MATHEMATICAL MODELLING OF FLOOD PROPAGATION FOR THE DELIMITATION OF RURAL, SEMIURBANIZED AND URBANIZED ZONES WITH INUNDATION RISK

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SUMMARY

A mathematical model of subcritical flow simulation is described in this paper. It's capable for flood propagation in water courses with flood plains partial or totally obstructed. Also it permits the simulation in semiurbanized and urbanized zones , modelling the flow through the ground surface (major system) and through conduit networks (minor system). The model was calibrated by simulation of several events with recorded information and now this is being applied as a technical tool in the risk zone delimitation of floods with different return period in the Luduena Stream, water course of the Rosario District.

In this work the governing equations of the model are exposed. Later the numeric formulation, the solution of the equations in finite differences, the boundary and initial conditions, and the computational structure are presented. Then the state of advance of a model application is described. This application consists in the modelling on: 15 *km* of water course, 1.5 km on big underground conduits, embracing a zone of 40 km^2 in the rural and urban zone of the cities of Rosario and Funes. Lastly the conclusions of the work are exposed.

1. INTRODUCTION

The Rosario district, at the south of the Santa Fe state, Argentina, is characterized by being crossed for water courses at north and south , which have their nascent in rural zones with beds not defined, downstream they descend toward the Parana river with the bed more marked and draining water permanently. Upon arriving these hydric systems to the down basins, they cross big inhabited orbs as Rosario and other cities, for where their floodplain comes urbanized in growing degree, until it becomes fully inhabited zone, in which the overflow produces serious material affectations and at the same time human losses. In order to solve this hydric hydric problematic, public organisms faced the construction of several structural works in views to the resolution. At the present, with the works projected in the culmination state it is necesary the definition of no-structural rules to continue with the legislation that englobe them, they permit an appropriate occupation and use of the floodplains soil in order to avoid damages and losses of human being for the future, always inside a context of planning of hydric resources to regional level. In this context, is proposed the configuration of inundation risk map associated with the return periods of the events. For this delimitation mathematical models have begun to implement for the flow simulation. In this work a cell model is exposed, it results capable for the subcritical flow simulation (or slightly supercritical) on superficial propagation , by bed ,floodplain and by street at urban zones; in like manner for underground collector networks. The model is based on two governing equations: those based on continuity and those ones on discharge laws between cells or compartments. The model hypothesis agree with the flow characteristics in the regional water courses, where the rivers and streams are conformed by a main permanent bed and a tempeorarily occupied flood plain by the flow. This flood plain is crossed by highway embankments, railroads,etc, which allow to divide it in a certain number of interconnected compartments {cells). Also the flows regjster small variations of water levels through the time. On the other hand, in order to evaluate the influence of the velocity energy variation of the flow in certain zones, it is incorporated an approach of the inertial term. This term evaluates the convective acceleration forces in the governing equations. In concerning to the routing in urban zones, the model was developed to simulate flood propagation on streets (denominated major system of urban drainage), and on close conduit networks (minor system). This model results capable to simulate flows under pressure for values lightly greater than the atmospheric pressure (up to 1.5 kg/cm^2).-

The model uses a numeric resoluting algorithm where it doesn't exist any type of restrictions relating to the linkings between the several constituent cells of the model. it is not necessary to respect any condition of cell arrangement. Hence, the model provides any alternative of topological and hydraulic discretization between their components. The computational system was carried out in Fortran77 Language - Lahey 3.0, obtaining a system extremely quick.

2. GOVERNING EQUATIONS

The flood propagation on main beds, on floodplains, on urban streets or in conduits, clearly has two-dimensional characteristics. This model simulate the twodimensional propagation by means of flow exchange between cells with any coordinate system contained in the plane, but with exchange laws of unidimensional type.In the Fig.1a and 1b are presented, as an illustrative manner, a reach of water course conformed by main bed and flood plain, and an urban reach conformed for the great and minor systems. The equations that govern the model are those of continuity and those of discharge between linked cells.

2.1. Continuity Equations

In the formulation of the continuity equations is supposed that the whole i cell corresponds to a characteristic water level z_i which is assumed in the cell center (Fig.2). Also, it is assumed that the water surface is horizontal between the borders of the cell and that its is z_i. There are two fundamental hypothesis on which the governing equations are based:

(1) The volume V_i of water stored in the i cell are directly related to the water level z_i of the cell: $\mathbf{V_i = V\{z_i)}$

(2) the $Q_{i,k}^n$ discharge between two cells adjacent **i** and **k** at one time given $n\Delta t$ is a function of the energy levels: $z_i^n + a_i V_i^2/2g$ and $z_k^n + a_k V_k^2/2g$.

