ON THE NUMBER OF FRACTIONS TO COMPUTE TRANSPORT OF SEDIMENT MIXTURES

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ABSTRACT

This paper deals with the determination of the minimum number of granulometric classes that should be used to calculate sediment transport of non-uniform grain-size materials. This is very important to reduce computational efforts in hydro-morphological mathematical models based on sediment continuity by fractions. In the case of log-normal river bed grain-size distributions the increment of the geometric standard deviation itself is already sufficient to describe the corresponding augment of the number of classes. However, strongly non-uniform river bed sediments (like those encountered in gravel-bed rivers) show completely different statistical characteristics when compared with the former ones. Herein specific relationships are developed which confirm that in these cases both the geometric standard deviation and the skewness of the grain-size distribution must be taken into account to define the minimum number of grain-size classes.

Key Words: Non-uniform sediments, Sediment transport of mixtures, Number of fractions

1 INTRODUCTION

Many hydro-morphological mathematical models neglect the influence of river bed material heterogeneity and its time and space changes during transport and related erosion/deposition processes. In these models a representative diameter of the river bed grain-size distribution (for example d_{50}) is specified as initial data in each computational point of the modeled domain. Different d_{50} can be assigned to each grid point but temporal changes in bed material gradation cannot be simulated. As a result, models based on this type of bed schematization can be applied in rivers with uniform bottom sediment compositions (geometric standard deviation σ_g <1.3) and in which the compositions itself does not change substantially in time. These assumptions are quite restrictive and in many cases it is impossible to neglect river bed material heterogeneity, like in gravel-bed rivers.

After the introduction of the active layer concept by Hirano (1971), hydro-morphological models have expanded their capabilities by allowing the simulation of longitudinal sorting, dynamic armoring processes, etc. In these models sediment continuity equation is solved for each granulometric class present in the active layer. As a consequence, fractional sediment transport rates must be calculated by using an appropriate sediment transport formula. Fractional transport rates are basically calculated by splitting-up the granulometric curve in a certain number of grain-size classes and considering the corresponding hiding-exposure effects on each grain-size (Parker et al., 1982a, 1982b). Many formulas developed in the context of uniform sediments have been modified in order to calculate sediment transport by fractions (White et al., 1982; Ribberink, 1987; Di Silvio et al., 1989; Basile, 1994; Basile et al., 1994). The modification consists in the introduction of the fraction associated to each grain-size class and the corresponding hiding-exposure coefficient. In any case, still remain open to the modeler criteria the selection of the number of grain-size classes in which split-up the granulometric curve. As a large number of classes produce a huge increase in computational efforts, modelers tend to select arbitrarily as few classes as possible but in this way significant errors in sediment transport calculations must be expected.

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Ribberink (1987) and Khin (1989) analyzed the minimum number of grain-size classes necessary for modelling non-uniform sediments. Khin recommends the selection of the number of classes only as a function of the geometric standard deviation of grain-size distributions. Her conclusions were derived for steady flow conditions, dimensionless bed shear stress (referred to the arithmetic mean diameter of the mixture) close to the critical one for incipient motion and log-normal grain-size distributions.

However, strongly non-uniform grain-size distributions, like those encountered in gravel-bed rivers, deviates substantially from symmetric S-shaped log-normal size distributions (Shaw et al., 1982; Parker, 1991). In fact, they generally display a concave upward shape except near the coarsest tail. Thus, they are strongly asymmetric and deviate considerably from log-normal. In addition, bimodality is also a common feature in some gravel-bed river reaches. In these situations the selection of grain-size classes as a function only of geometric standard deviation could lead to erroneous results.

Herein more general relationships between the number of grain-size classes and the corresponding statistical parameters of grain-size distributions were derived. To achieve this goal a wide spectrum of grain-size distributions was analyzed. Symmetric S-shaped curves with low spreads (sand-bed rivers) and strongly asymmetric curves with high values of geometric standard deviations (gravel bed-rivers) were considered. In the last case unimodal and bimodal grain-size distributions were analyzed. The sediment transport equations of Engelund & Hansen (1967) and Meyer-Peter & Müller (1948) both modified to calculate fractional transport rates were implemented.

