A MODEL OF CELLS FOR HYDROLOGICAL-HYDRAULIC MODELING

GERARDO A. RICCARDI
Consejo de Investigaciones and CURIHAM, Facultad de Ciencias Exactas, Ingeniería y Agrimensura, Universidad Nacional de Rosario
José Hernandez 986
2000 Rosario
Argentina
e_mail: riccardi@fceia.unr.edu.ar

SUMMARY
A hydrological-hydraulic quasi-2D multilayer simulation model is presented. It models the dominant hydrological processes involved in the rainfall-runoff transformation in rural and urban environments. The modelling structure is based on the well-known schemes of interconnected cells, with the original addition of a large set of linking laws. These allow the flow simulation in both surface runoff and networks of closed conduits. The model was developed trying to embrace different scales of rural and urban storm drainage from streets to river basins, and to cover different levels of detail, from river basins with a resolution of 0.10 elements/km^2 to drainage micro basin with 600 elements / km^2. The conceptual model and a synthesis of several applications of relevance are exposed in this work. The results obtained were entirely satisfactory, showing an appropriate capability of reproduction of the real process of rainfall-runoff transformation flowing in a single layer and in two interconnected layers (surface runoff and networks of closed conduits).

1. Introduction

The processes involved in the rainfall-runoff transformation in rural and urban environments have been widely studied by scientists, for better understanding the phenomena, and by engineers, for the hydraulic design of different structural components of drainage systems. There exists a large number of modelling systems with proved quality that allow the representation of such processes; among the most important we may mention S11S [1], MOUSE [5], SWMM [6], ILLUDAS [12], OTTHYMO [13], etc. However, there are still limitations in the modelling of the rainfall-runoff processes. Some suggested subjects of research and development are: development of reliable but not overly complicated hydraulic models, modelling with appropriate levels of detail, scale effects, modelling to small scale, integrated models and continuous modelling [2], [7]. Included in the above-mentioned areas, a system for hydrological-hydraulic numerical simulation is presented for both rural and urban environments. The simulation structure has been capable of simulating with different levels of detail and to different scales, from simulation in small conduits to micro and macro urban basins and rural basins.

2. Model Formulation

The modelling system is based on the scheme of cells originally proposed by Cunge [4]. In previous work we have enlarged the field of application of this scheme [8] - [11]. The system allows now the simulation of the rainfall-runoff processes with multidirectional and multi-layer flow dynamics. At each unit of subdivision of the surface layer it is possible to simulate the processes as precipitation, losses for interception and depression storage, and infiltration. The runoff can be propagated by means of a set of discharge from the kinematic approach of the momentum equation to an approach to the dynamic equation. This approach allows the flow routing through rivers, channels, flood valleys, urban streets and networks of closed conduits. Besides, to embrace diverse flow alternatives the model incorporates discharge laws for bridges, weirs, storm inlets, junctions, changes of section, pumping stations, etc. The model was denominated CELDAS8. The governing equations are those of continuity and different simplifications of the momentum equation, which was transformed to obtain the discharge between the linked cells.

2.1 Continuity Equation

It is supposed that to every cell $\boldsymbol{i}$ corresponds a water level $z_i = z_i(t)$ the cell centre (Fig. 1) and that he water surface is horizontal and its value is z_i. The continuity equation can be written in differential form [4]:

$$A_{S_i} \frac{dz_i}{dt} = P_i(t) + \sum_{k=1}^{j} Q_{k,i}(t) \qquad (1)$$

$P_i(t)$ represents processes as rainfall, interception, surface storage, infiltration and external exchange of flow.

2.2 Laws of discharge between the cells

2.2.1 Simple river link

It is used for flow with preponderance of the gravity, hydrostatic pressure and friction forces. The discharge expression $\boldsymbol{Q_{k,i}^{(n)}}$ is derived of the momentum equation considering negligible inertia and using the Strickler-Manning resistance formula [4]:

$$Q_{k,i}^{(n)} = sign\,(z_k^{(n)} - z_i^{(n)})\,\frac{1/\eta\, R_{h\,k,i}^{2/3}\quad At_{k,i}}{\sqrt{\Delta x_{k,i}}}\sqrt{|\,z_k^{(n)} - z_i^{(n)}\,|} \tag{2}$$

2.2.2 Quasi-dynamic link

It is used when the forces originated in convection mechanisms are important and can produce large variations of wetted areas of the cross sections. The deduction of the discharge is made neglecting the term of the local acceleration in the momentum equation [8] [9] [10]:

$$Q_{k,i}^{(n)} = \pm\frac{K_{k,i}}{\sqrt{\Delta x_{k,i}}}\sqrt{ABS\left[\frac{z_k^{(n)} - z_i^{(n)}}{1+\left[K_{k,i}/\sqrt{\Delta x_{k,i}}\right]^2/2g\ \left(A_i^{-2} - A_k^{-2}\right)}\right]} \tag{3}$$

2.2.3 Dynamic link

This link is used when all the physical processes described in the momentum equation are important. Starting from the complete momentum equation, an approximate expression of second degree in the variable discharge can be derived [11]:

$$a_1 Q_{k,i}^{\ 2} + a_2\ |Q_{k,i}|\, Q_{k,i} + b\, Q_{k,i} + c = \tag{4}$$

$$Q_{k,i}^{(n)} = -b + \frac{\sqrt{b^2 - 4(a_1 + a_2)c}}{2(a_1 + a_2)}\ \ si\ c < 0 \quad (5.a) \qquad Q_{k,i}^{(n)} = -b - \frac{\sqrt{b^2 - 4(a_1 - a_2)c}}{2(a_1 - a_2)}\ \ si\ c > 0 \quad (5.b)$$

