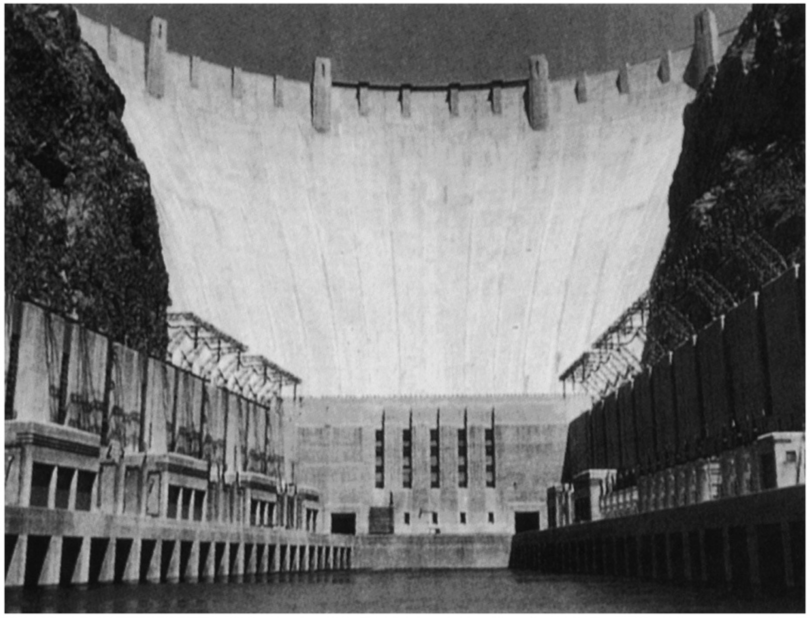
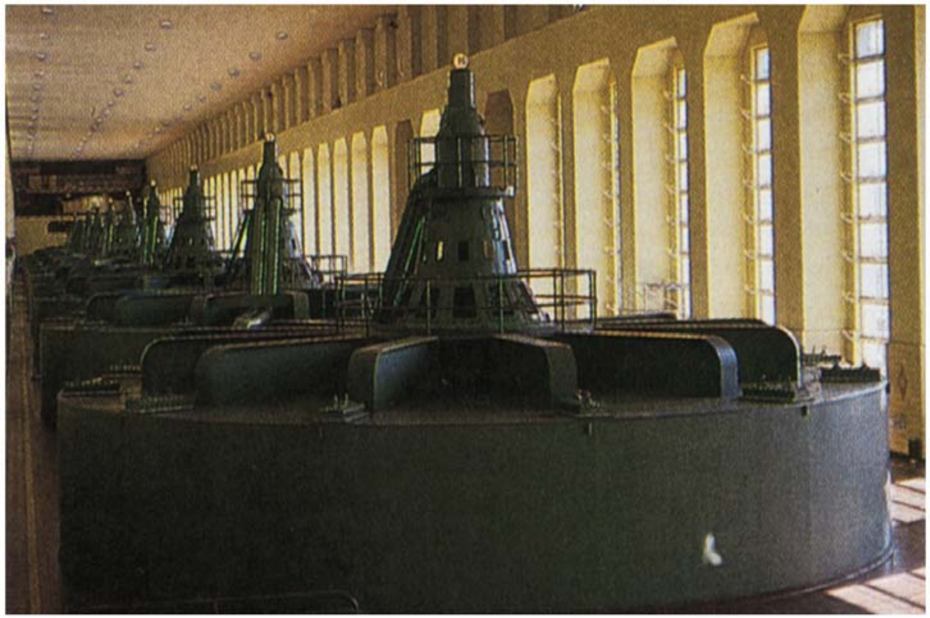