2 STATISTICAL PARAMETERS OF GRAIN-SIZE DISTRIBUTIONS

In order to derive the statistical parameters of grain-size distributions an appropriate scale, as a function of the logarithm of diameters, must be introduced. The sedimentological ϕ scale is defined in the following way:

$$\phi = -\log_2(d) \tag{1}$$

where d is the diameter expressed in mm. Considering a cumulative grain-size distribution F_i discretized in N classes bounded by ϕ_i grain-sizes with i ranging from 1 to N+1 and arranged in descending order of ϕ. For i=1 to N a representative diameter $\overline{\phi}_i$ for the i-th class and the associated fraction by weight f_i can be defined as:

$$\overline{\phi}_i = 0.5\left(\phi_i + \phi_{i+1}\right) \tag{2}$$

$$f_i = F_{i+1} - F_i \tag{3}$$

where the following condition is satisfied: $\sum_{i=1}^{N} f_i = 1$.

The most relevant statistical parameters of grain-size distributions in the ϕ scale are the mean size ϕ_m the standard deviation σ and the skewness s_K. These parameters are given by the expressions:

$$\phi_m = \sum_{i=1}^{N} \overline{\phi}_i f_i \tag{4}$$

$$\sigma = \left[\sum_{i=1}^{N} \left(\overline{\phi}_i - \phi_m\right)^2 f_i\right]^{1/2} \tag{5}$$

$$s_K = \sum_{i=1}^{N} \left(\overline{\phi}_i - \phi_m\right)^3 f_i \tag{6}$$

Combining Eqs. (5) and (6) the dimensionless skewness is defined by:

$$\beta = s_K^2 \big/ \sigma^6 \tag{7}$$

Moreover, from Eqs. (1), (4) and (5) the geometric mean diameter d_g and the geometric standard deviation σ_g are expressed as:

$$d_g = 2^{-\phi_m} \quad , \quad \sigma_g = 2^{\sigma} \qquad (8a, 8b)$$

From Eq. (1) one gets $\bar{d}_i = 2^{-\bar{\phi}_i}$, thus the arithmetic mean diameter as a function of each $\bar{\phi}_i$ size is:

$$d_m = \sum_{i=1}^{N} 2^{-\bar{\phi}_i} f_i \qquad (9)$$

3 GRAIN-SIZE DISTRIBUTIONS PRESENTATION

In the present study thirteen grain-size distribution were analyzed. Four of them (S1, S2, S3 and S4) are typical distributions of sand-bed rivers, slightly skewed and with low geometric standard deviation. The remaining nine distributions are typical of gravel-bed rivers, strongly skewed and with high geometric standard deviation. In this last case five distributions are unimodal (G1, G2, G3, G4 and G5) and four are bimodal (B1, B2, B3 and B4). In Fig. 1 the different grain-size distributions are presented.

In Table 1 the corresponding statistical parameters for each grain-size distribution are resumed. In sand-bed rivers distributions (type S) the geometric mean diameter varies between 0.24 mm < d_g < 0.52 mm and the geometric standard deviation varies between 1.8 < σ_g < 2.2. Grain-size distributions S1 and S2 are both perfectly symmetric (β =0). Instead, distributions S3 and S4 are slightly asymmetric, both distributions presents the same value of dimensionless skewness β=0.02 but s_K<0 for size distribution S3 and s_K>0 for size distribution S4.

As far as unimodal gravel-bed rivers distributions are concerned (type G) geometric mean diameter varies between 10 mm < d_g < 78 mm, the geometric standard deviation varies between 5 < σ_g < 7 and the dimensionless skewness 0.5 < β < 1.7. With respect to bimodal size distributions (type B) the geometric mean diameter varies between 6.5 mm < d_g < 23 mm, the geometric standard deviation varies between 8 < σ_g < 12 and the dimensionless skewness 0.3 < β < 0.85.