2.2.4 Kinematic link

This link is only used when the hydrodynamic information is propagated downstream [11]. The discharge is computed as a function of the upstream cell level:

$$Q_{k,i}^{(n)} = K_{k,i}\, S_{0_{k,i}}^{\ 1/2} \tag{6}$$

2.2.5 Weir link

It is used to represent links between cells with physical limits: cells separated by highway or railway embankments, connections between the main stream and flood plains. The discharge formulation used is the classical expression of the wide crest weir free (8.a) and submerged (8.b) [4]:

$$Q_{k,i}^{(n)} = \mu_1 b\sqrt{2g}\,(z_k^{(n)} - z_{(i)}^{(n)})^{3/2} \quad (8.a) \qquad Q_{k,i}^{(n)} = \mu_2 b\sqrt{2g}\,(z_k^{(n)} - z_w)\sqrt{z_k^{(n)} - z_i^{(n)}} \quad (8.b)$$

2.2.6 Bridge link

Is used the weir link considering bottom step zero and a discharge coefficient according to Ven Te Chow [3] for flow through constrictions. In addition, this link can be also used for big culverts. [8]

2.2.7 Head loss or control section link

This link is suitable for flow singularities with head loss due to abrupt changes in the cross section. Two flow conditions are possible free (9.a) and submerged (9.b) [11]:

$$Q_{k,i}^{(n)} = \sqrt{2g\,(z_k^{(n)} - z_{cri})/(Cd^{-2}\ A_{Cri}^{-2} - At_k^{-2})} \quad (9.a) \qquad Q_{k,i}^{(n)} = \sqrt{2g(z_k^{(n)} - z_i^{(n)})\,/\,(Cd_S^{-2}\, A_{Sc}^{-2} - At_k^{-2})} \quad (9.b)$$

2.2.8 Conduit link

It is used for connections between cells of closed conduits. The continuity and discharge equations for free surface flow are of the same type that those outlined in the approaches for diffusive wave, quasi-dynamics and dynamics [11]. In flow under surcharge, the surface area in eq. (1) is calculated as a minimum value corresponding to the surface that results of the Preissmann slot [11] along the conduit.

2.2.9 Storm inlet link

This link is used to represent the storm inlet of closed conduit components of drainage systems. The model allows simulating curb, gutter and combination inlets, undepressed and depressed, with and without grating [11]. The discharge is calculated by means of weir and hole laws, under free and submerged flow conditions:

$$Q_{k,i}^{(n)} = C_s\ \mu_l\ L_{su}\ \left(z_k^{(n)} - z_{su} \right)^{1.5} \quad (10a.) \qquad Q_{k,i}^{(n)} = C_s\ \mu_a\ L_{su}\ \left(z_k^{(n)} - z_{su} \right)\left(z_k^{(n)} - z_i^{(n)} \right)^{0.5} \quad (10b.)$$

2.2.10 Pumping link

It is used to represent elements of flow elevation by means of addition of external energy; such is the case of single pump or pumping stations [10]. It is necessary to specify the pumping sequence as a function of time t_1 - $Q_{k,i\,1}$; t_2 - $Q_{k,i\,2}$; t_3 - $Q_{k,i\,3}$ or as a function of the level at the upstream cell:. $z_{k\,1}$ - $Q_{k,i\,1}$; $z_{k\,2}$ - $Q_{k,i\,2}$

2.3 Numerical Formulation, Boundary and initial conditions

An implicit method of finite difference for the numerical resolution is used [4]:

$$A_{S_i} \frac{\Delta z_i}{\Delta t} = P_i + \sum_{k=1}^{j} Q_{k,i}^{(n)} + \sum_{k=1}^{j} \frac{\partial Q_{k,i}^{(n)}}{\partial z_i} \Delta z_i + \sum_{k=1}^{j} \frac{\partial Q_{k,i}^{(n)}}{\partial z_j} \Delta z_k \qquad (11)$$

$\boldsymbol{A_S}$ **,** $\boldsymbol{P_i}$ *y* $\boldsymbol{Q_{k,i}}$ are know at time $\boldsymbol{t = n\,\Delta t}$ and $\boldsymbol{\Delta z_i}$ y $\boldsymbol{\Delta z_k}$ are unknowns. For N cells, the system is established for N unknown functions $\boldsymbol{z_i}$ of the independent variable *t* . The solution exists and is unique [4] if the initial conditions $\boldsymbol{z_i}$ *(t)* can be prescribed. The numerical resolution is carried out by an algorithm based on the matricial resolution by the method of Gauss-Seidel. Three types of boundary conditions can be represented: *a) water level as function of time:* ***z(t)****; b) Discharge as a function of time*: ***Q(t)*** ; *c) level-discharge relationship*: ***Q = f(z).*** The model requires water levels at all the cells at the initial time. For dynamic links initial discharges should be defined at such links. The term $\boldsymbol{P_i}$ ***(t)*** in eq. (1) allows the addition or extraction of external flow at each cell. Usually it is used to represent effluents, overflows, extraction for irrigation, etc.

3 Applications

3.1 Urban basin

The simulation of the so-called urban basin "Conduit 27" located in Rosario city (Fig. 2) is described [11]. The basin has a 4.23 km^2 area and slopes ranging between 1 and 3 ‰. The drainage system is combined. The property runoff corresponding to rainfalls of recurrence smaller than that of the design drains flow through the combined sewer. In events of medium and large storms, the runoff drains by the combined sewer and by streets toward the storm inlets of the trunk conduit. The underground system of closed conduits is constituted by two trunk conduits and multiple secondary branches. The linking between the surface and underground systems consists in approximately 300 m of storm inlets. There is an 85% of impervious area. A continuous increment of the impervious area of the basin has produced a decrease of the maximum storm that supports the system without floods from 1 to 2 years of recurrence.

