Modeling, Simulation and Motion Control of Marine Vehicles

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- Deterministic modeling of Marine vehicles.

Part II:

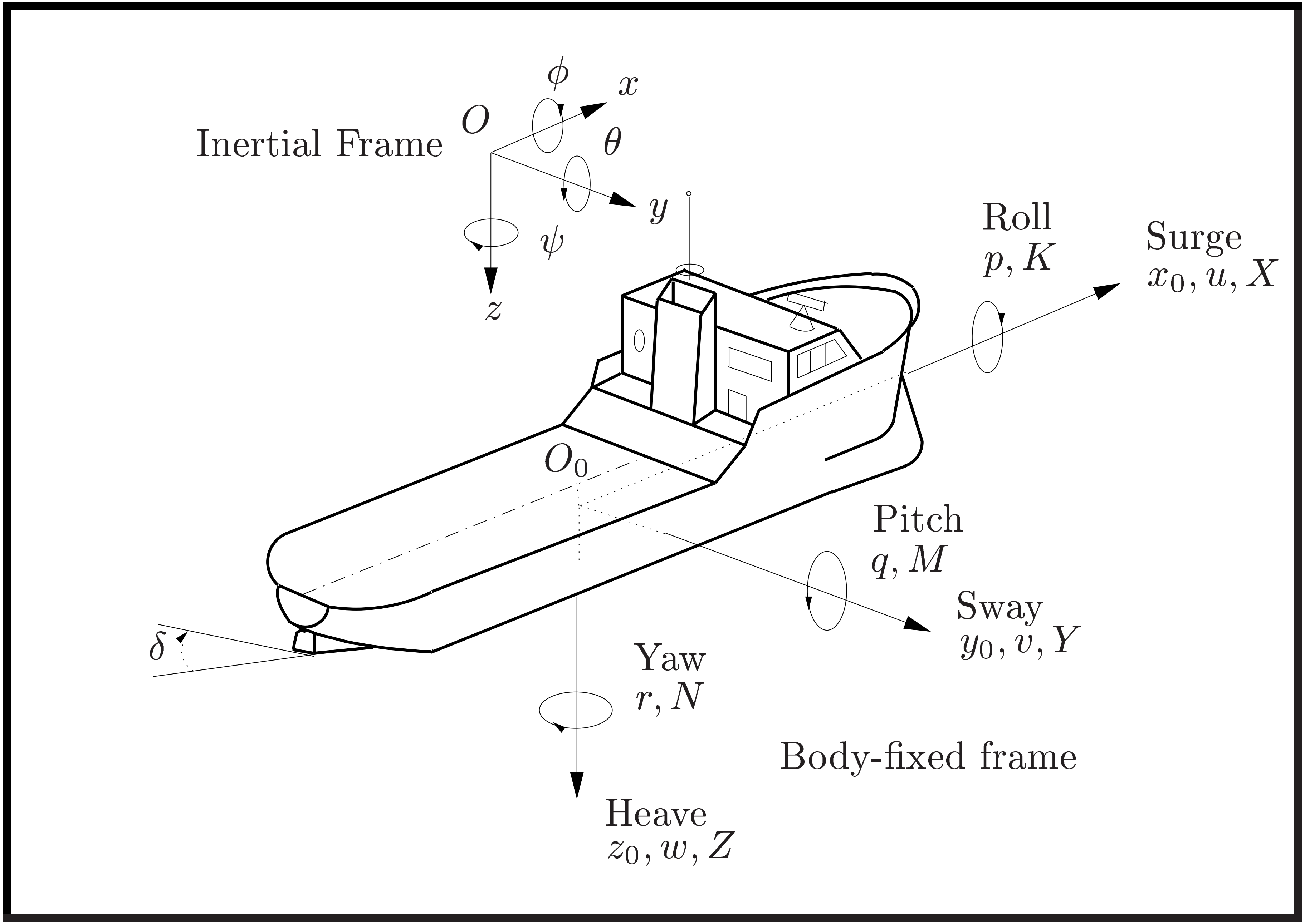
- Stochastic motion modeling: ship motion in seaway.

Part III:

- Rudder Roll Stabilization.

Part I: Modeling of Marine Vehicles

- Motion description: kinematics and dynamics.
- Motion control problems.
- Deterministic ship modeling.


Inertial Frame
O
ϕ
x
θ
y
ψ
z
Roll
p, K
Surge
x_0, u, X
O_0
Pitch
q, M
Sway
y_0, v, Y
δ
Yaw
r, N
Body-fixed frame
Heave
z_0, w, Z

Kinematics

Let us define the position-orientation vector η and linear-angular velocity vector ν with respect to the body-fixed frame:

$$\eta \triangleq \begin{bmatrix} x & y & z & \phi & \theta & \psi \end{bmatrix}^{\mathrm{T}}$$

$$\nu \triangleq \begin{bmatrix} u & v & w & p & q & r \end{bmatrix}^{\mathrm{T}}$$

Then, the position-orientation rate vector $\dot{\eta}$ is related to ν via:

$$\dot{\eta} = J(\eta)\,\nu\,,$$

where $J(\eta)$ is a transformation matrix that depends on the Euler angles $(\phi\,,\theta\,,\psi\,)$ and is of the form (Fossen, 1994):

$$J(\eta) = \begin{bmatrix} J_1(\phi\,,\theta\,,\psi) & \mathbf{0}_{3\times 3} \\ \mathbf{0}_{3\times 3} & J_2(\phi\,,\theta\,,\psi) \end{bmatrix}$$

Dynamics: Newtonian approach

The equations of motion of vehicle in the body fixed frame are given in a vector form by:

$$\begin{aligned} M_{RB}\dot{\nu} + C_{RB}(\nu)\nu &= \tau(\dot{\nu}, \nu, \eta), \\ \dot{\eta} &= J(\eta)\nu. \end{aligned}$$

The forces and moments vector τ, defined as

$$\tau = \begin{bmatrix} X & Y & Z & K & M & N \end{bmatrix}^{\mathrm{T}},$$

takes into account magnitudes generated by different phenomena.

Model in 6-DOF

$$\begin{bmatrix} m[\dot{u} - y_G\dot{r} + z_G\dot{q}] \\ m[\dot{v} - z_G\dot{p} + x_G\dot{r}] \\ m[\dot{w} - x_G\dot{q} + y_G\dot{p}] \\ I_{xx}\dot{p} + my_G\dot{w} - mz_G\dot{v} \\ I_{yy}\dot{q} + mz_G\dot{u} - mx_G\dot{w} \\ I_{zz}\dot{r} + mx_G\dot{v} - my_G\dot{u} \end{bmatrix} + \begin{bmatrix} m[-vr + wq - x_G(q^2 + r^2) + y_G pq + z_G pr] \\ m[-wp + ur - y_G(r^2 + p^2) + z_G qr + x_G qp] \\ m[-uq + vq - z_G(p^2 + q^2) + x_G rp + y_G rq] \\ (I_{zz} - I_{yy})qr + m[y_G(-uq + vp) - z_G(-wp + ur) \\ (I_{xx} - I_{zz})rp + m[z_G(-vr + wq) - x_G(-uq + vp) \\ (I_{yy} - I_{xx})pq + m[x_G(-wp + ur) - y_G(-vr + wq)] \end{bmatrix} = \begin{bmatrix} X \\ Y \\ Z \\ K \\ M \\ N \end{bmatrix},$$

Model in 4-DOF

$$\begin{bmatrix} m\dot{u} \\ m\dot{v} - mz_G\dot{p} + mx_G\dot{r} \\ -mz_G\dot{v} + I_{xx}\dot{p} \\ mx_G\dot{v} + I_{zz}\dot{r} \end{bmatrix} + \begin{bmatrix} m(-vr - x_G r^2 + z_g pr) \\ mur \\ -mz_G ur \\ mx_G ur \end{bmatrix} = \begin{bmatrix} X \\ Y \\ K \\ N \end{bmatrix}.$$

The magnitudes in τ can be classified as

$$\tau = \tau_{hyd} + \tau_{rudd} + \tau_{prop} + \tau_{ext},$$

where

- *hyd*: These forces and moments arise from the movement of the hull in the water.
- *prop*: These forces and moments come from the propulsion system, e.g., propellers and thrusters.
- *rudd*: These forces and moments arise due to the rudder, fins, etc. movement.
- *ext*: These are the forces and moments acting on the hull due to the environmental disturbances, e.g., wind, currents and waves.