Table 1 Statistical parameters of the analyzed grain-size distributions

Distribution	d_{16} (mm)	d_{50} (mm)	d_{84} (mm)	d_m (mm)	d_g (mm)	σ_g	β
S1	0.20	0.35	0.61	0.42	0.35	1.8	0.00
S2	0.15	0.35	0.82	0.47	0.35	2.2	0.00
S3	0.11	0.25	0.49	0.31	0.24	2.0	0.02
S4	0.26	0.50	1.12	0.65	0.52	2.0	0.02
G1	17.45	117.98	316.12	155.14	77.64	5.0	1.70
G2	6.35	58.35	244.44	110.72	40.89	6.0	0.80
G3	3.17	36.06	183.54	85.72	25.77	7.0	0.50
G4	2.21	24.25	94.52	45.63	16.32	6.0	0.50
G5	2.00	13.05	48.50	23.86	10.02	5.0	0.50
B1	0.40	12.26	50.80	23.73	6.50	8.0	0.30
B2	0.69	24.68	86.14	40.96	12.73	8.0	0.60
B3	0.53	46.31	161.27	77.21	17.39	12.0	0.55
B4	0.93	46.31	161.27	77.97	22.95	9.0	0.85

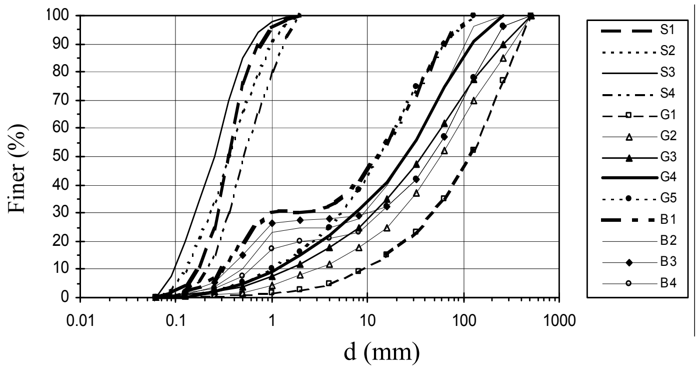

Fig. 1 Grain-size distributions

4 IMPLEMENTED SEDIMENT TRANSPORT EQUATIONS

4.1 Engelund and Hansen Equation (E&H)

The Engelund and Hansen (1967) equation was derived and verified with experimental data of uniform sediments with diameters ranging from 0.19 mm to 0.93 mm. Basile (1998) tested a modified version of E&H equation for heterogeneous sediments using experimental and field data regarding transport of sediment mixtures. Introducing the dimensionless transport for the i-th class W^*_i (Parker et al., 1982a), the modified version of E&H equation reads:

$$W_i^* = \alpha_{eh}\, f_i \left(\frac{u}{U_*}\right)^2 \tau_{*i}\, \xi_{oi} \tag{10}$$

with dimensionless sediment transport equal to:

$$W_i^* = \frac{T_i\left[(s-1)g\right]}{B\, U_*^3} \tag{11}$$

in which T_i is volumetric transport of i-th class; s= ρ_s / ρ specific gravity of sediments, with ρ_s sediment density and ρ water density; g is acceleration due to gravity; B is width; $U_*=(\tau_b / \rho)^{0.5}$ shear velocity, with τ_b bed shear stress; α_{eh}=0.05; f_i is fraction associated to the i-th class; u is mean flow velocity; and the dimensionless corrected shear stress referred to grain-size $\bar{d}_i$ is defined as:

$$\tau_{*i}^{corr.} = \tau_{*i}\, \xi_{oi} \tag{12}$$

where τ_{*i} is the dimensionless shear stress referred to grain-size $\bar{d}_i$ and ξ_{oi} is the corresponding hiding-exposure coefficient:

$$\tau_{*i} = \frac{\tau_b}{g(\rho_s - \rho)\bar{d}_i} \quad , \quad \xi_{oi} = \left(\frac{\bar{d}_i}{d_m}\right)^b \tag{13a,13b}$$

In Eq. (13b) d_m is the arithmetic mean diameter of bed material grain-size distribution (defined previously) and the exponent b varies between 0 and 1. These last values define two extreme transport behaviors. For b=0 there is no interaction between the different grain-size particles, each particle conserves its own intrinsic mobility (selective transport). For b=1 hiding-exposure effects are so strong that individual grain-size influence is canceled-out, thus all particles in the mixture are equally mobile (Parker et al., 1982a). Considering the quite satisfactory results obtained with Eq. (10) by adopting b=0.8 in Eq. (13b) (Basile, 1998), in the present analysis b was also set equal to 0.8.