CELDAS8 was implemented with 68 cells and 114 links (Figs. 3 and 4). The level of detail was 16 elements/ km^2. The calibration was made with a group of measured values of rainfall and water level registered at the main conduit. The calibrated Manning coefficient for runoff over tributary areas was η = 0.100-0.200. Into the secondary combined sewer the resistance was η = 0.020 and by avenue and main conduits η = 0.016. The discharge coefficient of the storm inlets was 0.60. The model implemented was used to quantify the improvement of the capacity of the drainage system by means of the substitution hypothesis of 30% of the impervious area (sidewalks, courtyards and secondary streets) for semipervious surfaces. The new porous surfaces were considered 30% impervious and 70 % permeable. In Fig. 5 the net rainfall and downstream hydrographs for 5 and 10 years recurrence storms in the current state and with the consideration of substitution of impervious for porous surfaces are illustrated. The maximum water levels calculated in the trunk conduit and over the avenue are shown in Figs. 6 y 7.

3.2 Determination of flood risk zones over Ludueña river floodplain

In this application the determination of inundation risk maps over the Ludueña River (Rosario, Argentina) are described [9],[10],[11]. This watercourse drains a 800 km^2 area, flowing toward the Paraná River. The floodplain under study includes a superficial 50 km^2 area over the low basin. A 75% of it is rural zone; a 15 % semiurbanized and the 10% is fully inhabited. There are around 300,000 inhabitants. The river length including its tributaries is 19 km, with a 1.2‰ average slope. In the main stream, overflow occurs for 80 m^3/s. In a reach 1.5 km length the watercourse is piped in five close conduits. These conduits have a 74 m^2

cross section, and the discharge maximum capacity is 350 m^3/s. The main goal of the study was to analyse the hydraulic behaviour of the system to determine the inundation risk maps for natural state (without works) and projected state (with structural works such as an upstream retention dam and new closed conduits). Those maps were determined for 50, 100 and 500 years of recurrence. In addition, a mapping for the maximum probable flood (MPF) as catastrophic event was also carried out. The topological and spatial discretization was done with 202 cells and 311 links (Figs. 8 and 9). The level of detail was 4 elements/km^2. The model was calibrated using data from historic floods, mainly with one measured in 1986 of 50 years recurrence.

During the model exploitation, the input peak discharges were 500 m^3/s (RP= 50 years), 700 m^3/s (RP= 100 years), 1,000 m^3/s (RP=500 years) and 1,700 m^3/s for the MPF. For the flood of 500 years recurrence the maximum relative error in water level over the main stream was 11% and over the flood plain 19%. The results for a 50 years recurrence without works and a 500 years recurrence with works are presented. The inflow from the high basin is shown in Fig. 10. The computed hydrographs at downstream boundary over urban zone, routing by the major system and by the conduits are illustrated in Fig. 11. The corresponding inundation maps are shown in Fig. 12. From the results obtained in both simulations it can be observed the positive effect of the structural works. Based on the risk area delimitation described, state and local government have planned the non-structural rules and developing the associated legislation.

3.3 River Ludueña basin

This application has consisted in using the CELDAS8 model as a subsystem of flow surface propagation in an integral system of hydrological simulation denominated SHAL [14]. This integral system is capable of simulating hydrological processes such as: precipitation, evapo-transpiration, surface storage, infiltration, surface runoff, percolation, flow in not saturated zones and groundwater flow.

The Ludueña river basin was modelled. It is located south east of Santa Fe province (Fig. 13). The length of the hydrographic network (including permanent and transitory watercourses) is 140 km and the basin area is 700 km^2 with an average slope of 1.0 ‰. The base flow is 0.50 m3/s, under ordinary flow condition is 80 m3/s and in extraordinary events (more than 50 years recurrence) discharges larger than 400 m3/s have been observed.

The discretization chosen was constituted by 100 cells and 111 links, reaching a mean level of detail of 0,14 elements/ km^2 (Fig. 14). At the downstream boundary a level-discharge relationship under a big road bridge was considered. The links between tributary cells and river cells were kinematic and the links between watercourse cells were simple river and bridges (culverts). The considered detail definition has allowed the fine representation of multiple geometric and hydraulic parameters of the components of the system.

In each simulated event the CELDAS8 model was loaded with the net rainfall computed by means of the subsystem of losses of the SHAL. The model was calibrated considering observed data from a serie of events. There are two meteorological stations with rainfall and hydrological parameters recorder. In addition, there are two measurement stations of water levels discharges under bridges. The comparison between observed and calculated hydrographs at the downstream boundary is shown in Fig. 15 and 16, corresponding to typical floods without and with floodplain.

The CELDAS8 model as a component of the SHAL was exploited for the analysis of diverse runoff alternatives [14]. In this way, the model application has been able to incorporate a technological tool to the planning of the regional water resources.

4. Conclusions

The modelling system presented here has demonstrated a satisfactory capability to reproduce the multiple processes involved in the rainfall-runoff transformation. It can simulate quasi-2D flow in single layer and flow over two interconnected layers. The simulation structure allows the representation with different levels of detail, allowing the maximum possible subdivision of the physical components with geometrical and hydraulic parameters consistent with the available data. The calibration parameters (in all testings and applications) were in agreement with the range of standard values encountered in the literature for flow resistance.

References

[1] Abbott, M. and Cunge, J. (1981), *A modelling system for the design and operation of storm-sewer network 11* Engineering applications of computational hydraulic storm, Vol. 1, Pitman, London.

