DIFERENCIAL DE VOLUMEN

Cartesianas $d\tau = dx\,dy\,dz$

Cilíndricas $d\tau = \rho\,d\rho\,d\varphi\,dz$

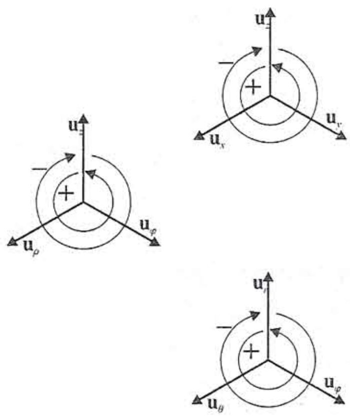
Esféricas $d\tau = r^2 \operatorname{sen}\theta\,dr\,d\theta\,d\varphi$

DELTA DE DIRAC

$$\nabla\cdot\left(\frac{\mathbf{r}}{r^3}\right) = 4\pi\delta(\mathbf{r})$$

$$\nabla^2\left(\frac{1}{r}\right) = -4\pi\delta(\mathbf{r})$$

Signos para los productos vectoriales



BASES VECTORIALES

Cartesianas

$$\begin{aligned}
\mathbf{u}_x &= \cos\varphi\,\mathbf{u}_\rho - \operatorname{sen}\varphi\,\mathbf{u}_\varphi = \operatorname{sen}\theta\cos\varphi\,\mathbf{u}_r + \cos\theta\cos\varphi\,\mathbf{u}_\theta - \operatorname{sen}\varphi\,\mathbf{u}_\varphi\\
\mathbf{u}_y &= \operatorname{sen}\varphi\,\mathbf{u}_\rho + \cos\varphi\,\mathbf{u}_\varphi = \operatorname{sen}\theta\operatorname{sen}\varphi\,\mathbf{u}_r + \cos\theta\operatorname{sen}\varphi\,\mathbf{u}_\theta + \cos\varphi\,\mathbf{u}_\varphi\\
\mathbf{u}_z &= \mathbf{u}_z = \cos\theta\,\mathbf{u}_r - \operatorname{sen}\theta\,\mathbf{u}_\theta
\end{aligned}$$

Cilíndricas

$$\begin{aligned}
\cos\varphi\,\mathbf{u}_x + \operatorname{sen}\varphi\,\mathbf{u}_y &= \mathbf{u}_\rho = \operatorname{sen}\theta\,\mathbf{u}_r + \cos\theta\,\mathbf{u}_\theta\\
-\operatorname{sen}\varphi\,\mathbf{u}_x + \cos\varphi\,\mathbf{u}_y &= \mathbf{u}_\varphi = \mathbf{u}_\varphi\\
\mathbf{u}_z &= \mathbf{u}_z = \cos\theta\,\mathbf{u}_r - \operatorname{sen}\theta\,\mathbf{u}_\theta
\end{aligned}$$

Esféricas

$$\begin{aligned}
\operatorname{sen}\theta\cos\varphi\,\mathbf{u}_x + \operatorname{sen}\theta\operatorname{sen}\varphi\,\mathbf{u}_y + \cos\theta\,\mathbf{u}_z &= \operatorname{sen}\theta\,\mathbf{u}_\rho + \cos\theta\,\mathbf{u}_z = \mathbf{u}_r\\
\cos\theta\cos\varphi\,\mathbf{u}_x + \cos\theta\operatorname{sen}\varphi\,\mathbf{u}_y - \operatorname{sen}\theta\,\mathbf{u}_z &= \cos\theta\,\mathbf{u}_\rho - \operatorname{sen}\theta\,\mathbf{u}_z = \mathbf{u}_\theta\\
-\operatorname{sen}\varphi\,\mathbf{u}_x + \cos\varphi\,\mathbf{u}_y &= \mathbf{u}_\varphi = \mathbf{u}_\varphi
\end{aligned}$$