4.2 Meyer-Peter and Müller Equation (MP&M)

The Meyer-Peter and Müller (1948) equation was derived and verified with experimental data regarding uniform and heterogeneous sediments with grain-sizes ranging from 0.4 mm to 28.65 mm. In the original version the authors proposed to calculate the overall transport rate of heterogeneous sediments by using the arithmetic mean diameter of size distribution. In the present formulation fractional transport rates are calculated by correcting the critical dimensionless shear stress with the hiding-exposure coefficient of Egiazaroff (1965) modified by Ashida and Michiue (1971):

$$W_i^* = \alpha_{mpm} f_i \left(1 - \frac{\tau_{*ci}^{corr.}}{\tau_{*i}}\right)^{3/2} \tag{14}$$

where α_{mpm}=8, W^*_i is defined by Eq. (11) and the dimensionless corrected critical shear stress for the i-th grain-size is:

$$\tau_{*ci}^{corr.} = 0.047\ \xi_{ci} \tag{15}$$

The hiding-exposure coefficient proposed by Egiazaroff (1965) and slightly modified by Ashida and Michiue for values of ($\bar{d}_i$ / d_m)≤ 0.4, can be expressed as follow:

$$\xi_{ci} = \begin{cases} 0.85\left(\bar{d}_i / d_m\right)^{-1} & if\ \left(\bar{d}_i / d_m\right) \le 0.4 \\ \left[1 + 0.782\ \log\left(\bar{d}_i / d_m\right)\right]^{-2} & if\ \left(\bar{d}_i / d_m\right) > 0.4 \end{cases} \tag{16}$$

5 COMPUTATIONAL PROCEDURE

In order to speed up the calculations a computational program was developed and tested. For a single grain-size distribution and a given hydrodynamic parameters a loop from N=2 to N=100 grain-size classes is performed. For each number of grain-size classes N in which the granulometric curve is discretized, the statistical parameters, the fractional and overall sediment transport rates and the transport composition are calculated. The overall sediment transport rate is obtained by summing-up each fractional transport T_i over N:

$$T(N) = \sum_{i=1}^{N} T_i \tag{17}$$

and the transport composition is: $f_{Ti} = T_i / T(N)$, i=1 to N.

The values of the statistical parameters and sediment transport calculated for N=100 (arbitrarily selected) are considered error free values or "true" values. Therefore, the associated error in the overall transport, as a function of the degree of grain-size distribution discretization, is calculated by means of the following expression:

$$E_T(N) = \frac{\left|T(100) - T(N)\right|}{T(100)} \tag{18}$$

6 RESULTS

In Fig. 2 the error in overall transport calculation as a function of N for E&H equation and grain-size distributions type G is presented. It can be observed that the error diminishes with the increment of grain-size fractions following approximately a rapidly decreasing exponential law. Fig. 2 shows that N increases considerably if errors lower than 5 % are specified. In order to derive the relationship between N and the statistical parameters of grain-size distributions, notably geometric standard deviation σ_g and dimensionless skewness β, herein this error is fixed in 5 %. By keeping the error fixed in that value it is also possible to establish a comparison between the different grain-size distributions and the minimum number of fractions required for each of them. For example, for grain-size distributions type G the minimum number of fractions varies from N=5 (size distribution G5) to N=9 (size distribution G1). It is interesting to note that even if size distributions G5 and G1 are characterized by the same value of the geometric standard deviation (σ_g =5), the number of fractions N increases with the augment of skewness. In fact, β=0.5 for grain-size distribution G5 and β=1.7 for size distribution G1. As far as bimodal grain-size distributions type B is concerned the minimum number of fractions varies from N=5 for size distribution B1 to N=7 for size distribution B4. With regard to grain-size distributions of sand bed rivers type S, N=2 (size distribution S2) and N=3 (size distribution S3). To obtain the functional relationship between N and the statistical parameters σ_g and β a multivariate regression analysis was carried out. For the complete set of grain-size distributions analyzed herein (type S, G and B) and for the E&H formula with $E_T(N)$=5 % the obtained regression equation is (correlation coefficient r^2=0.91):