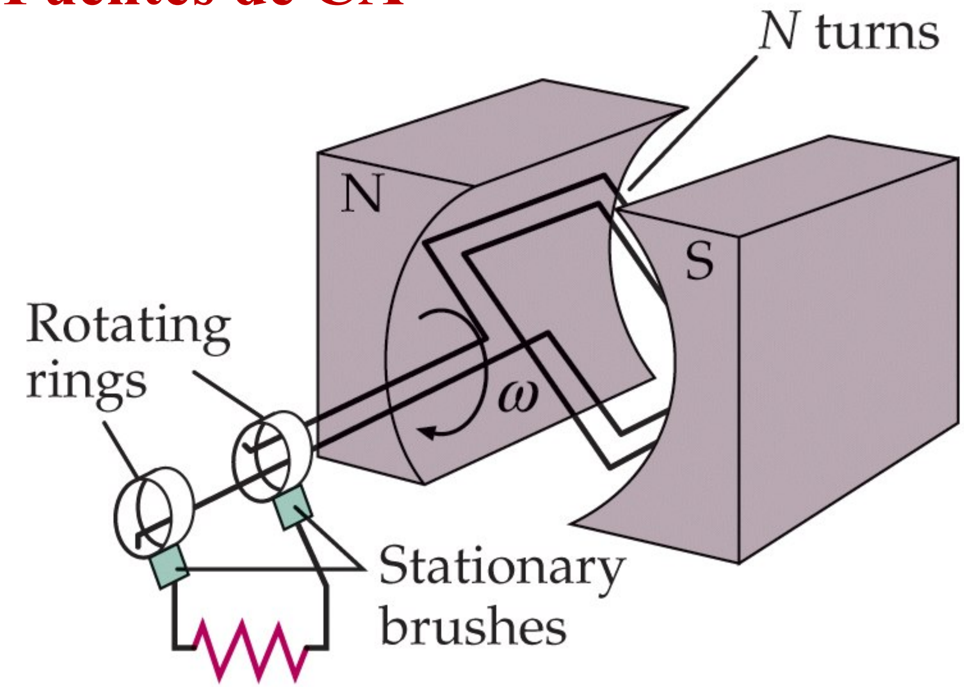
Capítulo 6