GRADIENTE

Cartesianas

$$\nabla\phi = \frac{\partial\phi}{\partial x}\mathbf{u}_x + \frac{\partial\phi}{\partial y}\mathbf{u}_y + \frac{\partial\phi}{\partial z}\mathbf{u}_z$$

Cilíndricas

$$\nabla\phi = \frac{\partial\phi}{\partial\rho}\mathbf{u}_\rho + \frac{1}{\rho}\frac{\partial\phi}{\partial\varphi}\mathbf{u}_\varphi + \frac{\partial\phi}{\partial z}\mathbf{u}_z$$

Esféricas

$$\nabla\phi = \frac{\partial\phi}{\partial r}\mathbf{u}_r + \frac{1}{r}\frac{\partial\phi}{\partial\theta}\mathbf{u}_\theta + \frac{1}{r\operatorname{sen}\theta}\frac{\partial\phi}{\partial\varphi}\mathbf{u}_\varphi$$

DIVERGENCIA

Cartesianas

$$\nabla\cdot\mathbf{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

Cilíndricas

$$\nabla\cdot\mathbf{A} = \frac{1}{\rho}\frac{\partial(\rho A_\rho)}{\partial\rho} + \frac{1}{\rho}\frac{\partial A_\varphi}{\partial\varphi} + \frac{\partial A_z}{\partial z}$$

Esféricas

$$\nabla\cdot\mathbf{A} = \frac{1}{r^2}\frac{\partial(r^2A_r)}{\partial r} + \frac{1}{r\operatorname{sen}\theta}\frac{\partial(\operatorname{sen}\theta A_\theta)}{\partial\theta} + \frac{1}{r\operatorname{sen}\theta}\frac{\partial A_\varphi}{\partial\varphi}$$

ROTACIONAL

Cartesianas

$$\nabla\times\mathbf{A} = \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right)\mathbf{u}_x + \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right)\mathbf{u}_y + \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)\mathbf{u}_z$$

Cilíndricas

$$\nabla\times\mathbf{A} = \left(\frac{1}{\rho}\frac{\partial A_z}{\partial\varphi} - \frac{\partial A_\varphi}{\partial z}\right)\mathbf{u}_\rho + \left(\frac{\partial A_\rho}{\partial z} - \frac{\partial A_z}{\partial\rho}\right)\mathbf{u}_\varphi + \frac{1}{\rho}\left(\frac{\partial(\rho A_\varphi)}{\partial\rho} - \frac{\partial A_\rho}{\partial\varphi}\right)\mathbf{u}_z$$

Esféricas

$$\nabla\times\mathbf{A} = \frac{1}{r\operatorname{sen}\theta}\left(\frac{\partial(\operatorname{sen}\theta A_\varphi)}{\partial\theta} - \frac{\partial A_\theta}{\partial\varphi}\right)\mathbf{u}_r + \frac{1}{r}\left(\frac{1}{\operatorname{sen}\theta}\frac{\partial A_r}{\partial\varphi} - \frac{\partial(rA_\varphi)}{\partial r}\right)\mathbf{u}_\theta + \frac{1}{r}\left(\frac{\partial(rA_\theta)}{\partial r} - \frac{\partial A_r}{\partial\theta}\right)\mathbf{u}_\varphi$$

LAPLACIANO

Cartesianas

$$\nabla^2\phi = \frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} + \frac{\partial^2\phi}{\partial z^2}$$

Cilíndricas

$$\nabla^2\phi = \frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial\phi}{\partial\rho}\right) + \frac{1}{\rho^2}\frac{\partial^2\phi}{\partial\varphi^2} + \frac{\partial^2\phi}{\partial z^2}$$