$$N = 0.19\sigma_g + 3.88\ \beta + 2.13 \tag{19}$$

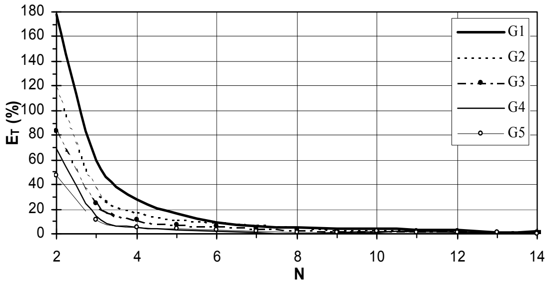

Fig. 2 $E_T(N)$ for grain-size distribution type G, E&H equation

In Figs. 3, 4, 5, 6, 7 and 8 the results obtained with MP&M equation for grain-size distributions S3, G1, G2, G3, G5 and B4 respectively are presented. These Figures show the error in overall transport as a function of N for different values of the dimensionless bed shear stress referred to the arithmetic mean diameter τ_{*M}. Can be clearly observed that, for E_T constant, an increment in τ_{*M} allows the use of a lower number of fractions. This behavior is connected with bed load transport equations based on excess shear stress concept for which transport rates becomes independent of grain-size for high values of the applied bed shear stress. Let us consider the conditions τ_{*M}=0.07 and $E_T(N)$=5 %, in this situation for grain-size distribution type G the minimum number of fractions varies from N=7 (size distribution G5) to N=14 (size distribution G1). Note that, as mentioned above for E&H equation, also in this case N increases with the augment of skewness because for size distributions G5 and G1 the geometric standard deviation remains constant. By comparing the results shown for size distributions G1, G2 and G3 it is observed the important role played by skewness. In fact, in those distributions the geometric standard deviation increases while dimensionless skeweness decreases (see Table 1), thus allowing the use of a lower number of fractions. With respect to grain-size distributions type B (bimodal) the study reveals that

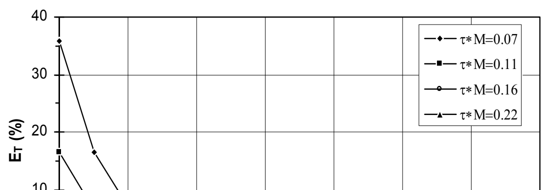

Fig. 3 $E_T(N)$ for grain-size distribution S3 and different values of τ_{*M} , MP&M equation

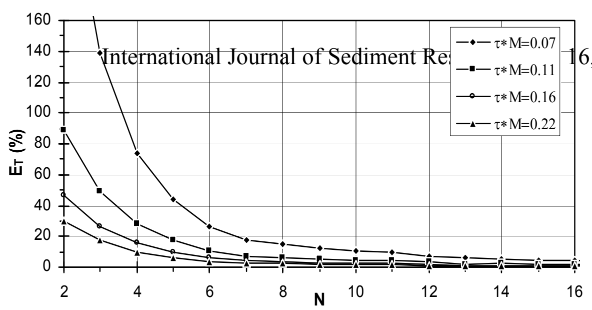

Fig. 4 $E_T(N)$ for grain-size distribution G1 and different values of τ_{*M}, MP&M equation

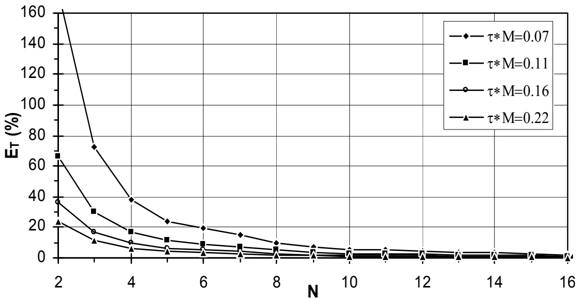

Fig. 5 $E_T(N)$ for grain-size distribution G2 and different values of τ_{*M}, MP&M equation