This formulation can be written in the following manner:

$Q_{i,k}^n = Q(z_i^n + a_i V_i^2/2g,\quad z_k^n + a_k V_k^2/2g)$. The forces originated in the local acceleration are neglected. If an **i** cell is considered **(Fig.2)** and an interval of time Δt is assumed for any knowed time $t_n = n\Delta t$, the water level in the **i** cell is z_i and the corresponding area of the water surface **(ABCD)** is equal to $A_s i(t_n)$ the water surface in the horizontal plane of the cell. At the time $t_{n+1} = t_n + \Delta t = (n+1)\Delta t$ the water level is $z_i(t_{n+1})$ and the water surface become A'B'C'D' equal to: $A_{Si}(t_{n+1}) = A_{Si}(t_n) + \Delta A_{Si}$. The increment of water level is due to the rainfall on the cell Pi (t) during a Δt interval , and also to the Q_i^n discharges fram the cells k adjacent and other contributions. The increment of the water volume stored in the cell i during a certain interval of time Δt dt may be defined from geometric conditions (eq.l) and from discharge conditions (eq. 2):

$$\Delta V_I = \int_{z_i(t_n)}^{z_i(t_{t+n})} A_{si}(z_i)dz_i \qquad (1)$$

$$\Delta V_I = \int_{t_n}^{t_{n+1}} P_i(t)dt + \sum_1^k \int_{t_n}^{t_{n+1}} Q_{i,k}(z_i, z_k)\partial t \qquad (2)$$

where the **k** suffix represents the total number of adjacent cells to **i**. Neglecting the terms of inferior order inferior and carrying out the approach of ler. order (p.e $\Delta A_{Si} / A_{Si} <<1$,) averaging the integrals (eq.2) in the time step Δt and comparing the volume increments defined by eg.(1) and (2), it is possible arrive to the continuity equation written under the differential form used by the model:

$$A_{S_i} \frac{dz_i}{dt} = P_{i_{(t)}} + \sum_i^k Q_{i,k}^n(z_i, z_k) \qquad (3)$$

There are many (eqs.3) as there are cells **i** in the model and, also as many unknow water level **z_i(t)**. Thus, for N cells the system of N ordinary differential equations is established for N unknow functions z_i of the independent variable **t**. The solution to this system exists and is unique if the set of initial condictions **z_i(t=O)** is prescribed (Cunge, 1975). Once this set is know, the functions **z_i(t)** may be computed numerically and the discharges **$Q_{i,k}$** can be found as they only depend upon the water levels **z_i, z_k**. The system of boundary conditions varyng in time must be prescribed too if the problem is to be well posed.

2.2 Laws of discharge between the cells

A group of exchange relationship between cells the model can use:

2.2.1 Simple river type links

In this case the Stricler-Manning formula i used. It is deduced from the momentun equation for flow with inertial forces negligible (eg. 4) where **h** is the water level, **z_f** the bottom elevation, and **K** is the conveyance coefficient defined by **K= k A $R^{2/3}$** with **k**: roughness coefficient of Strickler (1/η), **A**: cross-section, and **R**: hydraulic radius (Fig. 3). Discret:izating the **(eg.4)** for two adjacent cells **i** y **k**:

where **Δx** is the fixed distance between the cell centers. In eq.(5) the parameters, **k $A_{i,k}$** and **$R_{i,k}$** are functions of the water level **z** in the flow

$$\frac{\partial h}{\partial x} + \frac{\partial z_f}{\partial x} + \frac{Q|Q|}{K^2} = 0 \qquad (4)$$

$$Q = signo(z_k - z_i)\frac{K}{\sqrt{\Delta x}}\sqrt{|z_k - z_i|} \qquad (5)$$

section between **i** cells and **k**. Therefore one may write:

$$K = kA_{i,k} R_{i,k}^{2/3} = K(\overline{z_{i,k}}) \tag{6}$$

where $z_{i,k} = \alpha\, z_i + (1 - \alpha) z_k$ is the weighted level between the levels in the two cells and $K = K(z_{i,k}) = K(z_i, z_k)$ is recognized as the conveyance factor of the flow section . The coefficient of peso **a** is constant for each pair of given cells. For the evaluations of the discharge a function is defined:

$$\Phi(z_{i,k}) = \frac{K(z_{i,k})}{\sqrt{\Delta x}} = \frac{kAR^{2/3}}{\sqrt{\Delta x}} \tag{7}$$

$\Phi(z_i , z_k)$ must be established at first on the basis of the geometry and general roughness and then is adjusted during the calibration procedure.

