TABLA DEL SISTEMA P_{T2}

T02CO5.98 D.S.F.

FT

$$\frac{Kw_n^2}{s^2+2xw_ns+w_n^2}$$

ó

$$\frac{K}{T^2s^2+2xTs+1}$$

Donde :

$T = 1/w_n$

K: *ganancia estática*

x: *coeficiente de amortiguamiento*

w_n: *frecuencia natural no amortiguada*

EDO

$$\ddot{y}+2xw_n\,\dot{y}+w_n^2\,y=Kw_n^2\,u$$

ó

$$T^2\,\ddot{y}+2xT\,\dot{y}+y=K\cdot u$$

EE/ES

$$x_1 = y$$
$$x_2 = \dot{y}$$

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = -w_n^2x_1 - 2xw_nx_2 + Kw_n^2u$$

$$y = x_1$$

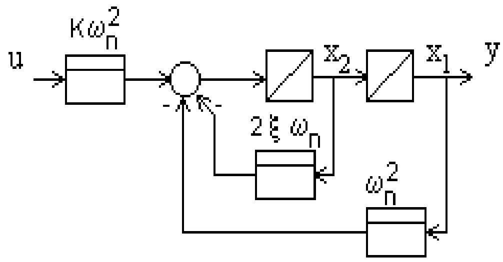
$$\dot{X} = AX + bu$$
$$y = c^T X$$

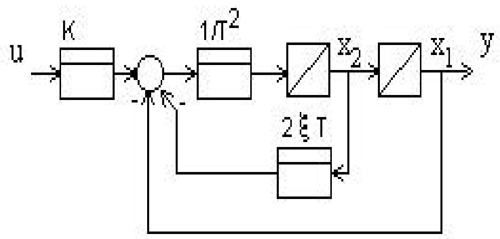
$$A=\begin{bmatrix} 0 & 1 \\ -w_n^2 & -2xw_n \end{bmatrix}$$

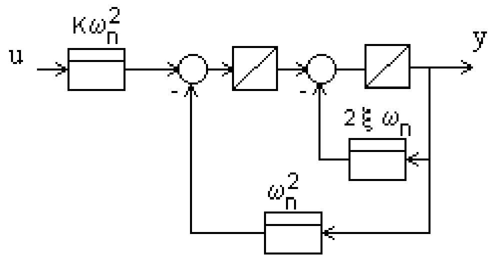
$$b=\begin{bmatrix} 0 \\ Kw_n^2 \end{bmatrix}$$

$$c^T = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

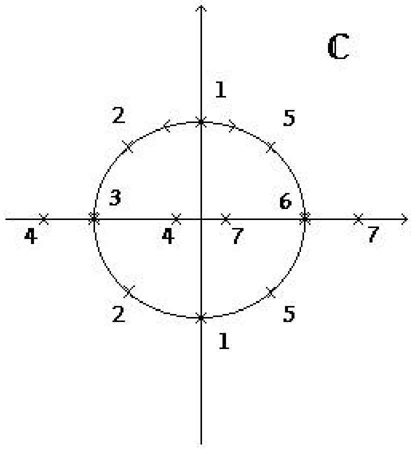
DB





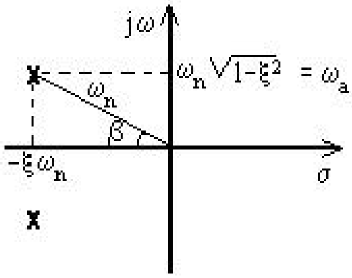


Respuesta al escalón del P_{T2} para diferentes parámetros de la FT

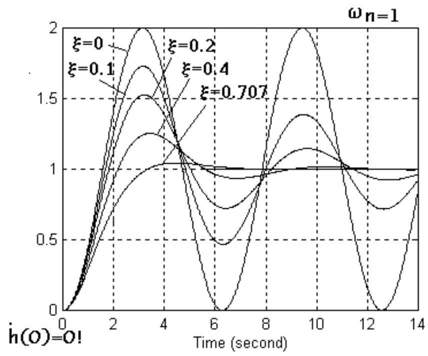
<table>
<tr><th>Polos</th><th>Amortiguación</th><th>Respuesta al Escalón h(t)</th></tr>
<tr><td rowspan="4">