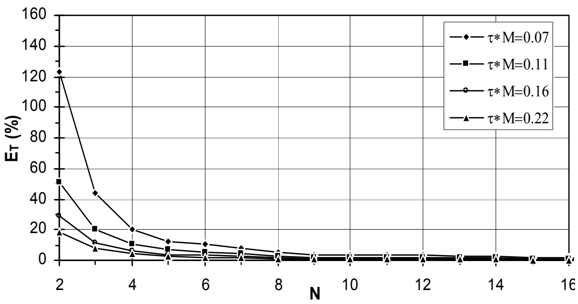

Fig. 6 $E_T(N)$ for grain-size distribution G3 and different values of τ_{*M}, MP&M equation

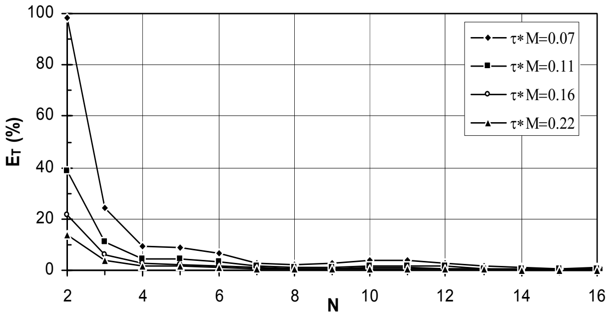

Fig. 7 $E_T(N)$ for grain-size distribution G5 and different values of τ_{*M}, MP&M equation

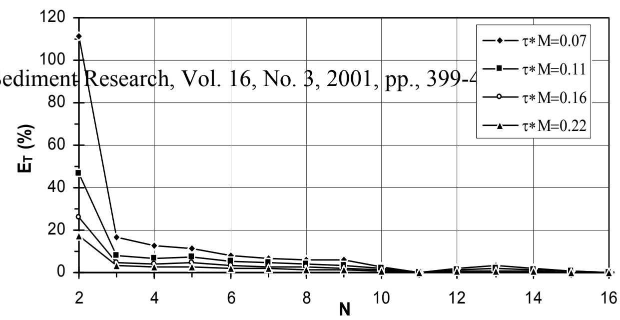

Fig. 8 $E_T(N)$ for grain-size distribution B4 and different values of τ_{*M}, MP&M equation

N varies from N=5 (size distribution B1) to N=9 (size distribution B4) and in the case of size distributions type S (sand bed streams) N varies between N=4 (size distribution S2) and N=5 (size distribution S3). Considering the complete set of grain-size distributions (type S, G and B) the regression equation obtained for MP&M formula with τ_{*M}=0.07 and $E_T(N)$ =5% is (correlation coefficient r^2=0.92):

$$N = 0.15\sigma_g + 5.48\ \beta + 3.96 \quad (20)$$

Eqs. (19) and (20) allows the determination of the minimum number of grain-size fractions as a function of two statistical parameters, which can be easily computed from bed material grain-size distributions, notably geometric standard deviation σ_g and dimensionless skewness β.

7 CONCLUSIONS

Grain-size distributions typical of gravel-bed rivers display important departures from those size distributions typical of sand bed rivers. In the case of strongly non-uniform river bed sediments, both geometric standard deviation and skewness play an important role in the definition of grain-size distribution discretization strategies for modelling sediment mixtures.

In morphological mathematical models for heterogeneous sediments, river bed schematization into a large number of grain-size classes leads to a huge increase in computational efforts. Therefore modelers tend to select arbitrarily as few grain-size classes as possible but doing so, as confirmed in this study, significant errors in sediment transport rates calculations could be committed.

In this study the relationships given by (19) and (20) were developed using a wide range of grain-size distributions which often are encountered in practical situations. By means of these relationships it is possible to determine the minimum number of grain-size fractions as a function of geometric standard deviation and dimensionless skewness of bed material grain-size distributions. This is twofold important because computational efforts in mathematical modelling of morphological changes in rivers with non-uniform sediments are reduced and contemporarily reasonably low errors in transport computations associated to grain-size distribution discretization are maintained.

Although Eq. (19) was specifically derived for the modified E&H formula, it can be applied in principle to estimate N when similar transport formulas without excess shear stress terms are implemented. Analogously, Eq. (20) can be used to estimate N when sediment transport formulas similar to the MP&M are utilized.

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