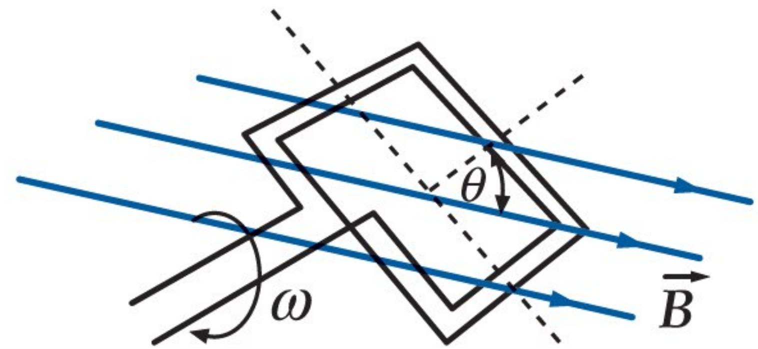
Circuitos de Corriente Alterna



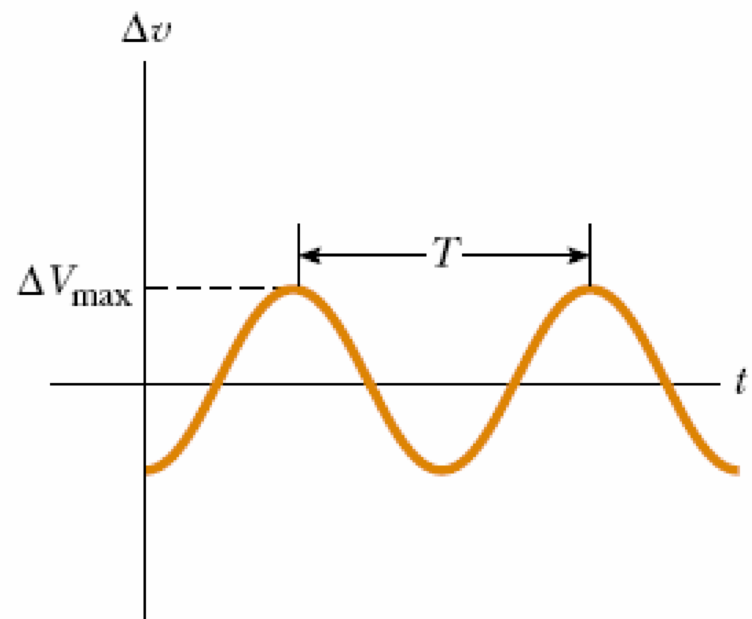


Fuentes de CA



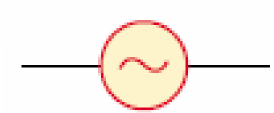


$$\Delta v = \Delta V_{\max} \sin \omega t$$



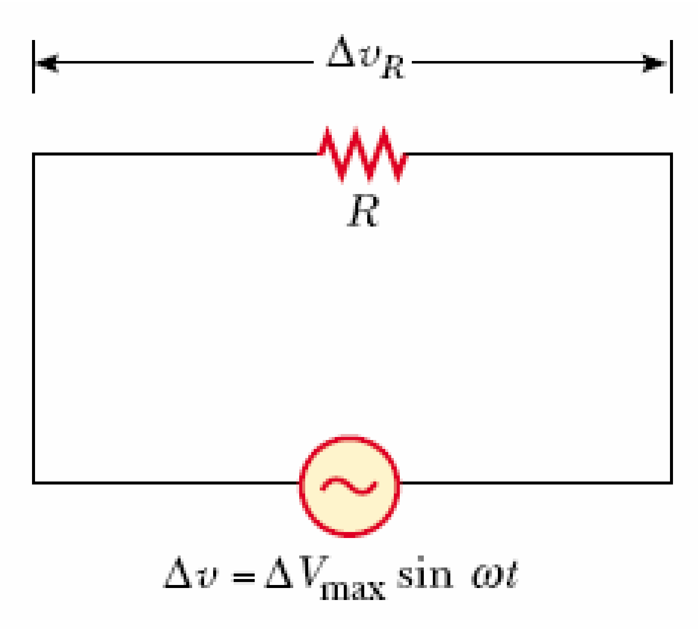
$\Delta V_{\max}$ **Voltaje máximo o amplitud**