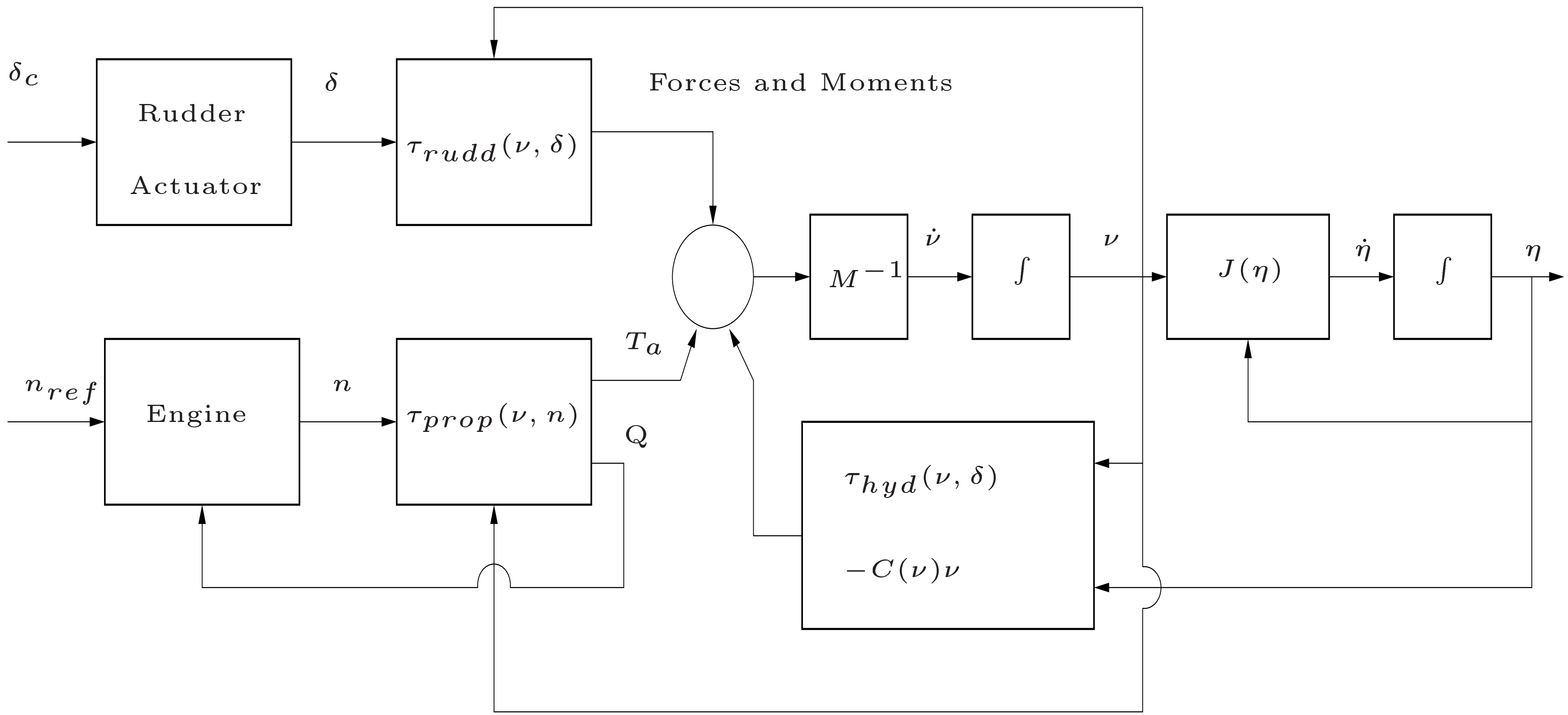


Figure 1: Marine vehicles' dynamics.

Hydrodynamic forces and moments.

The hydrodynamic forces and moments can be studied by considering two problems that study:

- The movement of the hull when there are no incident waves considered.
- The forces when hull is restrained from moving and there are incident waves. The second problem involves environmental forces like waves, wind and currents.

The hyd. forces and moments for the 1st problem are classified (Blanke, 1981)

- *Motion in an ideal fluid with no circulation*: In this analysis, only the displacement is considered, and it reveals the so-called added mass and inertia forces and moments and Munk moment.
- *Motion in an ideal fluid with circulation*: In this analysis the shape of the hull is relevant.
- *Motion in a viscous fluid*: This analysis reveals the presence of hydrodynamic resistance.
- *Gravitational and buoyancy forces*: These are the restoring forces and moments that depend on the Euler angles and act on the center of gravity CG and the center of buoyancy CB

The Hydrodynamic forces and moments are modeled as a nonlinear function, included in η:

$$\tau_{hyd} = \mathbf{f}(\dot{\nu}, \nu, \eta),$$

and can be expressed in a series expansion that is affine in the parameters or coefficients. For example, for Y_{hyd} force:

$$Y_{hyd} \approx Y_{\dot{v}}\dot{v} + Y_{vv}v^2 + Y_{r|v|}r|v| + \ldots$$

where the constant coefficients

$$Y_{\dot{v}} = \frac{\partial f_Y}{\partial \dot{v}} \qquad Y_{vv} = \frac{\partial^2 f_Y}{\partial v^2} \qquad Y_{r|v|} = \frac{\partial^2 f_Y}{\partial r \partial |v|}$$

are referred to as *hydrodynamic derivatives.*

Example: Multi-role Naval vessel (Christensen, and Blanke, 1993)

The surge equation is

$$X = X_{\dot{u}}\dot{u} + X(u) + X_{vr}vr,$$

The sway equation is

$$\begin{aligned} Y = {} & Y_{\dot{v}}\dot{v} + Y_{\dot{r}}\dot{r} + Y_{\dot{p}}\dot{p} \\ & + Y_{|u|v}\left|u\right|v + Y_{ur}ur + Y_{v|v|}v\left|v\right| + Y_{v|r|}v\left|r\right| + Y_{r|v|}r\left|v\right| \\ & + Y_{\phi|uv|}\phi\left|uv\right| + Y_{\phi|ur|}\phi\left|ur\right| + Y_{\phi uu}\phi\, u^2 \end{aligned}$$

The roll equation is

$$
\begin{aligned}
K = {} & K_{\dot{v}}\dot{v} + K_{\dot{p}}\dot{p} \\
& + K_{|u|v}\,|u|\,v + K_{ur}ur + K_{v|v|}v\,|v| + K_{v|r|}v\,|r| + K_{r|v|}r\,|v| \\
& + K_{\phi|uv|}\phi\,|uv| + K_{\phi|ur|}\phi\,|ur| + K_{\phi uu}\phi\,u^2 + K_{|u|p}\,|u|\,p \\
& + K_{p|p|}p\,|p| + K_p p + K_{\phi\phi\phi}\phi^3 - \rho g \nabla G_z(\phi)
\end{aligned}
$$

The yaw equation is

$$
\begin{aligned}
N = {} & N_{\dot{v}}\dot{v} + N_{\dot{r}}\dot{r} \\
& + N_{|u|v}\,|u|\,v + N_{|u|r}\,|u|\,r + N_{r|r|}r\,|r| + N_{r|v|}r\,|v| \\
& + N_{\phi|uv|}\phi\,|uv| + N_{\phi u|r|}\phi\,u\,|r| + N_{\phi u|u|}\phi\,u\,|u|
\end{aligned}
$$

Propulsion system and resistance

The propeller generates thrust, T_a to compensate the resistance forces $X(u)$. In static conditions

$$0 = X(u) + T_a$$

The resistance is made up of a number of different components:

- The frictional resistance, due to the motion of the hull in viscous fluid.
- The wave-making resistance, due to the energy carried away by the generated waves created on the surface.
- Eddy resistance due to energy carried away by eddies shed from the hull and appendages.