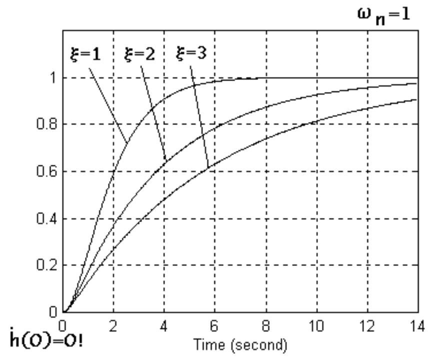
(1) $x = 0$
(2) $0 < x < 1$
(3) $x = 1$
(4) $x > 1$
(5) $-1 < x < 0$
(6) $x = -1$
(7) $x < -1$

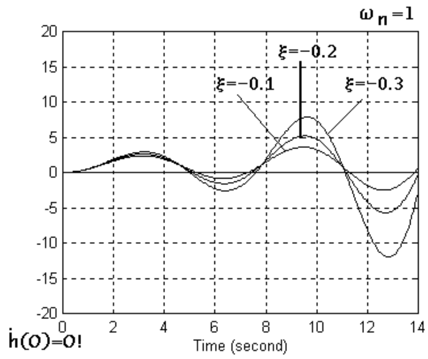
$x = 1 \longrightarrow s_{1,2} = -w_n$
$x > 1 \longrightarrow s_{1,2} = -x w_n \pm w_n \sqrt{x^2 - 1} = \frac{1}{T_{1,2}}$
$0 < x < 1 \longrightarrow s_{1,2} = -x w_n \pm j w_n \sqrt{1 - x^2} = -s \pm j w_a$

Ubicación en el plano complejo de los polos para 0<ξ<1


</td>
<td>ξ= 0
no amortig.
(inestable o marginalmente estable)

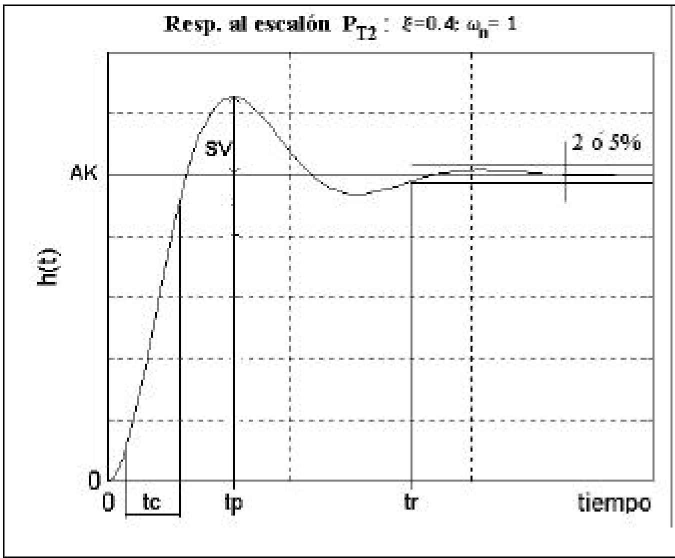
$0 < x < 1$
subamortig.</td>
<td>


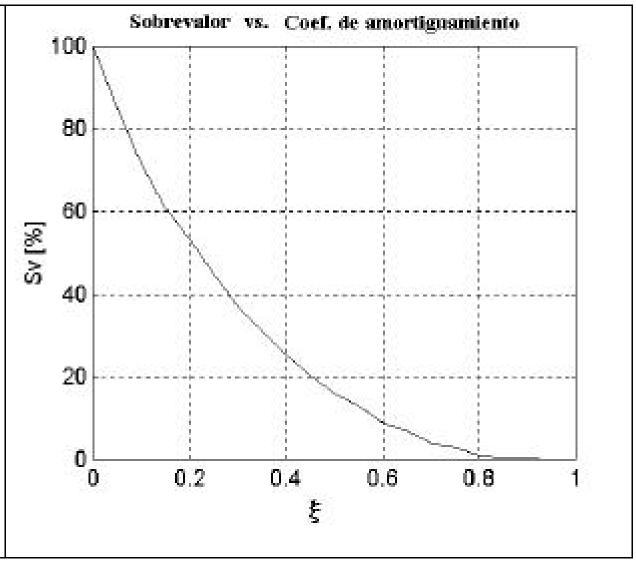
$h(t) = K - \frac{K}{\sqrt{1 - x^2}} e^{-x w_n t} \,\mathrm{sen}(w_a \cdot t + b)$
</td></tr>
<tr><td>$x = 1$
amortig. crítico</td>
<td>
$h(t) = K \;(1 - e^{-x w_n t} - x w_n \, t \, e^{-x w_n t})$