$\omega = 2\pi f = \dfrac{2\pi}{T}$ **frecuencia angular**



Símbolo

Resistores en un circuito de CA

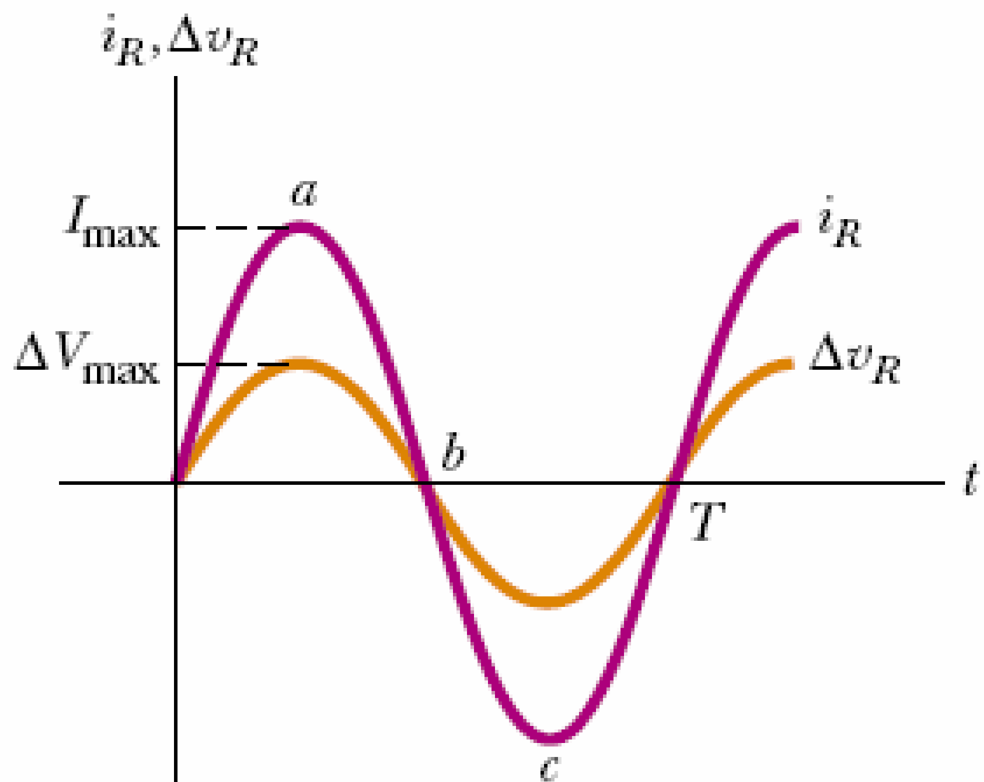


$$\Delta v = \Delta v_R = \Delta V_{max} \sin \omega t$$

$$i_R = \frac{\Delta v_R}{R} = \frac{\Delta V_{max}}{R} \sin \omega t = I_{max} \sin \omega t$$