To model the resistance, a polynomial in the surge speed is used:

$$X(u) = X_{u|u|} u|u|.$$

Interaction between Propeller and hull

When the ship moves in real fluid, the water around the stern acquires a forward motion in the direction of the motion of the hull. The moving water is called the *wake*, and the wake speed is

$$w = \frac{U - V_a}{U} \qquad and \qquad V_a = (1 - w)U.$$

A model for the propeller is then given by

$$T = T_{|n|n}|n|n + T_{|n|V_a}|n|V_a$$
$$Q = Q_{|n|n}|n|n + Q_{|n|V_a}|n|V_a \,.$$

Finally, there is an increase in the hyd. resistance and is modeled as

$$T_a = (1 - t)T \,.$$

Control surfaces: rudders and fins

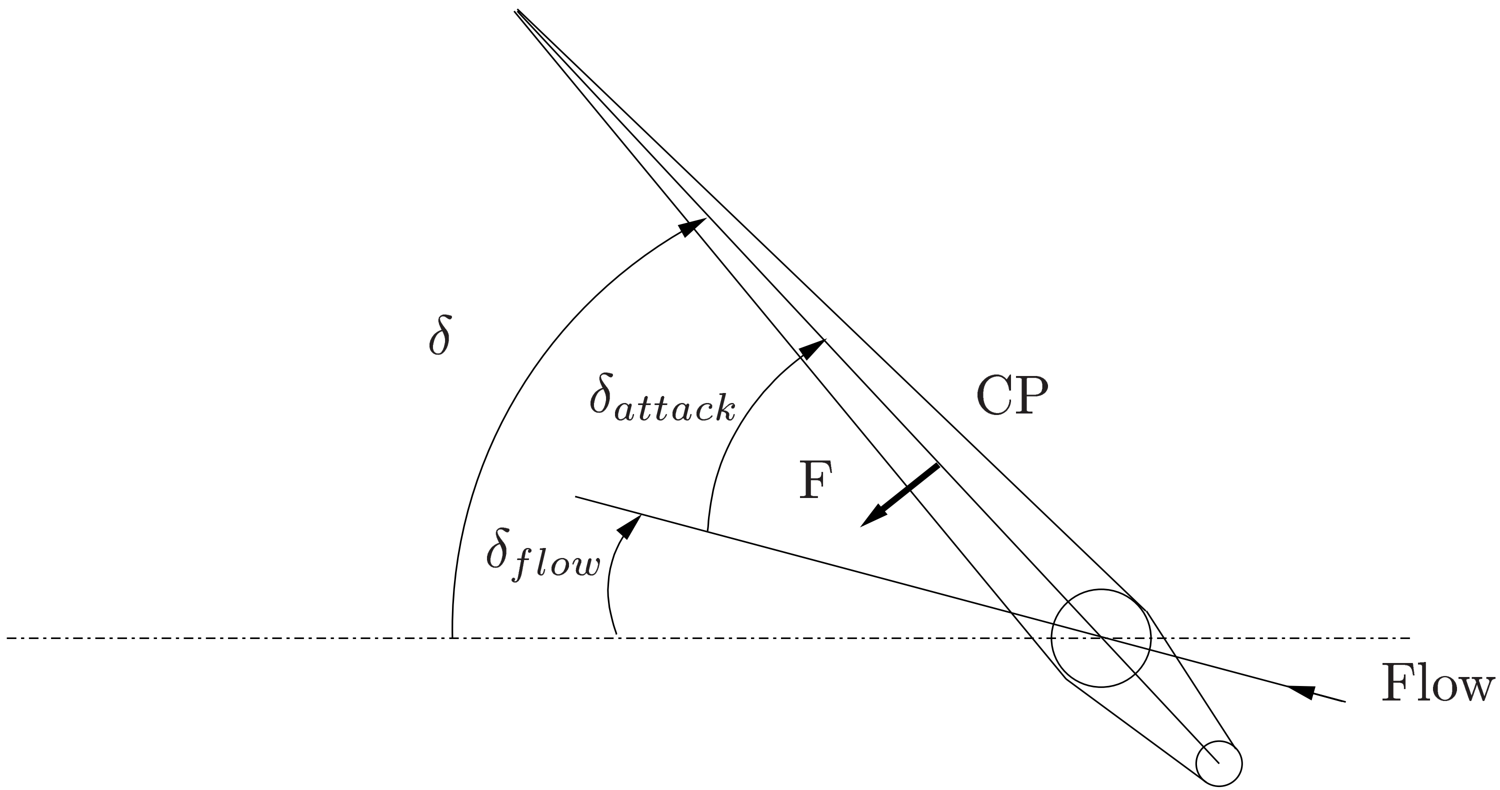


Figure 2: Rudder angles definition and convension.

Force on the rudder

$$F = \begin{cases} \frac{1}{2}\rho C_F A_r V_{av}^2 sin(\frac{\pi}{2}\frac{\delta_{attack}}{\delta_{stall}}) & \text{if } |\delta_{attack}| < \delta_{stall} \ , \\ \frac{1}{2}\rho C_F A_r V_{av}^2 sign(\delta_{attack}) & \text{if } |\delta_{attack}| \geq \delta_{stall} \ , \end{cases}$$

The angle δ_{attak} is calculated using the rudder angle δ the sway velocity v, the surge velocity u and the sway velocity at stern produced by the turn rate of the ship $(x_{cp} - x_G)r$ as

$$\begin{aligned} \delta_{attack} &= \delta - \delta_{flow} \\ &= \delta - \arctan(\frac{v + (x_{cp} - x_G)r}{u}). \end{aligned}$$

Forces due to the rudder acting on the hull

$$X_{rudd} = -F(u, V_{av}, v, r, \delta)\sin(\delta),$$
$$Y_{rudd} = F(u, V_{av}, v, r, \delta)\cos(\delta),$$
$$Z_{rudd} = 0.$$

and the moments are

$$[K_{rudd} \quad M_{rudd} \quad N_{rudd}]^{\mathrm{T}} = (\overline{CP} - \overline{CG}) \times [X_{rudd} \quad Y_{rudd} \quad Z_{rudd}]^{\mathrm{T}}.$$

The flow passing the rudder V_{av} is very much influenced by the propeller, and the average flow is

$$V_{av}^2 \approx V_a^2 + C_T T,$$

where

$$C_T \approx \frac{6.4}{\pi \rho h D_p}.$$

Generalization to any fin

The forces on the fin frame are given by

$$X_f = -F\sin(\delta_r),$$

$$Y_f = F\cos(\delta_r),$$

$$Z_f = 0,$$

The forces acting on the center of gravity are obtained via the following transformation (Fossen, 1994):

$$[X \quad Y \quad Z]^{\mathrm{T}} = Rot(\lambda, \theta_{tilt})[X_f \quad Y_f \quad Z_f]^{\mathrm{T}},$$

where

$$Rot(\lambda, \theta_{tilt}) = \cos(\theta_{tilt})\mathbf{I} + (1 - \cos(\theta_{tilt}))\lambda\lambda^{\mathrm{T}} - \sin(\theta_{tilt})\mathbf{S}(\lambda).$$

The moments are calculated as:

$$[K \quad M \quad N]^{\mathrm{T}} = (\overline{CP} - \overline{CG}) \times [Y \quad Y \quad Z]^{\mathrm{T}}.$$

State space No-linear models

The non-linear state space model has the general form

$$\dot{x} = H^{-1} f(x, \delta)\,,$$

where

$$f(x, \delta) \triangleq f_{hyd}(x) + f_{rudd}(x, \delta) + f_{acc}(x)\,.$$

For example, for a model in 4-DOF, x is usually taken as

$$x = \begin{bmatrix} u & v & r & p & \phi & \psi \end{bmatrix}^{\mathrm{T}}\,.$$

The matrix H is given by

$$H = \begin{bmatrix} (m - X_{\dot{u}}) & 0 & 0 & 0 & 0 & 0 \\ 0 & (m - Y_{\dot{v}}) & -(mz_G + Y_{\dot{p}}) & (mx_G - Y_{\dot{r}}) & 0 & 0 \\ 0 & -(mz_G + K_{\dot{v}}) & (I_{xx} - K_{\dot{p}}) & -K_{\dot{r}} & 0 & 0 \\ 0 & (mx_G - N_{\dot{v}}) & -N_{\dot{p}} & (I_{zz} - N_{\dot{r}}) & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$f(x, \delta) = \begin{bmatrix} X^*_{hyd}(x) + X_{rudder}(x, \delta) + m(vr + x_G r^2 - z_G pr) \\ Y^*_{hyd}(x) + Y_{rudder}(x, \delta) - mur \\ K^*_{hyd}(x) + K_{rudder}(x, \delta) + mz_G ur \\ N^*_{hyd}(x) + N_{rudder}(x, \delta) - mx_G ur \\ p \\ r\cos(\phi) \end{bmatrix}$$

Linearized models

It is a common practice to decouple the surge equation from the others to analyze the linearized models considering a given service speed $\overline{u}$. Then, for $z \triangleq \begin{bmatrix} v & r & p & \phi & \psi \end{bmatrix}^{\mathrm{T}}$,

$$\dot{z} = H^{-1}A \quad z + M^{-1}B \quad \delta.$$

where,

$$A = \left.\frac{\partial f_{hyd}(z,u)}{\partial z}\right|_{\overline{z},\overline{u}} + \left.\frac{\partial f_{rudder}(z,u,V_{av},\delta)}{\partial z}\right|_{\overline{z},\overline{u},\overline{V}_{av},\overline{\delta}} + \left.\frac{\partial f_{acc}(z)}{\partial z}\right|_{\overline{z}}$$

$$B = \left.\frac{\partial f_{rudder}(z,u,V_{av},\delta)}{\partial \delta}\right|_{\overline{x},\overline{u},\overline{V}_{av},\overline{\delta}},$$

The matrix H is as defined before, but without the first row and first column, and $\overline{z} = [0 \quad 0 \quad 0 \quad 0 \quad 0]^{\mathrm{T}}$ and $\overline{\delta} = 0$.