[2] Cao, C., Yen, B.C. and Benedini, M.(eds.) (1993), *Urban storm drainage proceeding U.S.-Italy*

Bilateral Seminar, Water Resources Publications, Highlands Ranch.
[3] Chow, V.T. (1959*), Open-Channels Hydraulics,* Mac Graw-Hill, New York.
[4] Cunge, J. (1975), *Two dimensional modelling of flood plains,* in: Mahmood K. and Yevjevich V., eds., Unsteady flow in open channels, Water Resources Publications, Fort Collins, 705-762.
[5] Danish Hydraulics Institute (1996), *MOUSE User Manual, V 4.0,* Horsholm.
[6] Huber, W.C. and Dickinson, R.E. (1988), *Storm Water Management Model User´s Manual, V4*, EPA, Environmental Protection Agency, Athens, Georgia.
[7] O'Loughlin, G., Huber, W. and Chocat, B. (1996), *Rainfall-runoff processes and modelling,* Journal of Hydraulic Research, 34, 733-752 .
[8] Riccardi, G. (1994), *Aplicación de un modelo matemático de celdas en el Arroyo Saladillo,* XVI Congreso Latinoamericano de Hidráulica, IAHR-LAD, Santiago, Chile (in spanish).
[9] Riccardi, G. (1997*), The mathematical modelling of flood propagation for the delimitation of inundation risk zones,* in: Sustainability of Water Resources under Increasing Uncertainty (D. Rosbjerg et al.,eds.) IAHS Publication Nro 240, Wallingford, 355-364.
[10] Riccardi, G. (1997), *El mapeo de riesgo de inundación por medio de la modelación matemática hidrodinámica,* Ingeniería del Agua, 4 , 45-56. (in spanish)
[11] Riccardi, G. (1998), *Testeos y aplicaciones del modelo de simulación hidrológica-hidráulica CELDAS8,* Annual Report, Consejo de Investigaciones, Univ. Nacional de Rosario, Rosario.(in spanish)
[12] Terstriep, M. and Stall, J.B. (1974), *The Illinois Urban Drainage Area Simulator ILLUDAS*, B 58, Illinois State Water Survey, Urbana.
[13] Wisner, P.E. (1983), *IPSWM Urban Drainage Modelling Procedures, The Implementation of Storm Water Management Program,* University of Ottawa
[14] Zimmermann, E. and Riccardi, G. (1995),*Modelling of fluxes of water in flatland,* XX General Assembly European Geophysical Society, Hamburg.

Notations

A_{Si}	Superficial wetted area of the cell i (m^2)
z_i and z_k	Water level at the cells *i and k* (m)
$Q_{k,i}$	Discharge between cells k and i ($m^3 s^{-1}$)
$\Delta x_{k,i}$	Distance between centres of cells k e i (m)
$K_{k,i,}$	Conveyance coefficient of the link between cells k and i ($m^3 s^{-1}$)
$Rh_{k,i}$	Hydraulic ratio of the link between cells k and i (m)
$At_{k,i}$	Cross sectional of the link between cells k and i (m^2)
η	Manning resistance coefficient ($s\ m^{-4/3}$)
At_i , At_k	Cross wetted areas of the cells k and i (m^2)
a_1 , a_2 , b, c	Coefficients, depending on $Q_{k,i}^{(n-1)}$, $z_i^{(n-1)}$, $z_k^{(n-1)}$ (m^{-3}), (s^{-1}), ($m^3 s^{-2}$)
z_w	Elevation bottom weir (m)
μ_1 , μ_2	Weir discharge coefficients for free and submerged flow
b	Weir width (m)
z_{cri}	Critical level in control section (m)
A_{cri}	Wetted area of the control section for critical level (m^2)
A_{Sc}	Wetted area in the control section (m^2)
Cd	Discharge coefficient in control section
g	Aceleration of gravity ($m\,s^{-2}$)
A_{cll}	Full cross section of closed conduit (m^2)
μ_l , μ_a	Discharge coefficients of storm inlets (weir or hole) for free and submerged flow
L_{su}	Opening length (m)
Cs	Coeficient of geometric and hydraulic characteristics in storm inlet
z_{su}	Elevation bottom of gutter (m)
Q_{ll}	Discharge in full section conduit ($m^3 s^{-1}$)
$Q_{p;}$ Q_b	Peak and base discharge ($m^3 s^{-1}$)
$t_{p;}$ t_b	Peak and base time (s)
$D; L$	Diameter and length of conduit (m)
i_l	Longitudinal slope
RP	Return period (years)
t	Time (s)