Esféricas

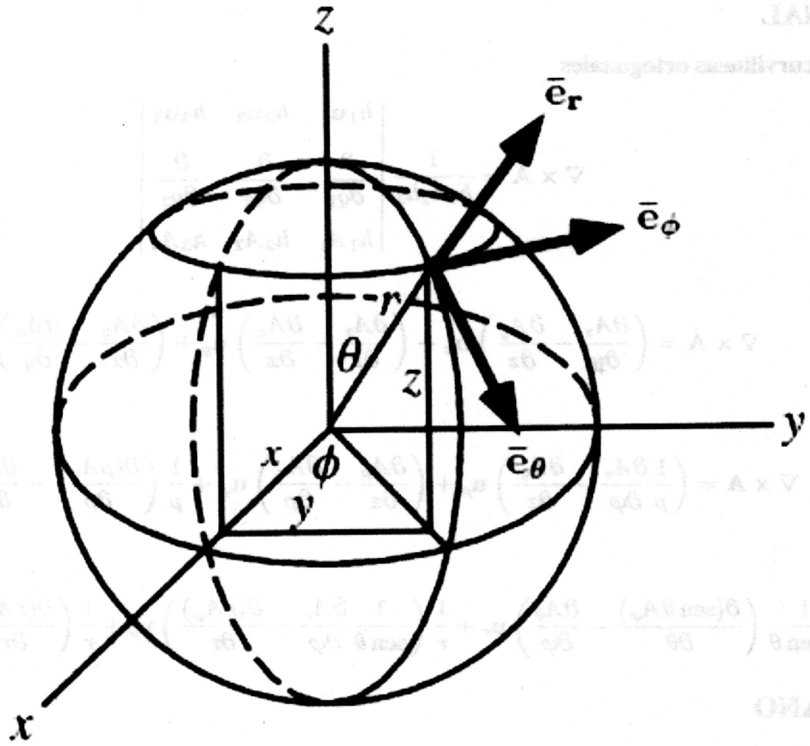
$$\nabla^2\phi = \frac{1}{r}\frac{\partial^2}{\partial r^2}(r\phi) + \frac{1}{r^2\operatorname{sen}\theta}\frac{\partial}{\partial\theta}\left(\operatorname{sen}\theta\frac{\partial\phi}{\partial\theta}\right) + \frac{1}{r^2\operatorname{sen}^2\theta}\frac{\partial^2\phi}{\partial\varphi^2}$$

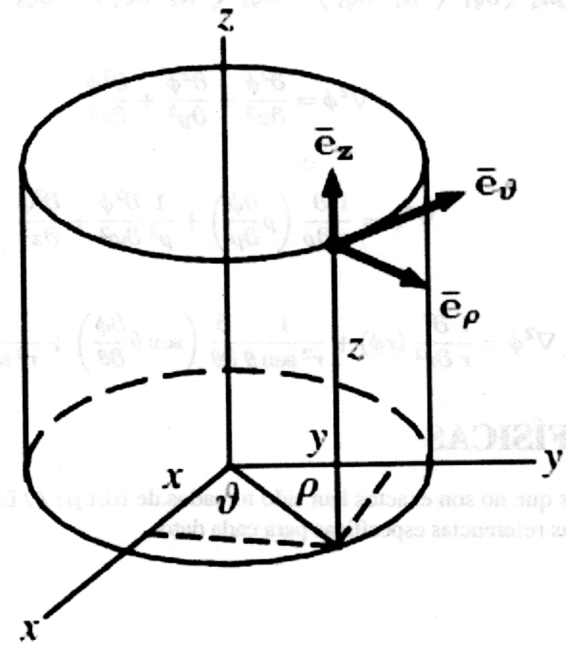
Polinómios de Legendre

n	$P_n(\theta)$
0	1
1	$\cos\theta$
2	$\frac{1}{2}(3\cos^2\theta - 1)$
3	$\frac{1}{2}(5\cos^3\theta - 3\cos\theta)$

Armónicos Cilíndricos

1,	$\ln r$,
$r^n\cos n\theta$,	$r^{-n}\cos n\theta$,
$r^n\operatorname{sen} n\theta$,	$r^{-n}\operatorname{sen} n\theta$.


z
$\bar{e}_r$
$\bar{e}_\phi$
r
θ
z
y
x
ϕ
y
$\bar{e}_\theta$
x


z
$\bar{e}_z$
$\bar{e}_\vartheta$
$\bar{e}_\rho$
z
y
y
x
ρ
ϑ
x