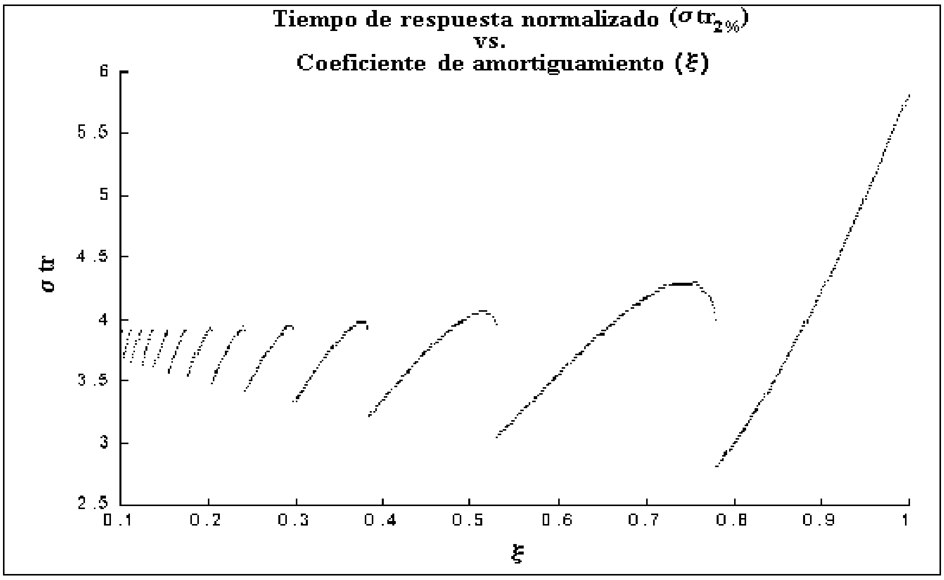
$h(t) = K - \frac{K}{T_1 - T_2} \left(T_1 e^{-\frac{t}{T_1}} - T_2 e^{-\frac{t}{T_2}} \right)$
</td></tr>
<tr><td>$x > 1$
sobreamort. o aperiódico</td><td></td></tr>
<tr><td>$x < 0$
inestable</td>
<td>


</td></tr>
</table>

Características de la respuesta al escalón ($0 < \xi < 1$):







$$SV = A \cdot K \cdot e^{\frac{-p \cdot x}{\sqrt{1-x^2}}}$$ (Sobrevalor)

A: amplitud del escalón de entrada
K: ganancia estática

$$D\% = 100 \cdot e^{\frac{-p \cdot x}{\sqrt{1-x^2}}}$$

$tp = \pi / \omega_a$ (Tiempo de Pico o Peak-time)

$tr \approx 4/ \sigma$ (criterio 2%) (Tiempo de Respuesta o Settling-time)
$tr \approx 3 / \sigma$ (criterio 5%)

$tc_{\,0\text{-}100\%} = (\pi - \beta) / \omega_a$ (Tiempo de Crecimiento o Rise-time)
$tc_{\,10\text{-}90\%} \approx 2.5/ \omega_a$

Respuesta frecuencial del P_{T2}

$$u(t) = A\,\mathrm{sen}(w \cdot t) \Rightarrow$$

$$y(t) = A \cdot \left|G(s = jw)\right| \cdot \mathrm{sen}\left[w\,t + \underline{\angle\, G(jw)}\right] = |y| \cdot \mathrm{sen}(w \cdot t + \Phi)$$

$$G(jw) = \frac{K}{-\frac{w^2}{w_n^2} + j2x\frac{w}{w_n} + 1}$$

$$|y|_{dB} = 20\log(AK) - 20\log\left[\sqrt{\left(1 - \frac{w^2}{w_n^2}\right)^2 + \left(2x\frac{w}{w_n}\right)^2}\right]$$

$$\Phi = -\,\mathrm{arctg}\left[\frac{2x\frac{w}{w_n}}{1 - \frac{w^2}{w_n^2}}\right]$$