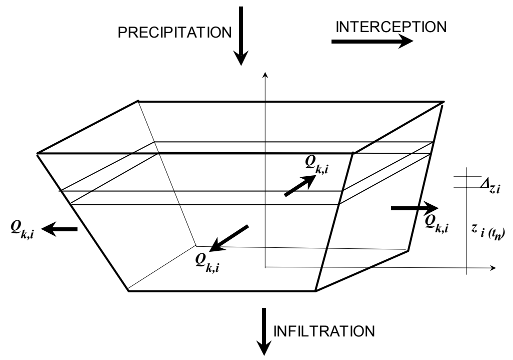

Fig. 1.Continuity equation of a cell

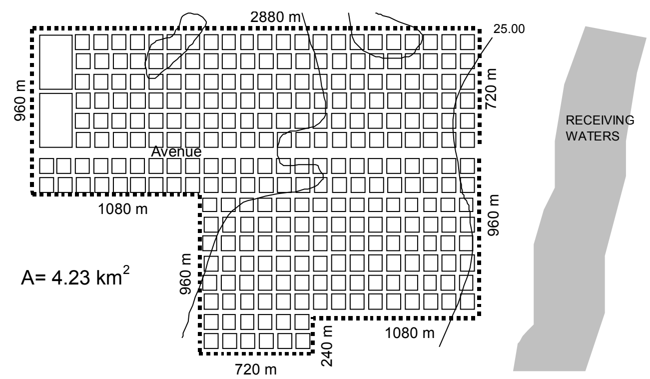

Fig. 2 Urban drainage basin

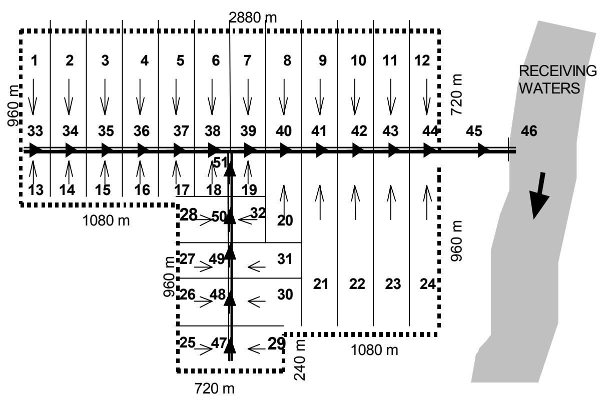

Fig. 3. Topological discretization of surface layer

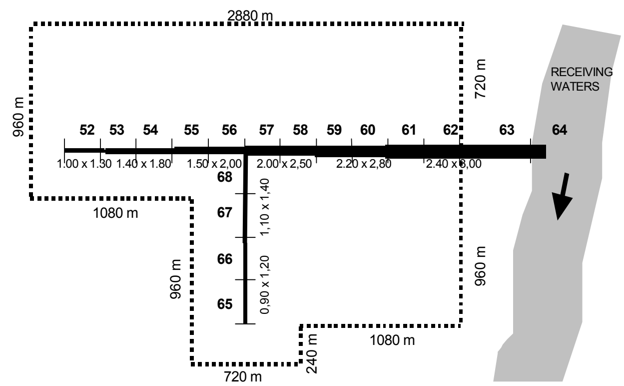

Fig. 4. Primary network of close conduits

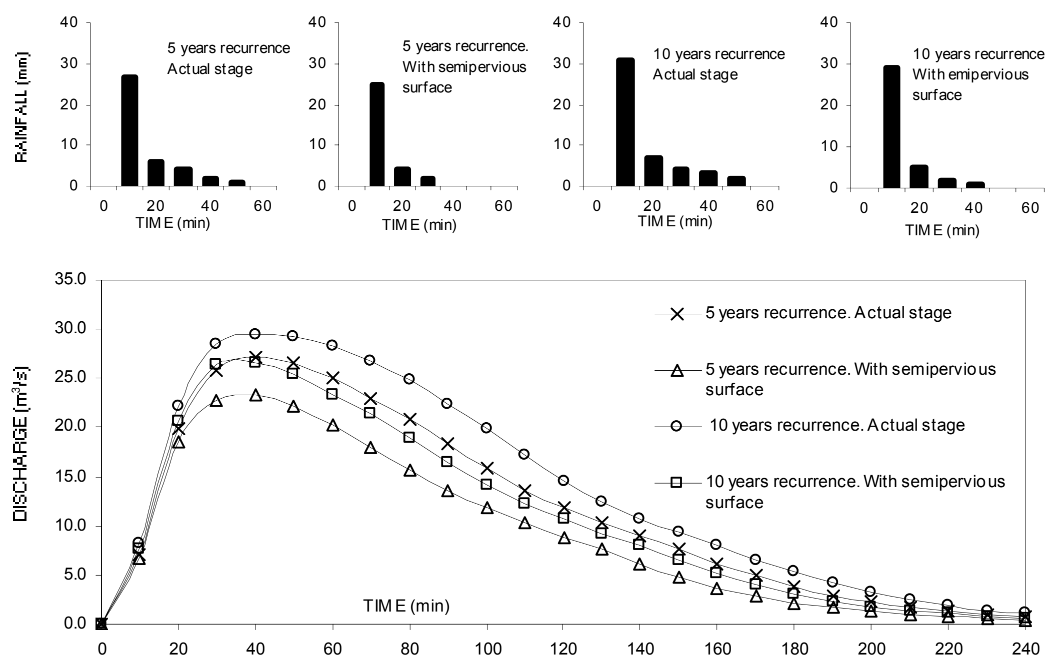

Fig. 5. Calculated net hyetographs and hydrographs. Actual and with semipervious surface stages