Motion control problems and associated variables

Control application	Subset of ν	Subset of η	F M
Surface ships:			
Forward speed	u, v, r		X
Course Autopilot	r	ψ	N
Autopilot + RRS	v, p, r	ϕ, ψ	Y, K, N
Dynamic Positioning	u, v, r	x, y, ψ	X, Y, N
Submarines:			
Pitch and Depth	u, w, q	z, θ	Z, M

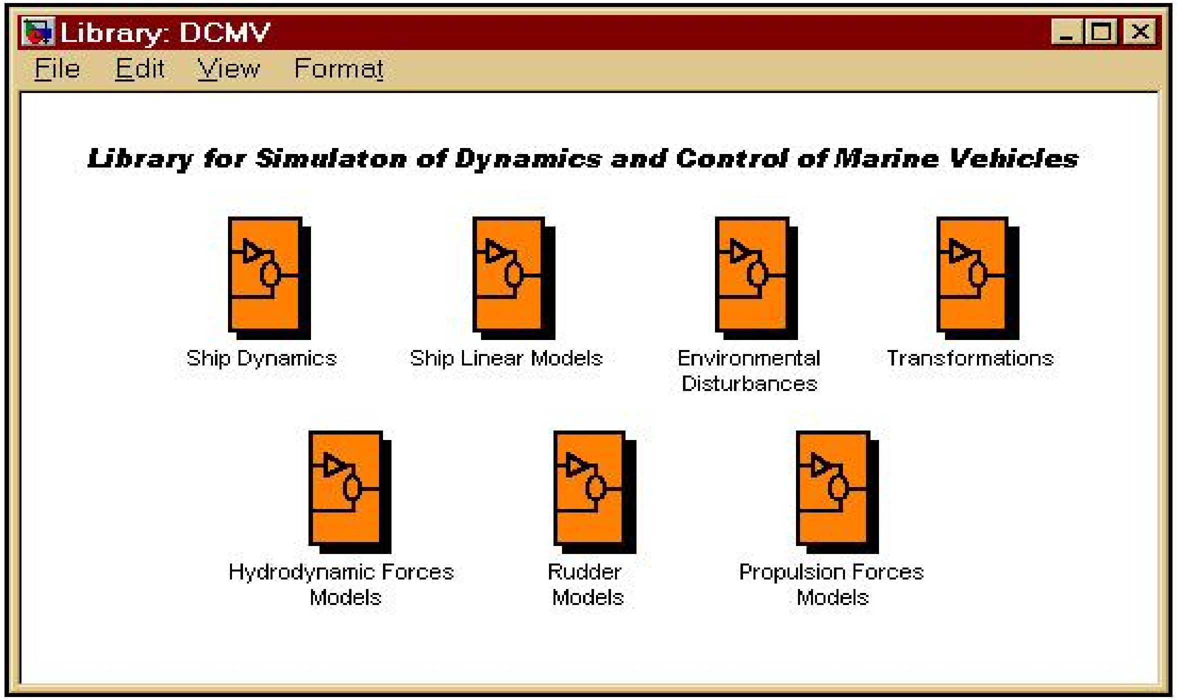
Matlab/Simulink based Toolbox: DCMV

DCMV is a Matlab/Simulink based toolbox for simulation and control of marine vehicles.

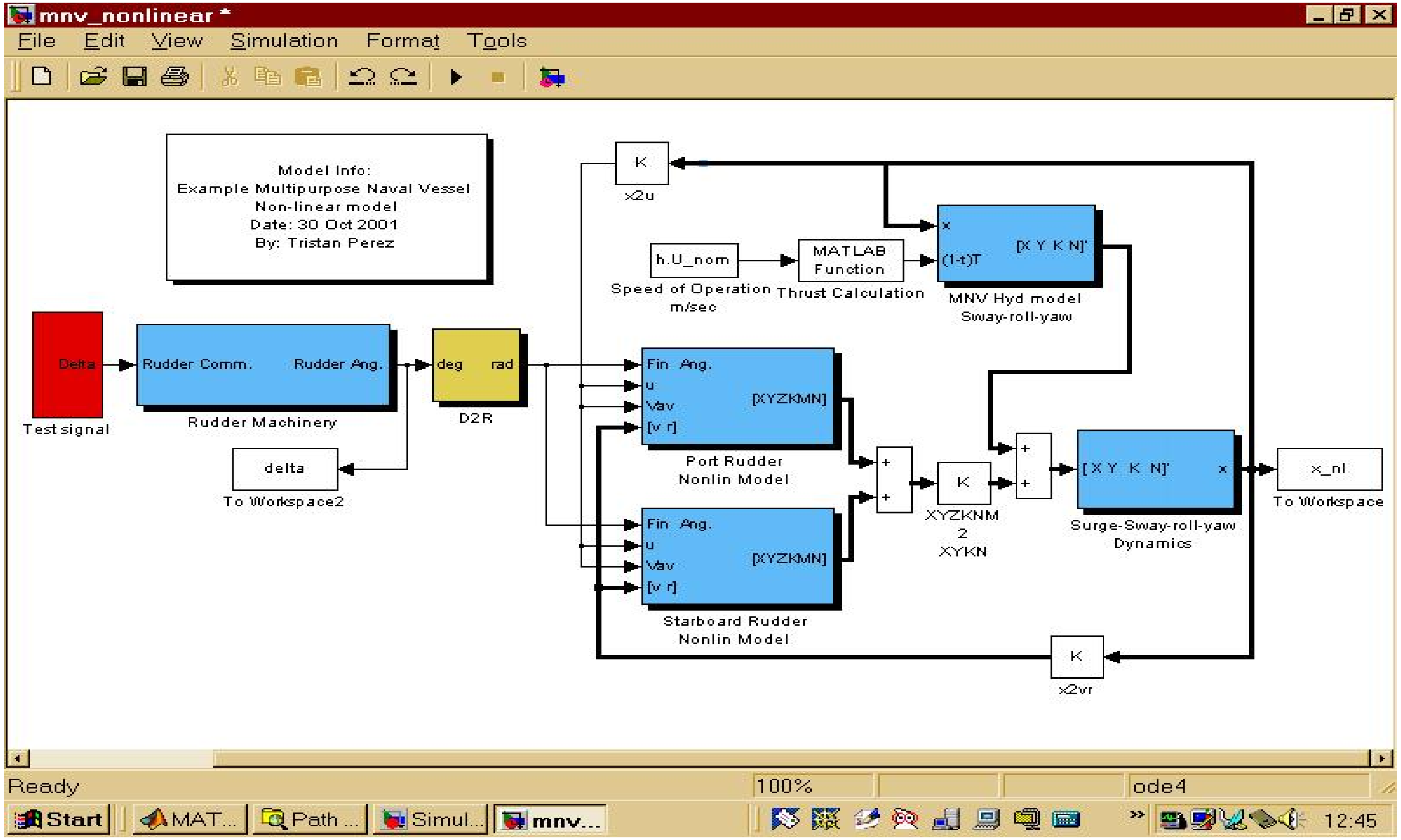
Features:

- Matlab prototype and specific functions.
- Simulink based library with masked blocks
- Modular approach.
- models in 4 and 6-DOF.