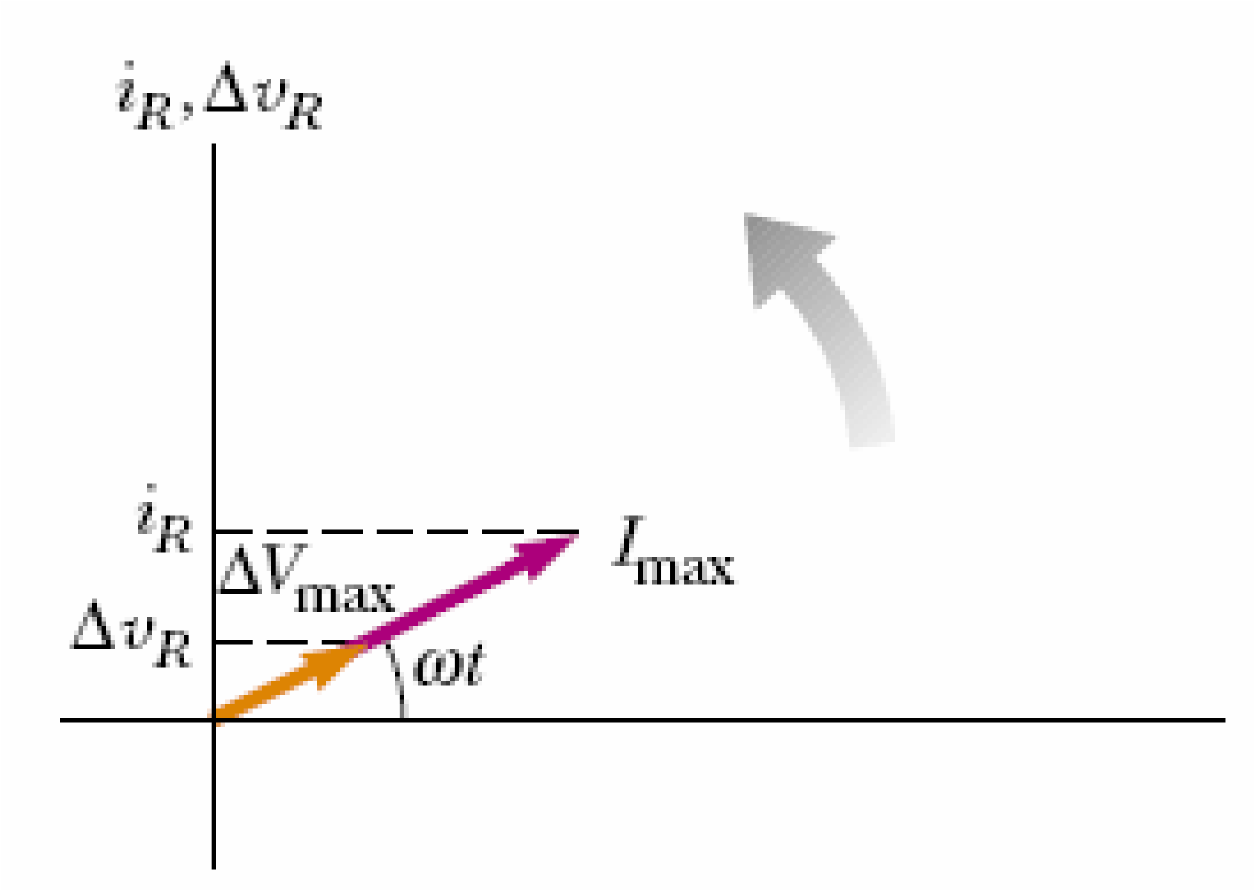
$$I_{max} = \frac{\Delta V_{max}}{R}$$

$$\Delta v_R = I_{max} R \sin \omega t$$



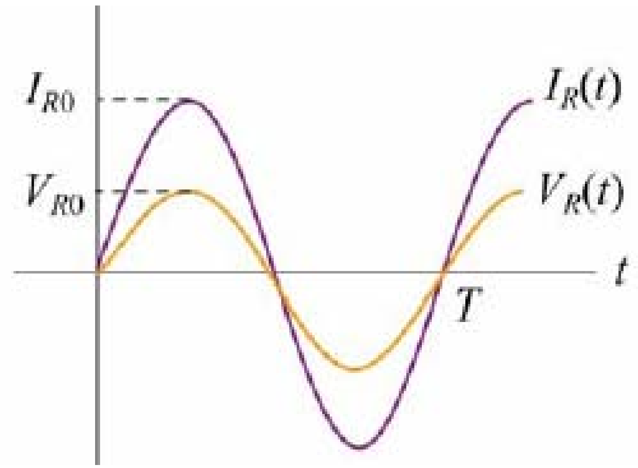
Corriente y voltaje alcanzan valores máximos en el mismo instante de tiempo: se dice que están en fase

Se representan con vectores rotatorios ⇨ **Fasores**



La potencia disipada en el resistor (calor Joule)

$$I_R(t) = \frac{V_R(t)}{R} = \frac{V_{R0}\sin\omega t}{R} = I_{R0}\sin\omega t$$



$$P = I^2_{R0}\ R \quad ?$$

en CA $\longrightarrow$ P_{media}

$$\langle I_R(t)\rangle = \frac{1}{T}\int_0^T I_R(t)dt = \frac{1}{T}\int_0^T I_{R0}\sin\omega t\ dt = \frac{I_{R0}}{T}\int_0^T \sin\frac{2\pi t}{T}\ dt = 0$$