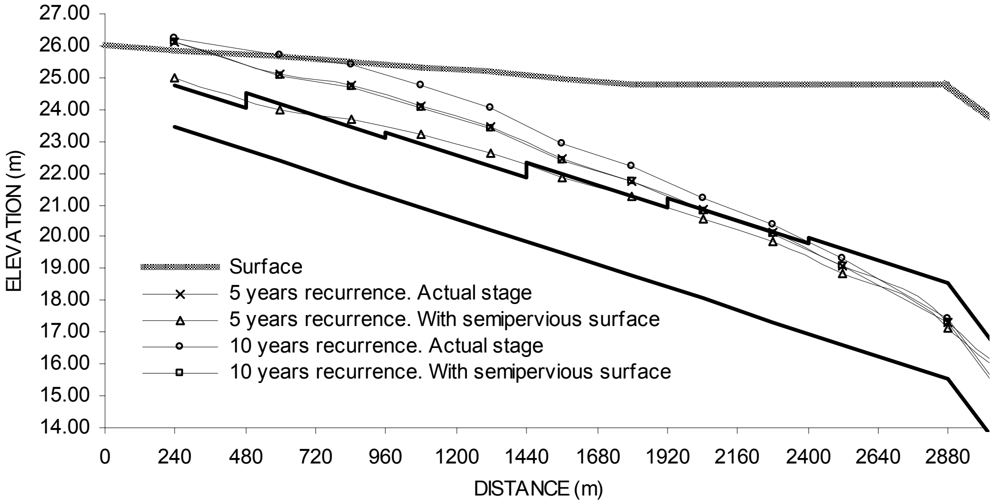

Fig. 6. Calculated water levels in trunk conduit

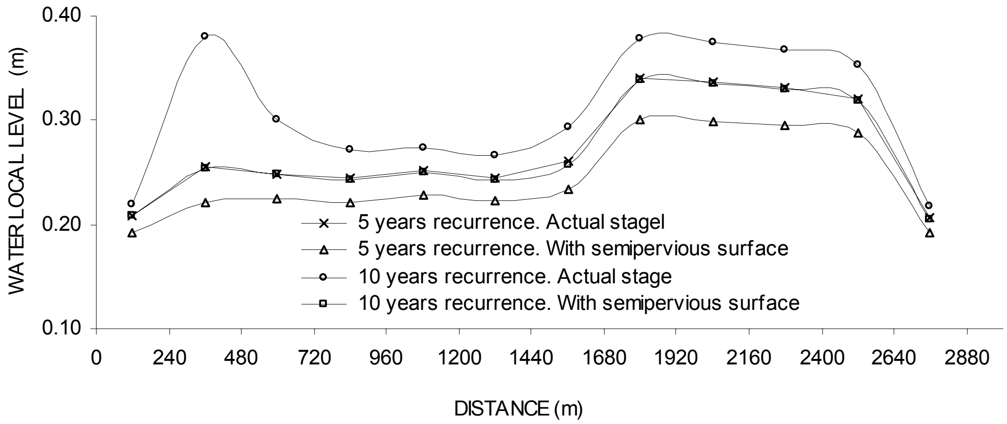

Fig. 7. Calculated water levels over the main avenue

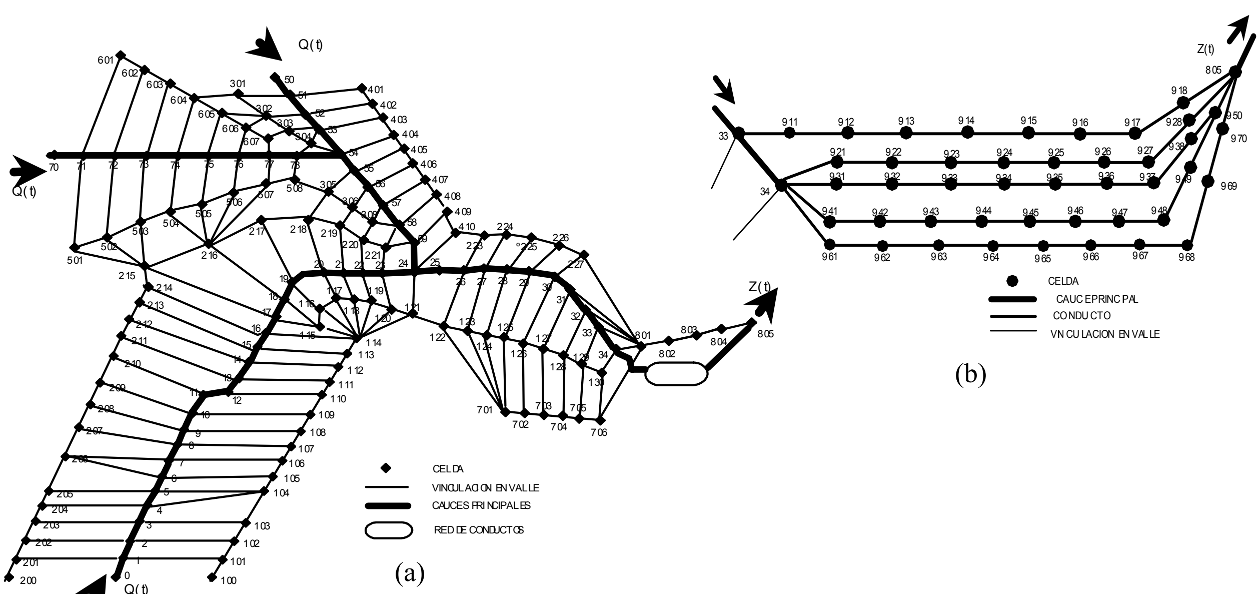

Fig. 8. Topological model representation. (a) Complete zone; (b) Closed conduit network