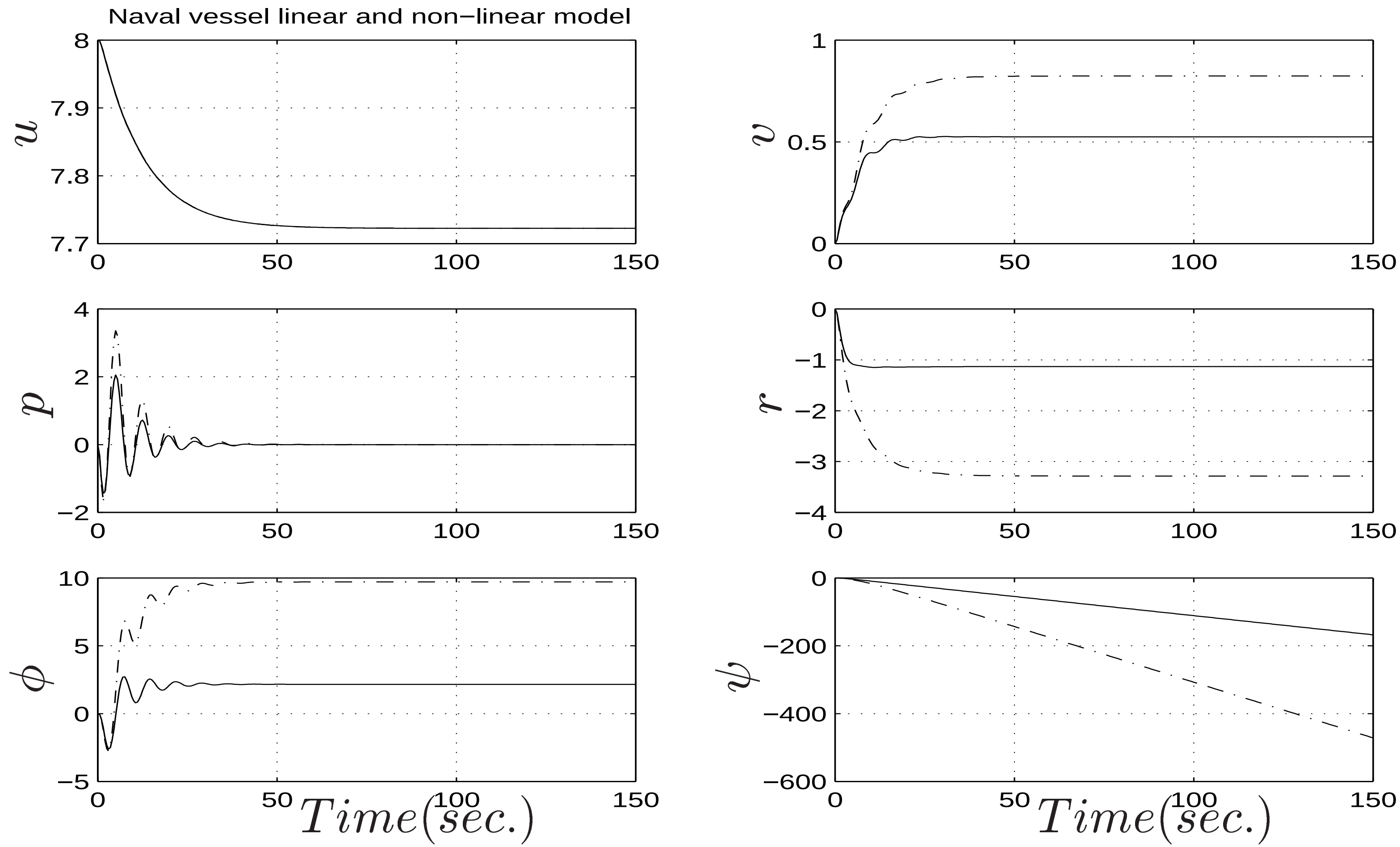
Matlab/Simulink based Toolbox: DCMV



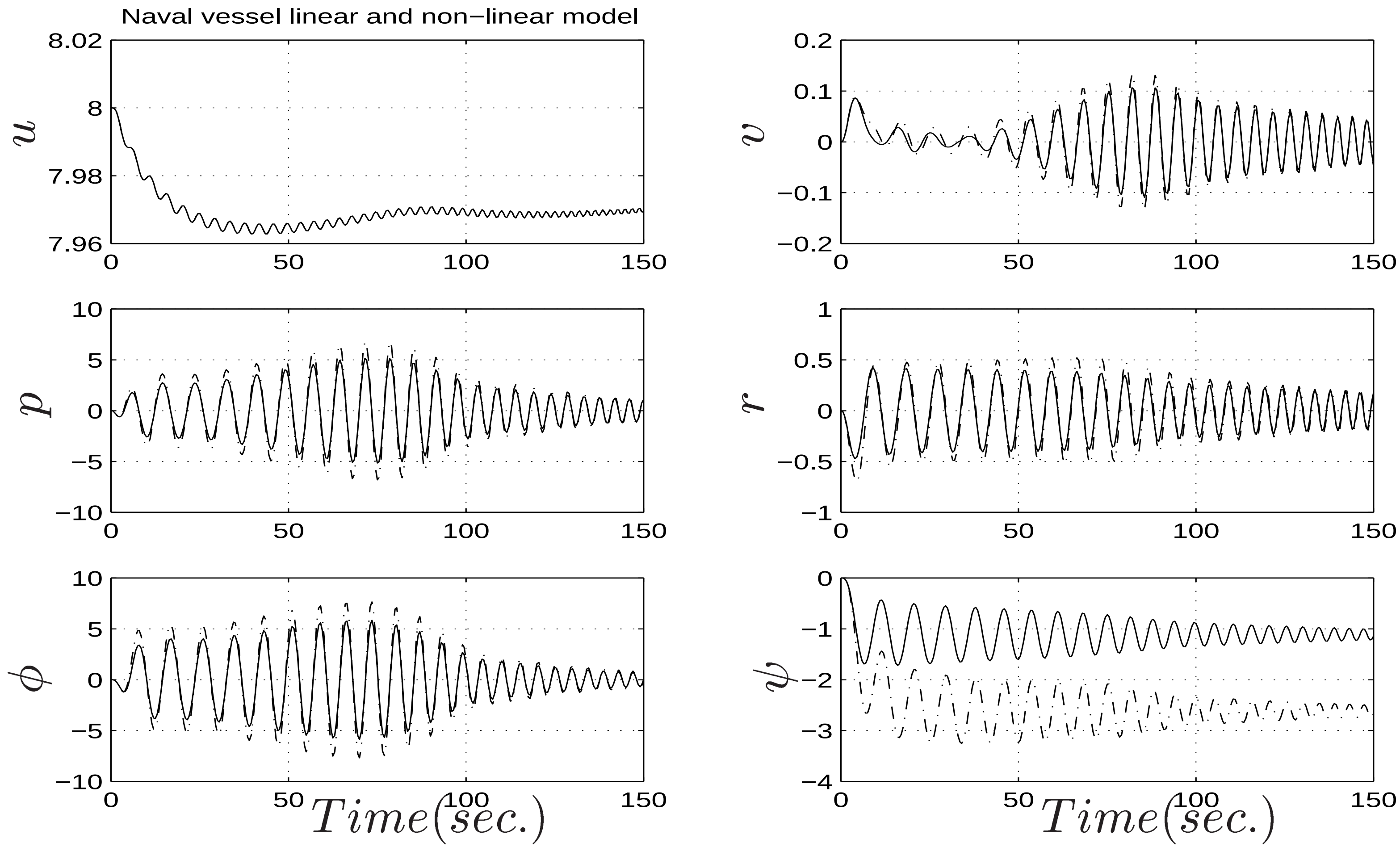
Simulink model: Naval vessel



Simulation results (low frequency)



Simulation results (high frequency)



Part II: Waves and ship motion in seaway

- Description of different sea patterns: regular, irregular, short and long crested.
- Simple harmonic progressive waves.
- Probabilistic description of sea elevation: wave spectral density functions.
- Ship motion in seaway: receptance functions.
- Simulation techniques

Ocean characteristics

- The irregularity of the elevation of the sea surface in presence of waves is indeed a fenomenon of stochastic nature.
- Over a wide area and for a period of half-hour or more the sea can be considered nearly statistically steady or stationary.
- Analysis of wave records indicate that under such conditions the sea elevation is Gaussian.
- If there is no wave breaking, the sea can be described by a superposition of a very large number of independent random components.

Wave generation and sea patterns

Waves are generates by the interaction of wind and the water surface.

- **Fully-developed sea.** When the wind speed is steady, while fetch and duration is increased, the sea conditions eventually reach a statistical stationary characteristic and its called a fully-developed sea.
- **Long-crested sea.** Outside the storm area, wave components become nearly parallel and the length of the crest becomes longer then the wave length. In this conditions the sea is called long-crested.
- **Short-crested sea.** Close to the storm area or when there are other disturbances (coastal reflection) there is angular dispersion and the wave components arrive from different directions. this gives rise to a short-crested sea.

Gravity Waves

A two-dimensional wave progressing at an angle χ with respect to the inertial axis, is described by its elevation ζ at a certain position x, y at time t by

$$\zeta(x, y, t) = \zeta_0 \cos(kx \cos\chi + ky \sin\chi - \omega_w t + \theta)$$

where

- k is the wave number.
- λ is the wavelength;
- ω_w is the wave frequency seen from a fixed position.
- ζ_o is the wave amplitude
- θ is an arbitrary phase angle.

Waves: phase velocity

The phase velocity of the wave, c, is the velocity with which the wave crest move relative to ground. Assuming a gravity wave and infinite depth of water, following relations hold

$$c = \sqrt{\frac{g\lambda}{2\pi}}; \quad k = \frac{2\pi}{\lambda}; \quad \omega_w = \sqrt{gk} = \frac{g}{c}$$

The last expression is known as the dispersion of gravity waves. The phase velocity is inversely proportional to its frequency; this means that long waves propagate faster than short ones.

A ship advancing in a seaway in following seas will overtake some short waves, while it will be overtaken by some long ones.

Point irregular seas

Irregular seas are described by superposition of wave components.