2.2.2 Weir type links

This type link is used usually in order to represent to linkings between cells where evidence a physical limit between them. Such it is the case of cells separated by highway embankment, roads, etc. Also are used as linking between the cells of the main course and the flood plains. Is used the classical formulation of the wide crest wier (Fig. 4). In the [ec]. (8a. and 8b.) the discharge formula for the case $z_k > z_i$ is presented :

$$Q_{i,k} = \Phi_F (z_k - z_w)^{3/2}$$ 1) Vertedero de descarga libre (8.a)

$$Q_{i,k} = \Phi_D (z_i - z_w)\sqrt{z_k - z_i}$$ 2) Vertedero sumergido (8.b)

Generally the coefficients Φ_F and Φ_D may be defined as:

$\Phi_F = \mu_1 b\sqrt{2g}$ $\Phi_D = \mu_2 b\sqrt{2g}$ b being the effective width of the weir and μ the discharge coefficients.

2.2.3 Cuasi-inertial river type links

The deduction of the discharge between two adjacent cells is based in the complete momentun equation (eq. 9). Deriving the second term, considering slow variations of water levels and flows through the time, and with an appropriate discretization in cells of the course, which implies that the variationals terms of flow concerning **t** and to **x** of the flow could be neglected, the eg.9 can be written as (eq. 10) :

$$\frac{\partial Q}{\partial t}+\frac{\partial}{\partial x}\left(\frac{Q^2}{A}\right)+gA\frac{\partial z}{\partial x}+g\frac{AQ|Q|}{K^2}=0 \tag{9}$$

$$-\frac{1}{gA}\frac{Q^2}{A^2}\frac{\partial A}{\partial x}+\frac{\partial z}{\partial x}+\frac{Q|Q|}{K^2}=0 \tag{10}$$

by discretization of (eq. 10) and considering the variations of wet areas not greater than a 50% one can arrive to an explicit expression of the flow $Q_{i,k}$:

$$Q_{i,k}=\Phi_{i,k}\sqrt{ABS\left[\frac{Z_k-Z_i}{1+\frac{\Phi_{i,k}^2}{2g}\left(\frac{1}{A_i^2}-\frac{1}{A_k^2}\right)}\right]} \tag{11}$$

2.2.4 Bridge type links

For the computation of the $Q_{i,k}$ discharge, between two cells separate por un bridge was used the formulation that presented Chow (1959) in their article referred to flow through constrictions. According to Chow applying the equation of energy in the **k** sections and **p** (**k**: upstream, **p**: zone under bridge) one can express an explicit formulation of the discharge as :

$$Q_{i,k}=\sqrt{2g}\,C\,A_p\sqrt{\frac{\Delta h}{1-\alpha_k C^2\left(\frac{A_p}{A_k}\right)^2+2gC^2\left(\frac{A_p}{K_p}\right)^2\left(L+L_a\frac{K_p}{K_k}\right)}} \tag{12}$$

$\mathbf{h_f}$ is function of $\mathbf{K_k}$ and $\mathbf{K_p}$ that they are the conveyance coefficients of the arriving section to the bridge and under the bridge; $\mathbf{L_a}$ is the arrive reach longitude upstream (of the perturbed zone) and **L** is the constriction longitude; **Ak** and **Ap** are the area wet of the **k** sections (cells upstream) and **p** {under the bridge); **C** is a total coefficient of discharge that depends mainly of: Flow Constriction Percentage; Froude Nr.; Rounding of the edges; bridge skew; buttress characteristic; eccentricity; etc.

2.2.5 Loss energy type links

This type of link is capable for those ones flow singularities in with energy loss due to abrupt changes in the cross-section: contractions or expansions so much in vertical as in horizontal plane.