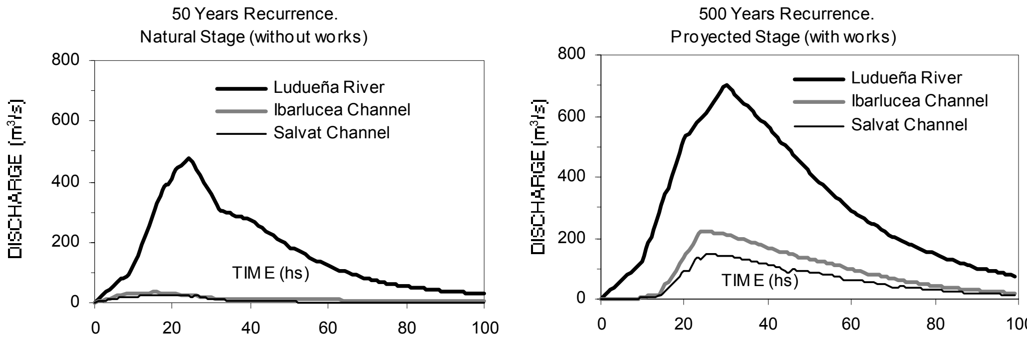

Fig. 9. Input hydrographs. 50 and 500 years recurrence

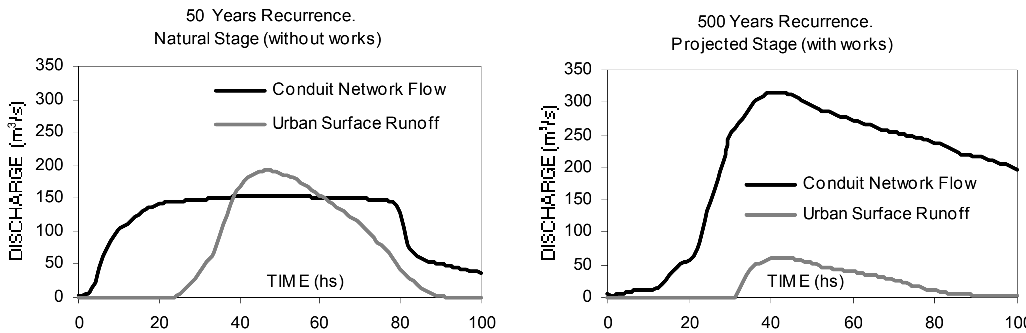

Fig. 10. Downstream hydrograph. 50 and 500 years recurrence

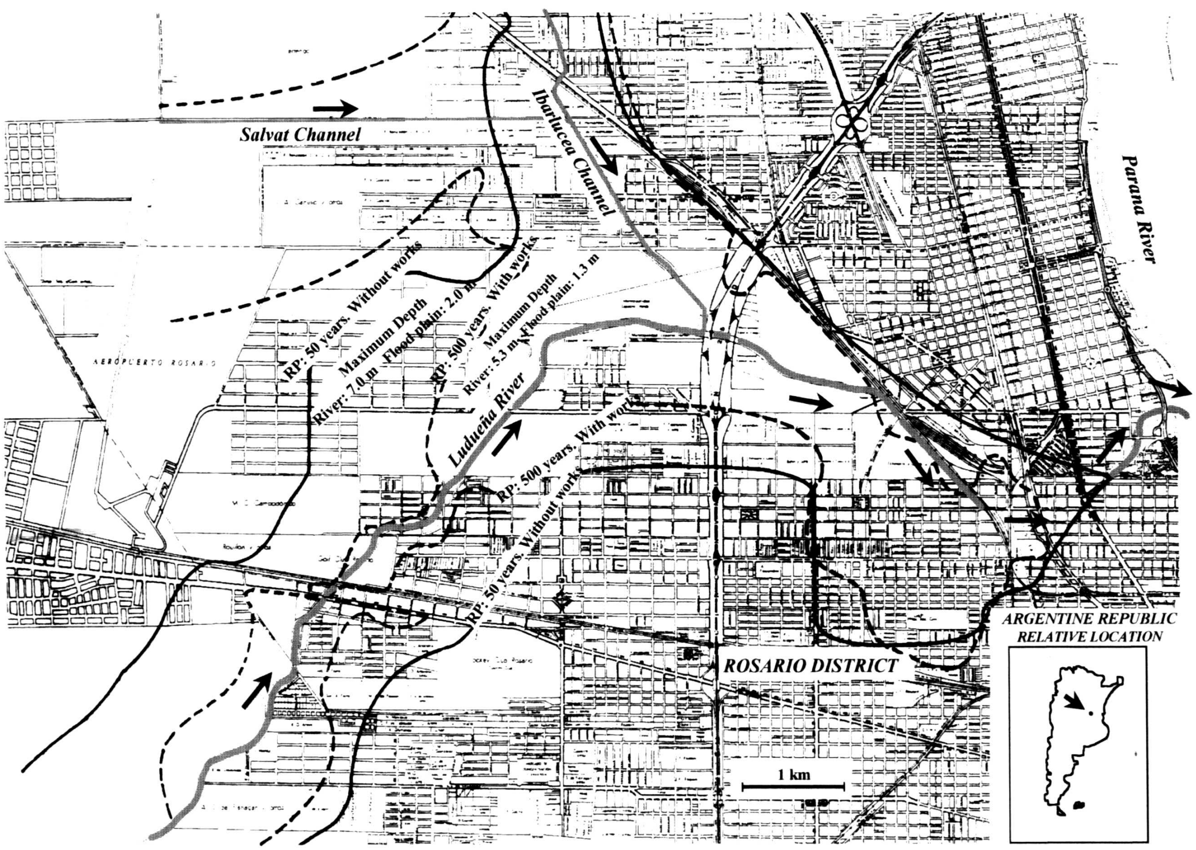

Fig. 11. Ludueña river flood map

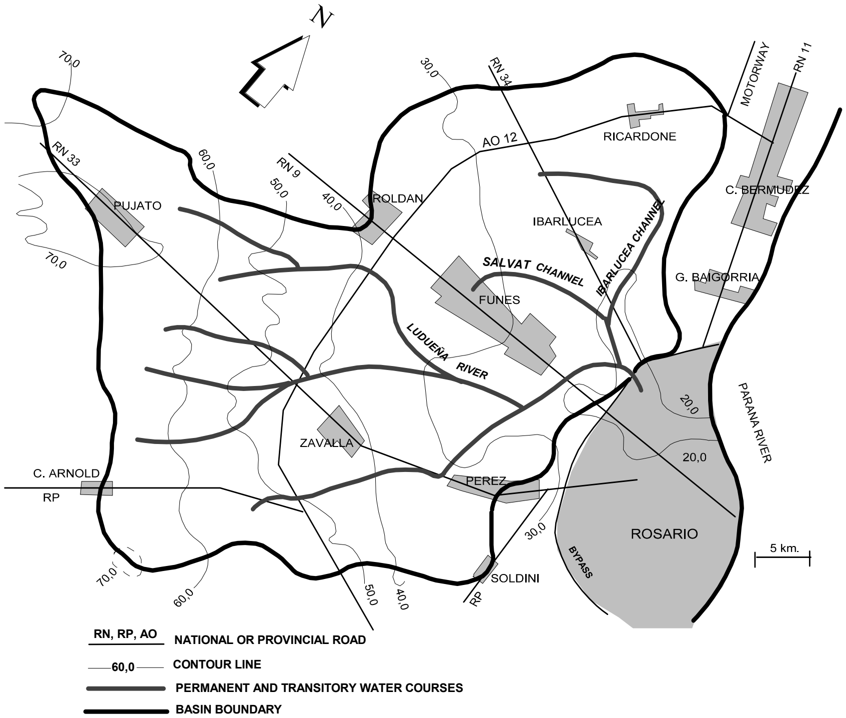

Fig. 12. The Ludueña river basin