The simple description is given at a single point $(x = 0, y = 0)$ assuming that all the waves have the same direction $\chi = 0$:

$$\zeta(t) = \sum_i \zeta_i(t) = \sum_i \overline{\zeta}_i \cos(-\omega_i t + \theta_i)$$

To define there components a point spectrum $S(\omega)$ is used such that:

$$var[\zeta(t)] = \int_0^\infty S(\omega) d\omega \qquad \zeta_i \approx \sqrt{2S(\omega_i)\delta_\omega}$$

These point spectrum gives a description of a long-crested sea.

Directional irregular seas

The seaway is better described by a directional spectrum. The total wave system is described by

$$\zeta(x, y, t) = \sum_i \sum_j \bar{\zeta}_{ij} \cos[(k_i(x \cos \chi_j + y \sin \chi_j) - \omega_i t + \theta_{ij})]$$

In a similar manner, for $(x = 0, y = 0)$ we have $S(\omega, \xi)$ such that

$$var[\zeta(t)] = \int_0^\infty \int_0^{2\pi} S(\omega, \chi) d\chi d\omega \qquad \zeta_{ij} \approx \sqrt{2S(\omega, \chi)\delta_\chi \delta_\omega}.$$

A directional spectrum is difficult to measure; therefore, it is generally assumed that

$$S(\omega, \chi) = S(\omega)M(\chi),$$

where, $M(\chi)$ is a spreading function.

Recommended spectral families

Some idealized point spectra have been devised for special purposes. Among them, we have the recommended

- ITTC:

$$G_{\zeta\zeta}\left(\omega\right)_{ITTC} = \frac{0.78}{\omega^5}\ \exp\left(\frac{-3.11}{\omega^4\ h_{1/3}^2}\right) \qquad (m^2 sec)$$

- ISSC:

$$G_{\zeta\zeta}\left(\omega\right)_{ISSC} = \frac{691 h_{1/3}^2}{4\omega^5 T_w^4}\ \exp\left(\frac{-691}{\omega_w^4 T_w^4}\right) \qquad (m^2 sec)$$

where $h_{1/3}$ is the significant wave height and T_w is the average wave period. For short-crested seas the recommended spreading function is

$$M(\chi) = \frac{2}{\pi}\cos^2(\chi)$$

Ship motion in waves

Frequency of encounter

$$\omega_e = \omega - \frac{\omega^2 U_0 \cos\chi}{g}.$$

Response operator and motion spectrum

$$G_{zz}(\omega_e, \chi, U) = \sum_{i=1}^{N} \frac{\left|R_{z\zeta}\left(\omega_i, \chi, U\right)\right|^2 G_{\zeta\zeta}\left(\omega_i, \chi\right)}{\left|1 - 2\omega_i \frac{U}{g} \cos\chi\right|},$$

with $N = 1$ for head seas and $N = 3$ for following seas.

Simulation of ship motion in waves

An approximation of a motion spectrum is obtained by

$$z(t) = \sum_{i=1}^{n} a_i \sin\left(\omega_{e,i} t + \varphi_i + \varphi_{init}\right)$$

Frequencies $\omega_{e,i}$ and phase angles φ_i are tabular values from the response operator. The initial phase φ_{init} is random. Then from

$$\tfrac{1}{2} a_i^2 = \int_{\omega_1}^{\omega_2} G_{zz}(\omega_e, \chi, U)\, d\omega_e = \int_{\omega_1}^{\omega_2} \left|R_{z\zeta}\left(\omega_w, \chi, U\right)\right|^2 G_{\zeta\zeta}\left(\omega_w, \chi\right)\, d\omega_w$$

we obtain

$$a_i = \sqrt{2}\ \left|R_{z\zeta}\left(\omega_{w,i}, \chi, U\right)\right|\ \sqrt{\sigma_w^2\left(\omega_1, \omega_2\right)}$$

where

$$\sigma_w^2\left(\omega_1, \omega_2\right) = \int_{\omega_1}^{\omega_2} G_{\chi\chi}\left(\omega_w, \chi\right)\ d\omega_w .$$

Part III: Rudder roll stabilization

- Problem description.
- Linear fundamental limitations.
- Control design.
- Future research directions.

Why doing roll stabilization and damping?

- Prevent cargo damage due to large accelerations.
- Ensure safety conditions for crew and passengers.
- Preserve performance.

Methods and Solutions

Stabilizer	applic.	Red. %	Cost	Remarks
Fins (fixed)	Mega yachts, naval	90	high	drag, noise
Fins (retrac.)	Cruise, Ferries, naval	90	very high	noise
Tanks (Free surface)	Work vessels, ferries	75	very low	space
Tanks (U tube)	Work/RO vessels,	75	high	space
RRS	Small and high speed	50-75	medium	robust gear
Bilge keels	Universal	25-30	-	Speed loss

The control problem of RRS

Application: Small high speed vessels with space constraints.

Objectives:

- Attenuate roll motion.
- Increase damping.
- Keep interference with yaw low.

Characteristics:

- Non minimum phase.
- Unstable
- Rank deficient: Single input two outputs.
- Constraints.
- Disturbances

Linear fundamental limitations of RRS

Here we study the characterization of the best achievable performance in terms of the intrinsic dynamics and structure of the system.

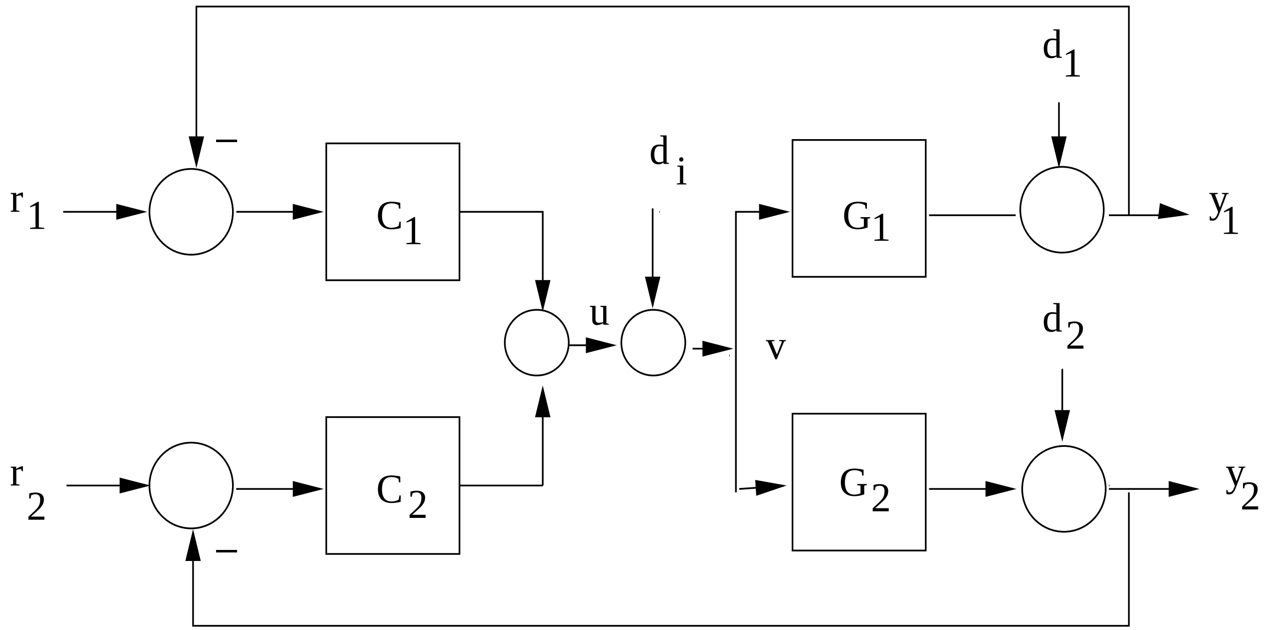
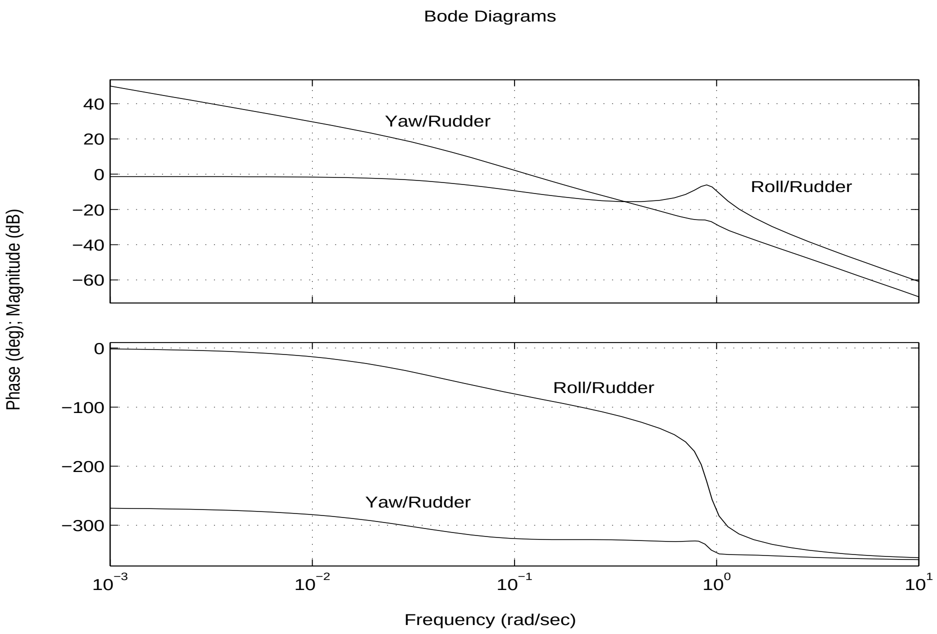


Figure 3: Control Scheme for RRS

Linear model

$$G_1 = \frac{\phi(s)}{\delta_c(s)} \qquad G_2(s) = \frac{\psi(s)}{\delta_c(s)}$$

The main features of these transfer functions are that G_1 is NMP and G_2 has a pure integrator as previously mentioned.