These particularities are present usually in sewers, collectors mouths, junctions box, etc. (Fig. 7). The discharge equations between two cells in a contraction (eq. 13) and expansion (ec.14) are defined as:

$$Q_{i,k} = \sqrt{2g}\sqrt{\frac{Z_k - Z_i}{\left(\frac{1+k_i}{A_i^2} - \frac{1}{A_k^2}\right)}} \quad (13)$$

$$Q_{i,k} = \sqrt{2g}\sqrt{\frac{Z_k - Z_i}{\left(\frac{1}{A_i^2} - \frac{1-K_k}{A_k^2}\right)}} \quad (14)$$

in which **k** establishes the portion of velocity energy of the contracted section, that gets lost in the link.

2.2.6 Frictional conduit type links

This type of linkings is used for the connection between cells of close conduits. The equation of used discharge is of the same type that the linkings exposed in II.2.1 II.2.4. The difference with those are given for the cases that the conduits work to pressure. In these cases the conveyance factor remains constant for values of hydraulic grade line more than the top conduit elevation : **$K=k\,A_{i,k}\,R^{2/3}_{i,k}$=constant** (Figs. 8). Also, in the cases of hydraulic grade line upper to top conduit, in the continuity eq. (3) the value of A_{Si} is minimum, corresponding to the Presissmann slot surface (Fig.9) {Cunge, 1980) along the whole close conduit.

3. NUMERICAL FORMULATION

For the numeric formulation of the model equations, in first place it is explicited the function of discharge between cells, then is introduced in the continuity eq. (3). It is assumed that the discharge $Q_{i,k}(zi(\tau), zk(\tau))$ is an intermediate discharge between $Q^n_{i,k}$ and $Q^{n+1}_{i,k}$ with varying between $n\Delta t \le \tau \le (n+1)\Delta t$. The intermediate value of $Q_{i,k}(\tau)$ is defined considering a weighting coefficient $0 \le \theta \le 1$ and is introduced in the continuity equation (3):

$$A_{s_i}\Delta z_i = \Delta t[\theta \sum\nolimits_1^k Q^{n+1}_{i,k} + (1-\theta)\sum\nolimits_1^k Q^n_{i,k} + P_i] \quad (15)$$

In order to avoid the arriving to a complex system, small water level variations Δz_i during the time interval Δt are assumed as hypothesis. This allows to develop the discharge formula in series of Taylor and neglect the terms of higer order. Therefore the eq. (15) can be written:

$$A_{s_i}\frac{\Delta z_i}{\Delta t} = P_i + \sum\nolimits_1^k Q^n_{i,k} + \theta\left[\sum\nolimits_1^k \frac{\partial Q^n_{i,k}}{\partial z_i}\Delta z_i + \sum\nolimits_1^k \frac{\partial Q^n_{i,k}}{\partial z_k}\Delta z_k\right] \quad (16)$$

The eq.(16) has as unknows to the increments Δz_i and Δz_k . It is can be show that is more appropriate for the resolution the use of an implicit metod of finite difference (Cunge 1975), it since enable the use of an computational time interval compatible with the rate of variation of the parameters that characterize the physical phenomenon. Thus the weighting coefficient can be taken as $\theta = 1$ to ensure a unconditionally stable system of finite differences equations, then the eq.(16) take the form:

$$A_{s_i} \frac{\Delta z_i}{\Delta t} = P_i + \sum_1^k Q_{i,k}^n + \sum_1^k \frac{\partial Q_{i,k}^n}{\partial z_i} \Delta z_i + \sum_1^k \frac{\partial Q_{i,k}^n}{\partial z_k} \Delta z_k \qquad (17)$$

the functions **A**$_{Si}$, **P**$_i$ and **Q**$_{i,k}$ are known in the time $t = n\Delta t$ and the increments Δz_i and Δz_k are the unknows. They will exist so much unknows as cells have the model and the system and it is completed with the boundary conditions.

The model uses an matrixial resolution algorithm based in the method of Gauss-Seidel. It is forme a system of **m x m** equations, where **m** : amount of internal cells of the model. In each step of time they are solved the matrixes in order to determine the corresponding Δz, with these values the water levels in each cell are calculated: $z_j^{n+1} = z_j^n + \Delta z_j$. Then is computed the discharges $Q_{i,j}^{n+1}$ in explicit form. This process is tnen repeated for the next interval of time.

4. BOUNDARY AND INITIAL CONDITIONS

The system of differential equations generated by the eq.(17), being of parabolic type, it is necessary and sufficient impose the **z(t)** levels in the geographical boundaries of the area to model. In the practice this is not always possible or convenient, for in real life three types of conditions may ocurr:

(i) Level given as a function of time: **z(t)**;

(ii) discharge given as function of: **Q(t)**;

(iii) Relationship is given between level and discharge: **Q=f(z).** The model requires the specification of the water levels in all the cells in the initial time of simulation.

