SUMMARY REGARDING THE TRANSIENT RESPONSE OF ONE-, TWO-, AND THREE-ELEMENT COMBINATIONS

A Single Elements

**Voltage Source**

- $e(t) [\text{(A)}]$ with $R$
- $e(t) [\text{(B)}]$
- $e(t) [\text{(C)}]$

**Current Source**

- $i(t) [\text{(A)}]$ with $R$
- $i(t) [\text{(B)}]$
- $i(t) [\text{(C)}]$
C Two Elements — $R, C$

\[ i_R, i_C \text{ same as in part (A)} \]

\[ e(t) = e^{-t/(RC)} \]

\[ i(t) = -\frac{A}{RC} e^{-t/(RC)} \]

\[ e(t) = E(1 - e^{-t/(RC)}) \]

\[ i(t) = I_R(1 - e^{-t/(RC)}) \]

\[ i_C = I_C e^{-t/(RC)} \]

\[ e(t) = e^{-t/(RC)} \]

\[ i(t) = I_R e^{-t/(RC)} \]

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\[ e(t) = e^{-t/(RC)} \]

\[ i(t) = I_R e^{-t/(RC)} \]
B. Two Elements—$R$, $L$

\[ e(t) = e_L(t) + e_R(t) \]

\[ i(t) = i_L(t) + i_R(t) \]

\[ e_L = -\frac{A}{L} V_0 e^{-\frac{RL}{R} t} \]

\[ i_L = \frac{A_0 V_0}{L} e^{-\frac{RL}{R} t} \]

\[ e_R = \frac{A}{L} V_0 e^{-\frac{RL}{R} t} \]

\[ i_R = \frac{A_0 V_0}{L} e^{-\frac{RL}{R} t} \]

\[ e_L, e_R \text{ same as in part (A)} \]

\[ i_L, i_R \text{ same as in part (A)} \]
Two Elements—$L, C$, 

\[ \omega_0 = \frac{1}{\sqrt{LC}} \]

\[ e_L = L \frac{di}{dt} = A u_0(t) - A \omega_0 \sin \omega_0 t \]

\[ e_C = \frac{1}{C} \int i \, dt = A \omega_0 \sin \omega_0 t \]

\[ e_C = E(1 - \cos \omega_0 t) \]

\[ e_L = E \cos \omega_0 t \]

\[ e_C, e_L \text{ same as in part (A)} \]

\[ i_C, i_L \text{ same as in part (A)} \]

\[ i_C = C \frac{de}{dt} = A u_0(t) - A \omega_0 \sin \omega_0 t \]

\[ i_L = \frac{1}{L} \int e \, dt = A \omega_0 \sin \omega_0 t \]

\[ i_C, i_L \text{ same as in part (A)} \]

\[ i_L = I(1 - \cos \omega_0 t) \]

\[ i_C \]

\[ i_L \]
SUMMARY REGARDING THE TRANSIENT RESPONSE

Three Elements—R, L, C

\( \omega_0 > \alpha \)

\[ \omega_0 = \frac{1}{\sqrt{LC}} \quad \alpha = \frac{R}{2L} \quad \omega_d = \sqrt{\omega_0^2 - \alpha^2} \]

\[ i(t) = E \frac{C \omega_0}{L \omega_d} e^{-\alpha t} \sin \omega_d t \]

when \( \alpha \ll \omega_0 \)

\[ i(t) = E \frac{C}{L} e^{-\alpha t} \sin \omega_d t \]

\( \epsilon_L, \epsilon_R, \epsilon_C \) as shown in part (A)

\[ i(t) = \frac{A}{L} \omega_0 e^{-\alpha t} \cos \left( \omega_d t + \cos^{-1} \frac{\omega_d}{\omega_0} \right) \]

when \( \alpha \ll \omega_0; \; i(t) \approx \frac{A}{L} e^{-\alpha t} \cos \omega_d t \)

\[ i(t) = I \frac{L \omega_0}{C \omega_d} e^{-\alpha t} \sin \omega_d t \]

when \( \alpha \ll \omega_0 \)

\[ e(t) = I \frac{L}{C} e^{-\alpha t} \sin \omega_d t \]

\( \eta_0, \eta_L, \eta_C \) as shown in part (A)
**F Initial Conditions**

1. **Initial voltages on condensers**

   Charged condenser $v_A(t) = V_0 u_{-1}(t)$ in series $i_A(t) = CV_0 u_0(t)$ in parallel

2. **Initial currents in coils**

   Initial current $i_A(t) = I_0 u_{-1}(t)$ in parallel $v_A(t) = LI_0 u_0(t)$ in series

If other sources are present in the circuit solve by superposition.

**PROBLEMS**

1. For the circuit shown below, determine $v(t)$ and $i(t)$ (a) for $i_A(t) = u_0(t)$, (b) for $i_A(t) = u_{-1}(t)$.

   ![Circuit Diagram 1](image1)

   ![Circuit Diagram 2](image2)

   **Prob. 1.**

   **Prob. 2.**
2. For the circuit shown above, determine \( v(t) \) and \( i(t) \) (a) for \( v_s(t) = u_s(t) \), (b) for \( v_s(t) = u_{-s}(t) \).

3. Given \( v_s(t) = u_{-s}(t) \). Determine \( v(t) \) and \( i(t) \).

4. Given \( i_s(t) = u_{-s}(t) \). Determine \( v(t) \) and \( i(t) \).

5. Given \( v_s(t) = u_{-s}(t) \). Determine \( v(t) \) and \( i(t) \).

6. Given \( v_s(t) = u_{-s}(t) \). Determine \( v(t) \) and \( i(t) \).

7. For the circuit shown below, \( R_1 \) represents the leakage resistance of the condenser of capacitance \( C \). What is the equation for the change on the condenser as a function of time after the switch \( K \) is closed?

8. Which pairs of the circuits in Probs. 1 to 6 inclusive are potential duals? Using this information, check your answers to these problems.

9. Switch \( S \) is closed for \( 0 < t < L/R \) and is open during the interval \( L/R < t < \infty \). Find analytic expressions for the voltage \( v(t) \) valid for \( 0 < t < L/R \) and again for \( L/R < t < \infty \).