Limitations due to the NMP zero

From the block diagram, the sensitivity of the roll loop is:

$$S_{11}(s) = \frac{1 + C_2(s)G_2(s)}{1 + C_2(s)G_2(s) + C_1(s)G_1(s)}$$

For the NMP zero $q = \sigma_q + j\omega_q$ of $G_1(s)$, $S_{11}(q) = 1$, and assuming closed loop stability the following integral constraint holds:

$$\int_{-\infty}^{\infty} \log |S_{11}(j\omega)| \frac{\sigma_q}{\sigma_q^2 + (\omega_q - \omega)^2} d\omega \geq 0$$

Design Interpretations: Now, suppose that the feedback loop has been designed to achieve

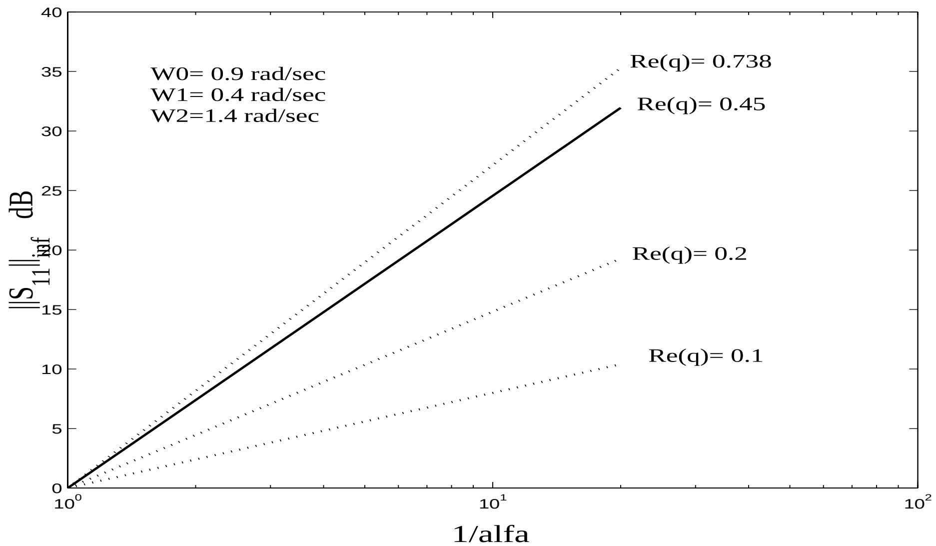
$$|S_{11}(j\omega)| \leq \alpha_1 < 1 \,, \qquad \forall \omega \in \Omega_1 \triangleq [\omega_1, \omega_2]$$

Sensitivity Constraints

Then, the infinity norm of the sensitivity function S_{11} has a lower bound:

$$\|S_{11}\|_\infty \geq \left(\frac{1}{\alpha_1}\right)^{\frac{\Theta_q(\omega_1,\omega_2)}{\pi-\Theta_q(\omega_1,\omega_2)}}$$

where $\Theta_{s_q}(\omega_1,\omega_2) = \arctan\dfrac{\omega_2-\omega_q}{\sigma_q} - \arctan\dfrac{\omega_1-\omega_q}{\sigma_q}$



Roll reduction vs. yaw interference fundamental limitations

To quantify the best achievable performance in the linear case, we propose to study the solution of the following optimal control problem:

$$\min J = \min_{C(s)} [\lambda \; var(\phi) + (1-\lambda) \; var(\psi) | S_{\phi\phi}, \lambda] \, ,$$

where $0 \leq \lambda \leq 1$ measures the importance of roll variance over the yaw variance:

- $\lambda = 0$ only yaw matters.
- $\lambda = 1$ only roll matters.

Roll Linear disturbance

We assume a linear approximation to the disturbance psd:

$$S_d(j\omega) = |F(j\omega)|^2 \sigma_n^2,$$

where

$$F(s) = \frac{K_w \ s}{s^2 + 2 \ \xi \ \omega_0 \ s + \omega_0^2}$$

with $K_w = 2\xi\omega\sigma_n$. Then

$$S_d(j\omega) = \frac{4(\xi\omega\sigma_n)^2\omega^2}{(\omega_0^2 - \omega^2)^2 + 4 \ (\xi \ \omega_0 \ \omega)^2} \ ,$$

with the property that

$$\max_{\omega} S_d(j\omega) = S_d(\omega_0) = \sigma_n^2 \ .$$

Controller parameterization

In matrix form

$$G(s) \triangleq \begin{bmatrix} G_1(s) \\ G_2(s) \end{bmatrix}, \qquad C(s) \triangleq \begin{bmatrix} C_1(s) & C_2(s) \end{bmatrix}$$

and

$$C(s) = [I - Q(s)G(s)]^{-1}Q(s)$$

where $Q(s)$ belongs to the set of all matrices with appropriate dimensions which are stable, proper, and having real-rational functions entries. To ensure internal stability there are interpolation constraints to be satisfied:

- $Q_1(0) = 0$ and $Q_2(0) = 0$
- $G_2(0)\ Q_2(0) = 1$

Quadratic Optimal synthesis

Using the Fourier transform, we obtain

$$J = \lambda\sigma_n^2 \int_{-\infty}^{+\infty} |F(j\omega) - G_1(j\omega)Q_1(j\omega)F(j\omega)|^2 \; d\omega$$
$$+(1-\lambda)\sigma_n^2 \int_{-\infty}^{+\infty} |G_2(j\omega)Q_1(j\omega)F(j\omega)|^2 \; d\omega$$

Finally, the problem reduces to

$$Q_1^{opt}(j\omega) = (j\omega)\widetilde{Q}_1^{opt}(j\omega) = (j\omega) \arg \min_{\widetilde{Q}_1(j\omega)\in\mathcal{S}} \|W(j\omega) - V(j\omega)\widetilde{Q}_1(j\omega)\|_2^2$$

where $\mathcal{S}$ is the ring of all proper stable transfer functions. Then using $Q_1^{opt}(j\omega)$ the cost is evaluated.