5. COMPUTATIONAL IMPLEMENTATION

The computational system was denominated CELDAS4. It is conformed for a series of assistants routines and a main program.

In first place a series of auxiliary routines carries out a treatment of the topographical and hydraulics information for all the cells and links that compose the model. Then a files with the data of the topographical and hydraulic characteristics tabulated according to the stage increments are generated. Later, it begin the main process in wich the program computes and stores the results in disk by means of tabular files: Water level - time in each cell; discharge, velocity, Froude Nr. - time in each links; hydrographs and limnigraph in incoming and salient links.

For the case of partial simulations the program allows to store the previous history and reruns the program from the last time in which the data are stored in the files.

The program was developed in Fortran Language V.3 - Lahey and it is composite by around 1500 statments and it can be operated in computers PC-AT-386 or higher. At the moment it is working in the development of routines for presentation of results in graphic and tabular form by interactive device.

6. ADVANCES IN THE APLICATION

The model this being applied in a sector of Luduena Stream in the Rosario and Funes districts. The zone is located at south of the State of Santa Fe. The longitude of water courses is of approximately 19 *km*, with a average slope of 1.2 per thousand. In the main course the overflowing is presents for 80 m^3 *sec*. The floodplain in study includes a tatal surface of around 40 km^2. A 75% it is rural zone, a 15% semiurbanized and the 10% it is urbanized zone fully inhabited. In a reach of 1.5 km of lenght the water course is tubed in 5 conduits. These conduits have a cross-section 73.3 m^2, and drain in the Parana river flowing to free surface by a channel of 0,8 *km* of lenght. The conveyance capacity of the conduit networks is 350 m^3/sec.

The topological and spatial discretization was been satisfied with 202 cells and 311 links. 54 cells have corresponded to open channel flow; 43 to conduit networks; 95 to flood plains of rural and semiurban zone and 10 to urban zone. The topographical information was considered over a grid of points with average distance of 100 meters, this information was digitalizaded for a precise determination of contorn map each 0.25 *m*. Also the cross-sections of water courses, sewers, conduits, bridges and other works of art have been measured in order to the hydric modelling composition. The hydraulic parameters considered in preliminary form: roughness, efficiency coefficients , etc. was take in account of previous simulations (Riccardi G., 1991,1992, 1993, 1994).

The model was calibrated in function of different floods and mainly with one happened in the year 1986 of 50 year of return periode. At the moment it is being operated for the generation of the risk zones associated with floods of 100 and 500 years of return periode. The calculated results are translated in Hydrographs, Elevation-graphs and Maps of Elevation Maximum. This last are built by the calculated maxim elevation and the digitalizaded information of the zone in study. In illustrative manner have been presented in Fig 10 to 15 the results of the Preliminary Runs for 100 years of return periode. It is showed the variation of rainfall over the catchment; the inflow from the high catchment; the hydrographs modelled in urban zones near the conduit mouths, over the major system and in the conduit networks; also it is presented the water level variations in urban zone and a longitudinal profile of maxim water levels in the minor and major system. In Fig. 16 can be observed the study area.

7. CONCLUSIONS

The cell model developed model has resulted completly capable for the flow simulation with twodimensional characteristic. It simulates with satisfactory approach the flow propagation by beds, flood plains, underground conduit networks and by major system of urban zones. The results of calibration have demostrated a very good approximation to the real phenomenon.

Also the model application has been able to incorporate a powerful technological tool to the resolution of the regional hydric problematic, alowing the delimitation of zones with inundation risk by means of the computed results.

