shown in Fig. 4.

The estimated transfer functions of the linear blocks in the Wiener models are shown in Table III, together with the transfer function associated with the BLA model.

The normalized[3] (to unit static gain) magnitude frequency responses of the linear blocks in the estimated Wiener-OB, Wiener-BLA, and Wiener-nlhw models are represented in Fig. 5. It can be observed that all the estimated linear blocks

[3]The normalization is required due to the cascade structure of the Wiener model. Without normalization, the parameters of the linear and nonlinear blocks can only be estimated *modulo* an arbitrary gain.

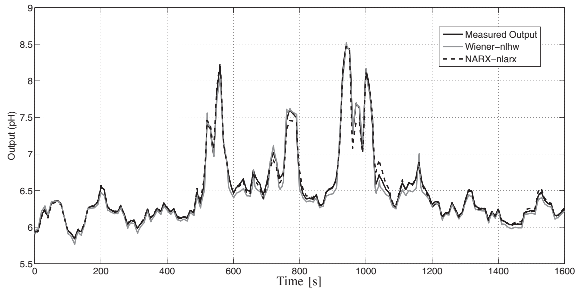

Fig. 4. Validation: Measured Output (black solid line), Outputs of the estimated Wiener-nlhw (grey solid line, BestFit = 87.4905 %), and NARX-nlarx (black-dashed line, BestFit = 89.1412 %) models.

TABLE III
ESTIMATED TRANSFER FUNCTIONS OF THE LINEAR BLOCKS IN THE WIENER MODELS, AND BLA TRANSFER FUNCTION.

Model	Method	$G(z)$
Wiener	OB	$\frac{0.0066z^3-0.0163z^2+0.0130z-0.0033}{z^4-3.50z^3+4.5431z^2-2.5849z+0.5418}$
Wiener	BLA	$\frac{0.0093z^2-0.0149z+0.0063}{z^3-2.6071z^2+2.2875z-0.6793}$
Wiener	nlhw	$\frac{z-0.7834}{z^3-1.337z^2+0.08087z+0.2606}$
BLA	n4sid	$\frac{0.0093z^2-0.0149z+0.0063}{z^3-2.6071z^2+2.2875z-0.6793}$

capture approximately the same dynamics. For the case of the Wiener-BLA model, this is an expected result since it has been proved that, when Gaussian inputs are employed, the dynamics estimated by the BLA model are equal to the ones captured by the linear block in the Wiener structure [24].

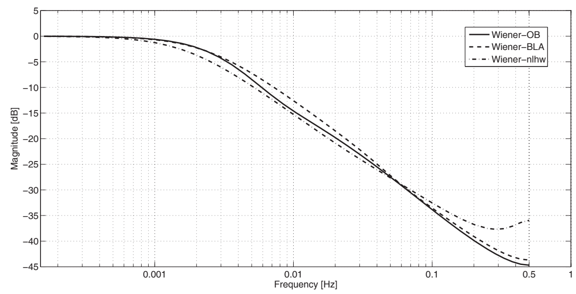

Fig. 5. Normalized (to unit static gain) magnitude frequency responses of the linear blocks in the estimated Wiener-OB (solid line), Wiener-BLA (dashed line), and Wiener-nlhw (dash-dotted line) models.

The estimated static nonlinearities for the Wiener-OB, the Wiener-BLA, and the Wiener-nlhw models are represented in Figures 6, 7, and 8, respectively.

VI. CONCLUSIONS

In this paper, several state-of-the-art nonlinear identification methods have been compared on a benchmark pH neutralization process. Wiener models estimated using methods based on orthonormal bases representations, as well as on methods based on support vector regression techniques were considered. The estimated Wiener model with the best accuracy was the Wiener-nlhw model, but with a relatively large number of parameters. The Wiener model with the best tradeoff between

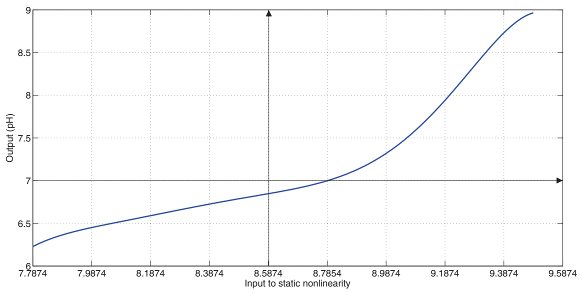

Fig. 6. Estimated static nonlinearity for the Wiener-OB model.

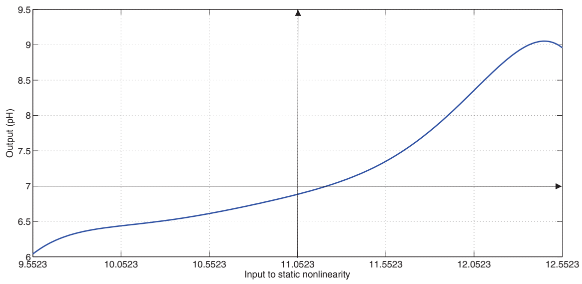

Fig. 7. Estimated static nonlinearity for the Wiener-BLA model.

fitting accuracy and model complexity was the Wiener-OB model. The overall best fitting accuracy was obtained by the estimated NARX model, with a reasonable model complexity. Even though there are many results on the use of Wiener models for control design (in particular for Nonlinear Model Predictive Control), this is not the case for NARX models and kernel models, and their suitability for control design must be still investigated.

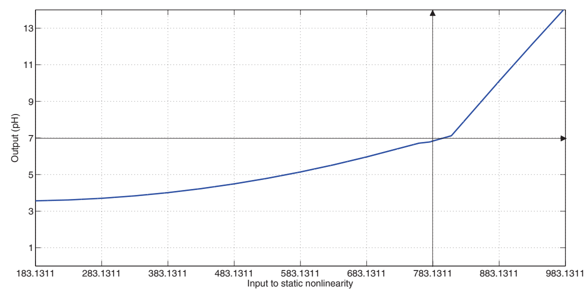

Fig. 8. Estimated static nonlinearity for the Wiener-nlhw model.

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