Graphic representation of the trade-off

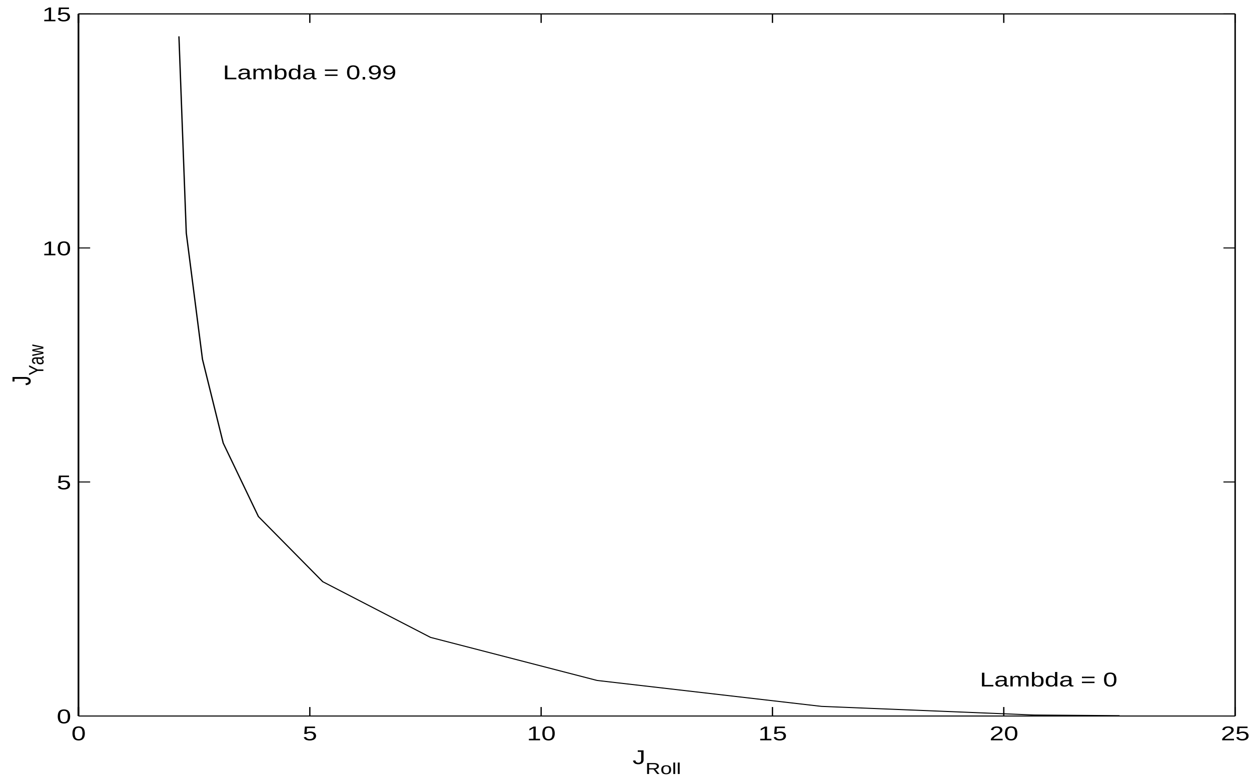


Figure 4: Graphic Representation of the Roll reduction vs. Yaw interference trade-off.

Constrained control design

The rudder mechanism, and operative conditions imposes constraints:

- Max rudder angle $|\delta| < \delta_{max}$.
- Max rudder rate $|\dot{\delta}| < \dot{\delta}_{max}$.
- Max yaw deviation $|\psi| < \psi_{max}$.

Solution approaches

- Serendipitous: constraints ignored in design.
- Cautious: Mechanisms are included after the design (AGC, anti-wind-up.)
- Tactical: Constraints are integral part of the design (MPC.)

Constrained model predictive control

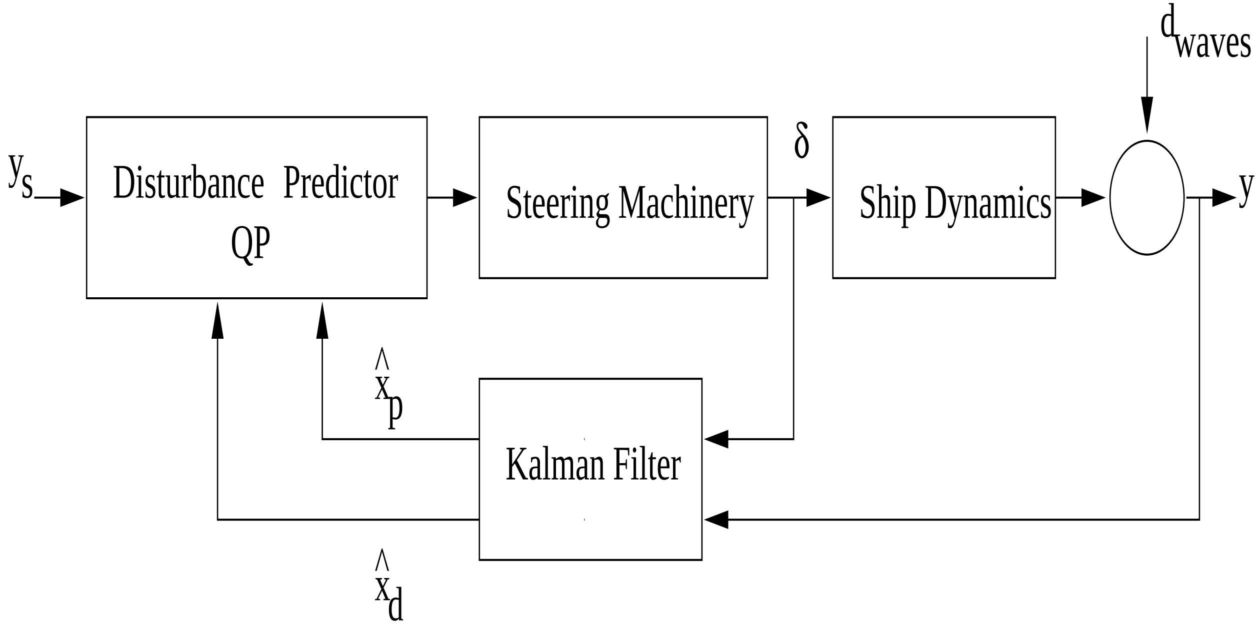


Figure 5: Schematic Block Diagram of the Control Structure

Simulation results

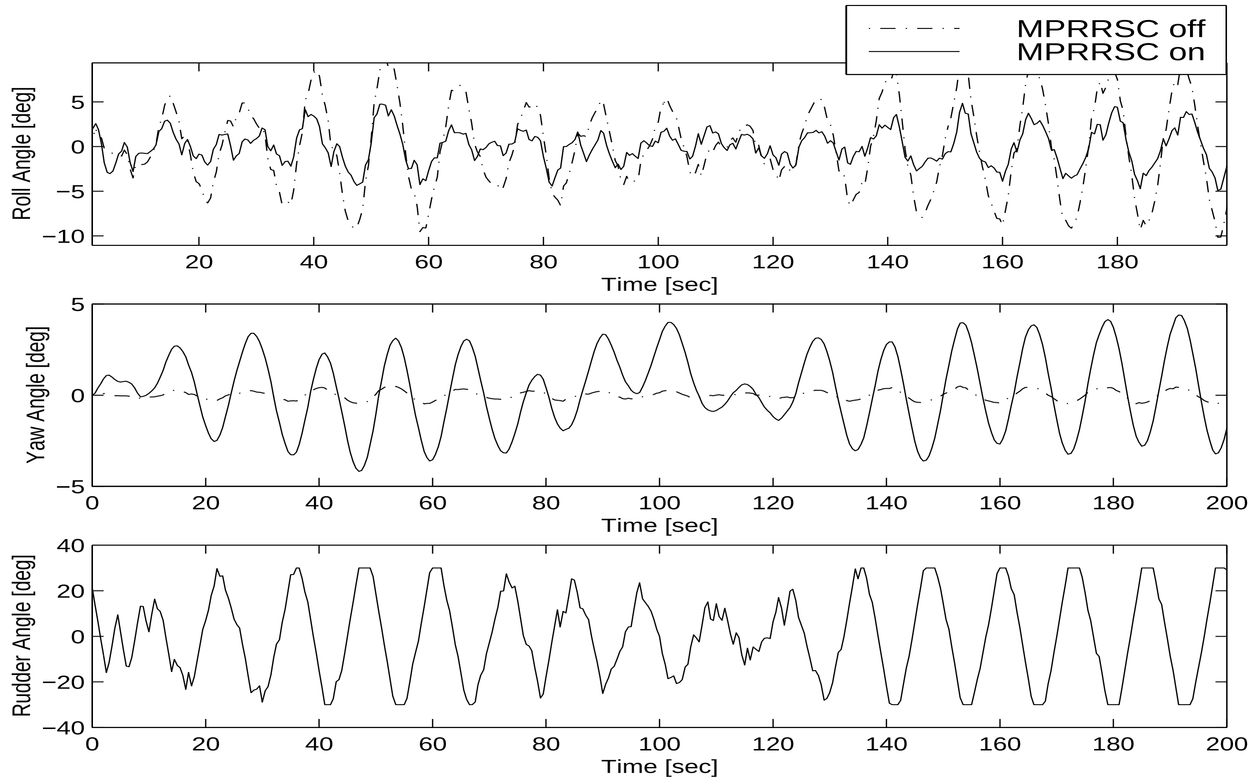


Figure 6: MPRRSC Performance. a- Roll Angle; b- Yaw Angle; and c- Rudder Angle.

Simulation results

In order to measure the roll reduction produced by MPRRSC, the following percent quantity is used:

$$RR\% = \frac{var(\phi)_{off} - var(\phi)_{on}}{var(\phi)_{off}} \times 100\,,$$

Max. Excursion / Max. Slew Rate	*RR%*
30 *deg* / 15 *deg/sec*	59
30 *deg* / 12 *deg/sec*	53
30 *deg* / 8 *deg/sec*	41
20 *deg* / 15 *deg/sec*	49
20 *deg* / 12 *deg/sec*	47
20 *deg* / 8 *deg/sec*	42