porque $\langle\sin\omega t\rangle = \frac{1}{T}\int_0^T \sin\omega t\ dt = 0$

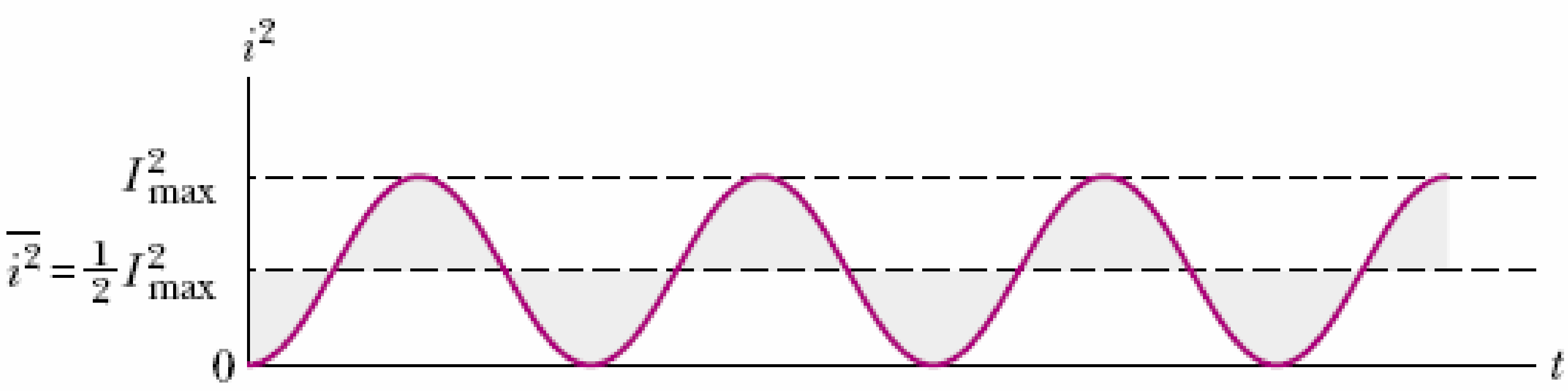
$$\langle \cos \omega t \rangle = \frac{1}{T}\int_0^T \cos \omega t \; dt = 0$$

$$\langle \sin \omega t \cos \omega t \rangle = \frac{1}{T}\int_0^T \sin \omega t \cos \omega t \; dt = 0$$

$$\langle \sin^2 \omega t \rangle = \frac{1}{T}\int_0^T \sin^2 \omega t \; dt = \frac{1}{T}\int_0^T \sin^2\left(\frac{2\pi t}{T}\right) dt = \frac{1}{2}$$

$$\langle \cos^2 \omega t \rangle = \frac{1}{T}\int_0^T \cos^2 \omega t \; dt = \frac{1}{T}\int_0^T \cos^2\left(\frac{2\pi t}{T}\right) dt = \frac{1}{2}$$

$$\langle I_R^2(t) \rangle = \frac{1}{T}\int_0^T I_R^2(t) dt = \frac{1}{T}\int_0^T I_{R0}^2 \sin^2 \omega t \; dt = I_{R0}^2 \frac{1}{T}\int_0^T \sin^2\left(\frac{2\pi t}{T}\right) dt = \frac{1}{2} I_{R0}^2$$



Es conveniente definir la corriente cuadrática media I_{rms} (rms: root-mean-square) también denominada corriente eficaz

$$I_{\mathrm{rms}} = \sqrt{\left\langle I_R^2(t) \right\rangle} = \frac{I_{R0}}{\sqrt{2}}$$

En forma similar para el voltaje:

$$V_{\mathrm{rms}} = \sqrt{\left\langle V_R^2(t) \right\rangle} = \frac{V_{R0}}{\sqrt{2}}$$

La potencia instantánea disipada en el resistor es:

$$P_R(t) = I_R(t) V_R(t) = I_R^2(t) R$$

Con lo cual, la potencia media sobre un periodo es:

$$\left\langle P_R(t) \right\rangle = \left\langle I_R^2(t) R \right\rangle = \frac{1}{2} I_{R0}^2 R = I_{\mathrm{rms}}^2 R = I_{\mathrm{rms}} V_{\mathrm{rms}} = \frac{V_{\mathrm{rms}}^2}{R}$$