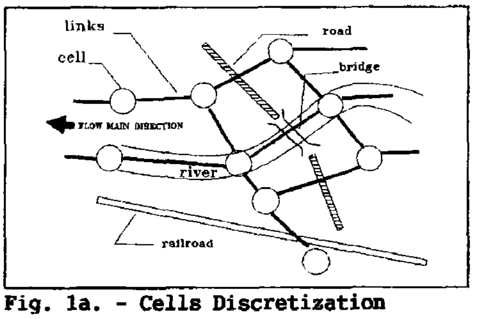

Fig. 1a. - Cells Discretization

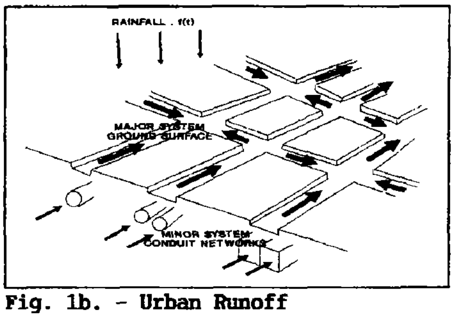

Fig. 1b. - Urban Runoff

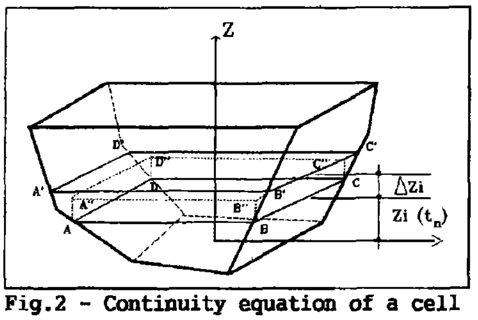

Fig.2 - Continuity equation of a cell

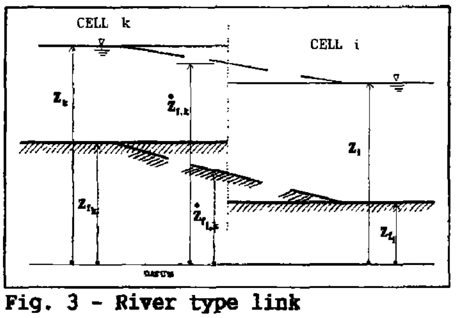

Fig. 3 - River type link

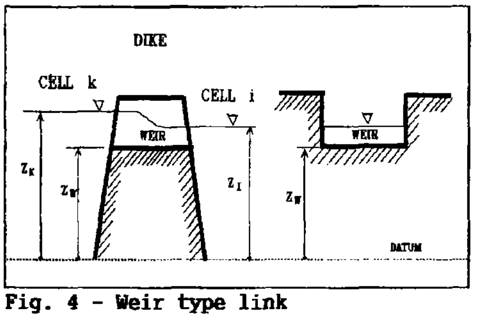

Fig. 4 - Weir type link

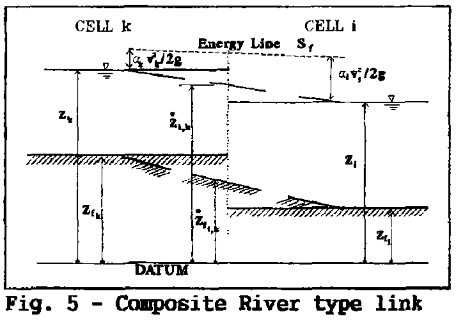

Fig. 5 - Composite River type link

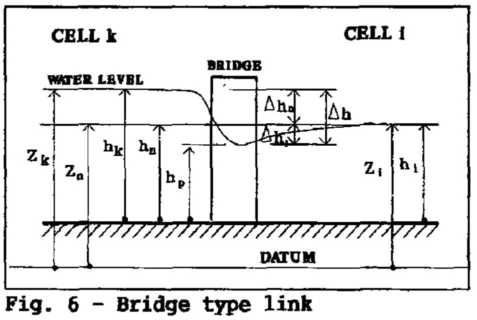

Fig. 6 - Bridge type link

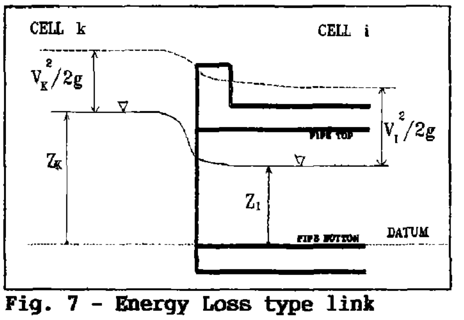

Fig. 7 - Energy Loss type link

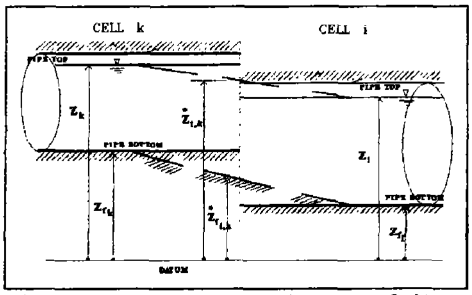

Fig. 8 - Frictional link in conduit

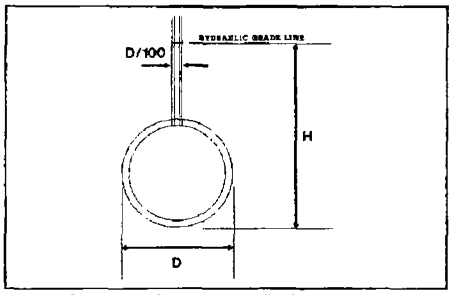