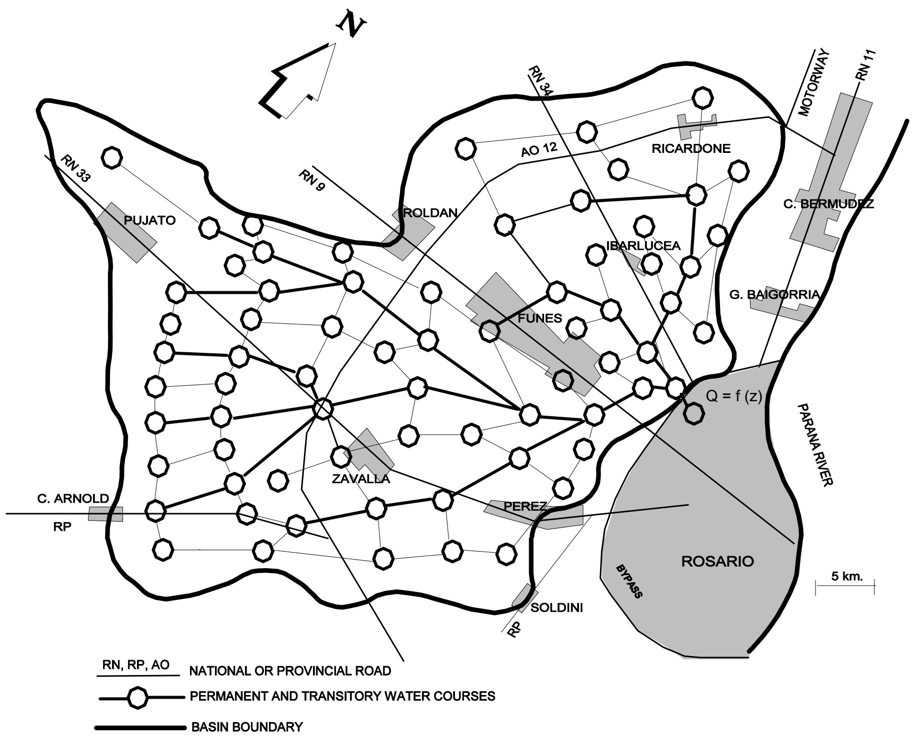

Fig. 13. Topological model representation of the Ludueña river basin

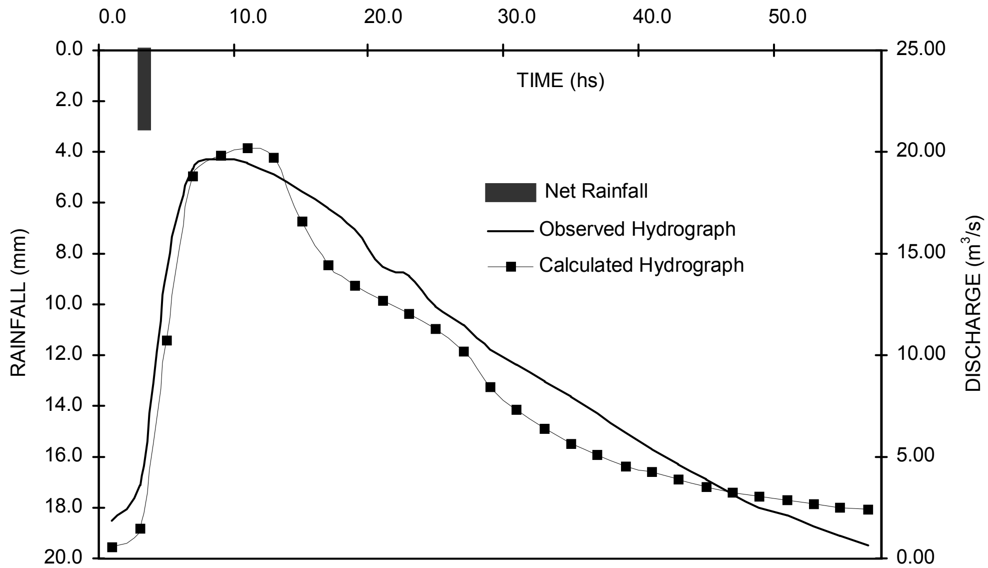

Fig. 14. Observed and calculated hydrographs for Ludueña river basin. Event without overflow

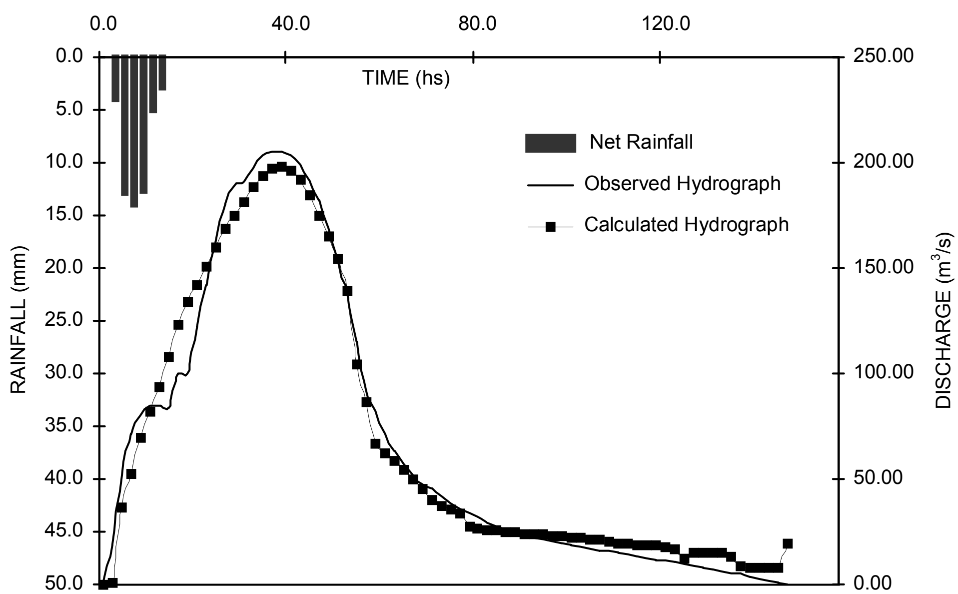

Fig. 15. Observed and calculated hydrograph for Ludueña river basin. Event with overflow