$$I_{rms} = \frac{I_{max}}{\sqrt{2}} = 0.707 I_{max}$$

$$\Delta V_{rms} = \frac{\Delta V_{max}}{\sqrt{2}} = 0.707\, \Delta V_{max}$$

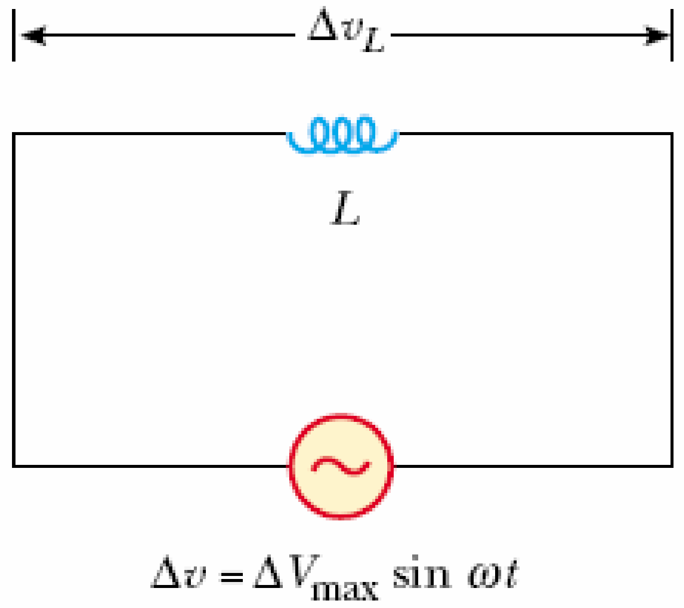
Son valores ficticios en CA que producen la misma potencia que en un circuito de CC

En la línea domiciliaria: $\mathbf{V_{rms} = 220\ V}$

$\mathbf{V_{max} = 312\ V}$

Amperímetros y voltímetros miden valores eficaces.

Inductores en un circuito de CA



$$\Delta v + \Delta v_L = 0$$

$$\Delta v - L\frac{di}{dt} = 0$$

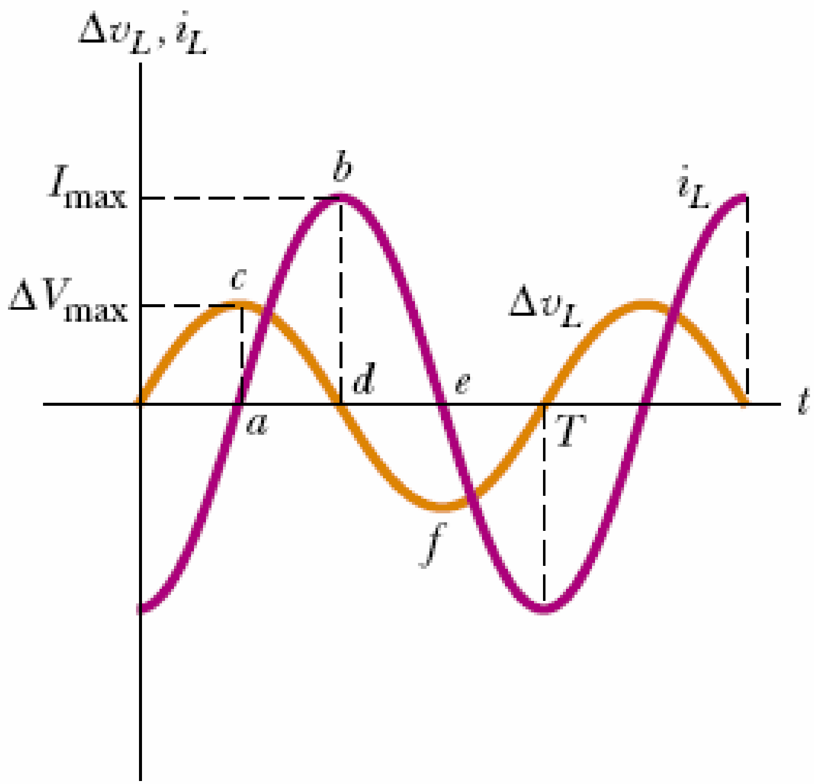
$$\Delta v = L\frac{di}{dt} = \Delta V_{max} \sin \omega t$$