Fig. 9 - Preissmann slot

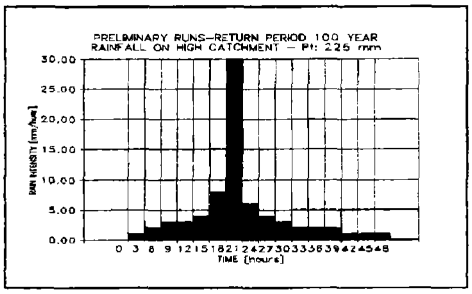

Fig. 10

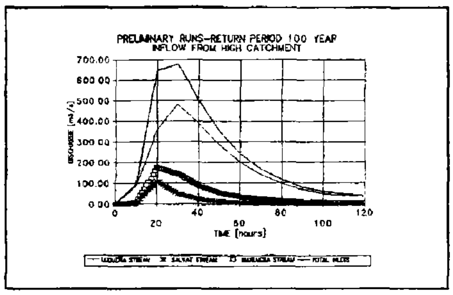

Fig. 11

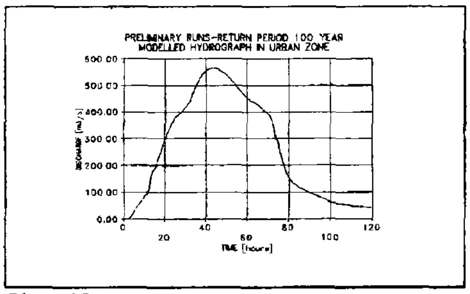

Fig. 12

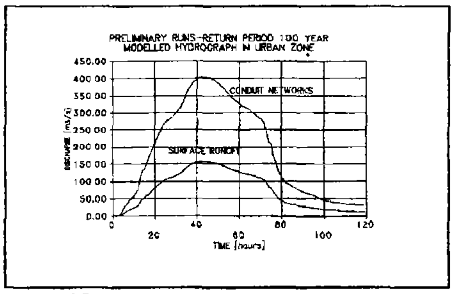

Fig. 13

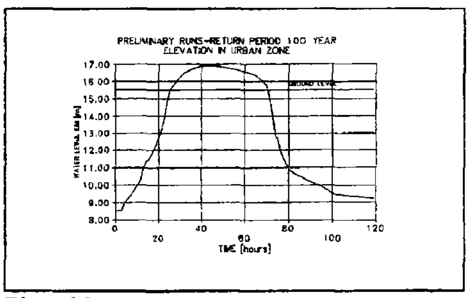

Fig. 14

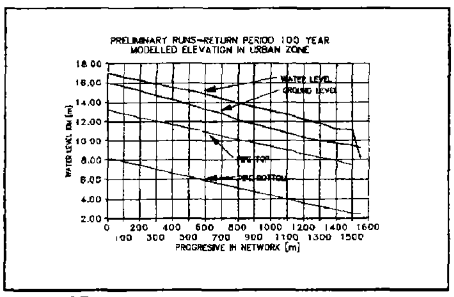

Fig. 15

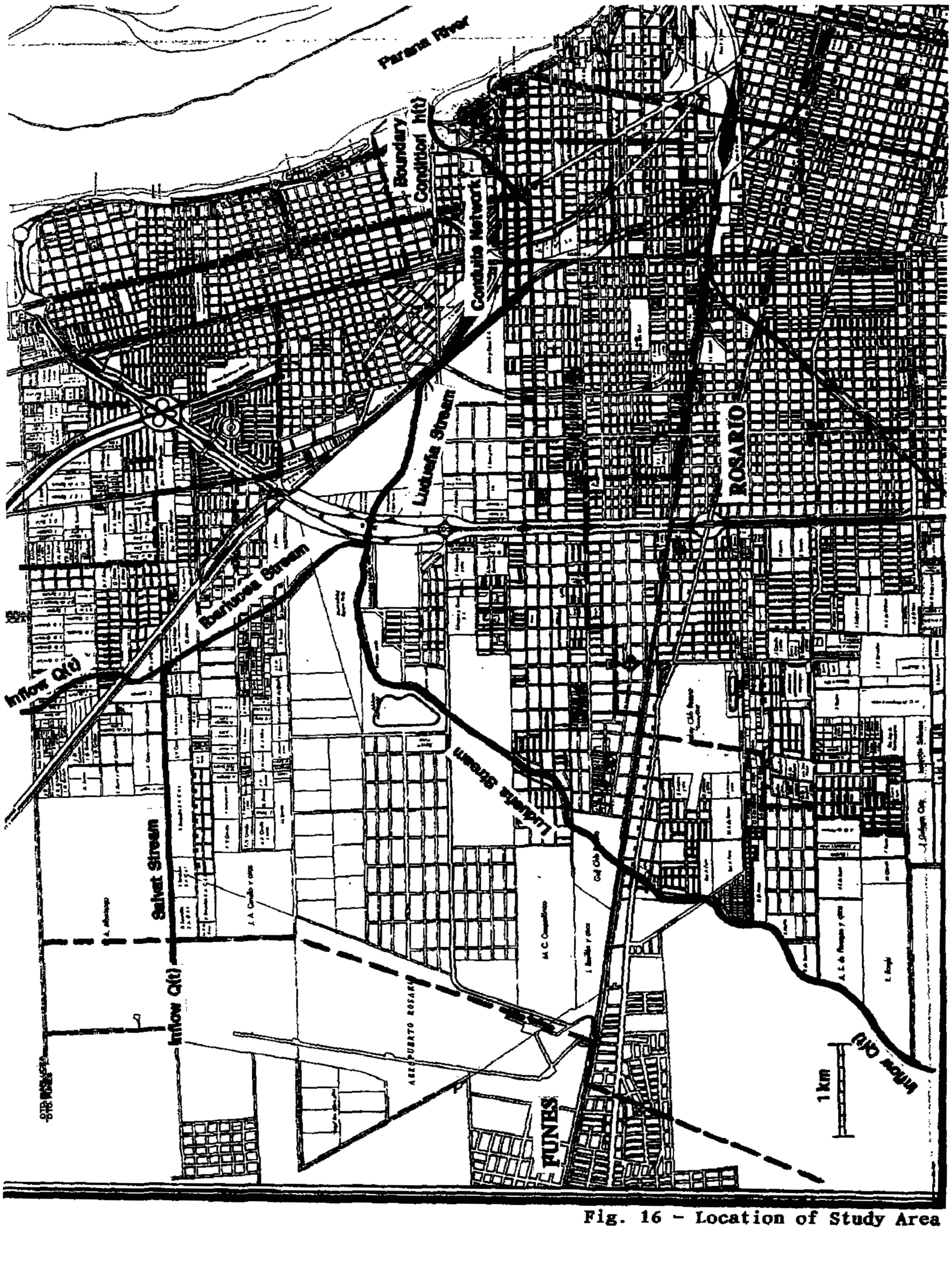

Fig. 16 - Location of Study Area

Fig.16. Location of Study Area

REFERENCES

Chow, VT (1959) Hydraulics of the Open Channels. Edit. Carton, Mexico

Cunge, J.A., Holly, F.M., Verwey, T.O. (1980) Practical Aspects of Computational Rivers Hydraulics, Pitman, 420 p.

Cunge, J.A. (1975) Two Dimensional Modeling of Flood Plains (Chapter 17- "Unsteady flow in open channels"). Publ. by Mahmood K and Yevjevich V., Mater Resources Publications, Fort- Collins, USA

Riccardi, G. (1991/92/93/94) Complex Mathematical Modelling Onedimensional and twodimensional of Free Surface Flows. Annual Reasearch Reports. Research Council of National University of Rosario

Riccardi, G. (1992) Application of a Bidimensional Mathematical Model of Cells in a reach of the Luduena stream in the zone of the Great Rosario. XV Latin American Congress of hydraulics, IAHR, Cartagena, Colombia.

Riccardi, G, (1994) A Hydrodynamic Mathematical Model Cuasi- Bidimensional of Cells for Cuasi-Dinamicos Flow. XVI Latin American Congress of hydraulics, IAHR, Santiago, Chile.