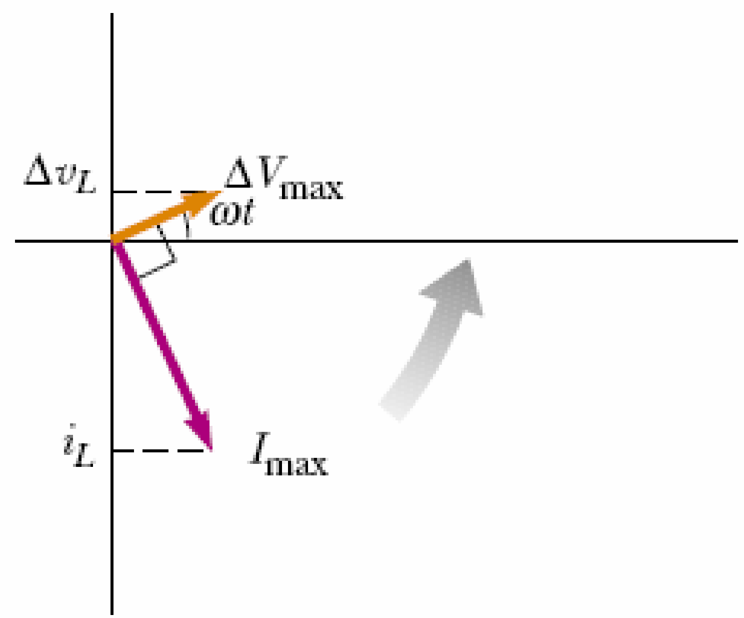
$$di = \frac{\Delta V_{max}}{L} \sin \omega t \, dt$$

$$i_L = \frac{\Delta V_{max}}{L}\int \sin \omega t \, dt = -\frac{\Delta V_{max}}{\omega L} \cos \omega t$$

Usando: $\cos \omega t = -\sin(\omega t - \pi/2)$

$$\boxed{i_L = \frac{\Delta V_{max}}{\omega L} \sin\left(\omega t - \frac{\pi}{2}\right)}$$





La corriente está retrasada respecto al voltaje en π/2

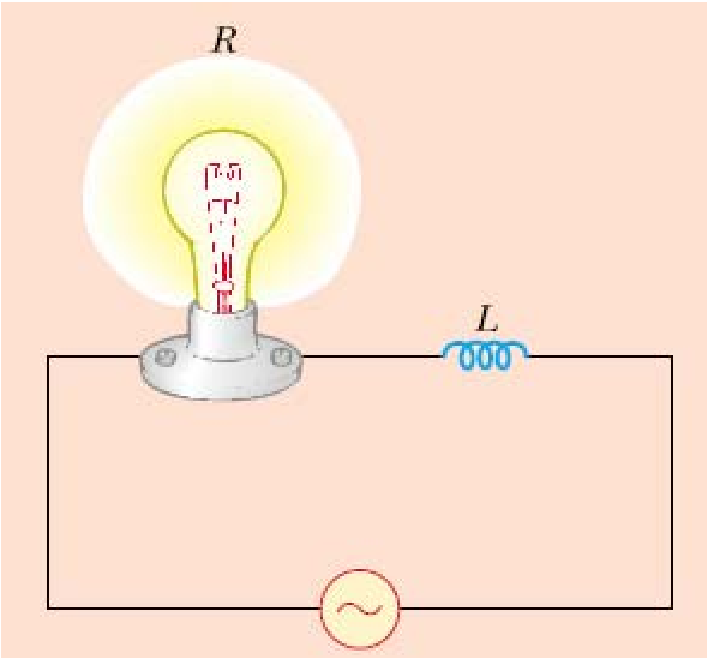
$$I_{max} = \frac{\Delta V_{max}}{\omega L}$$

$$\boxed{X_L \equiv \omega L}$$

reactancia inductiva

$$\boxed{I_{max} = \frac{\Delta V_{max}}{X_L}}$$

$$[\,X_L\,] = \Omega$$



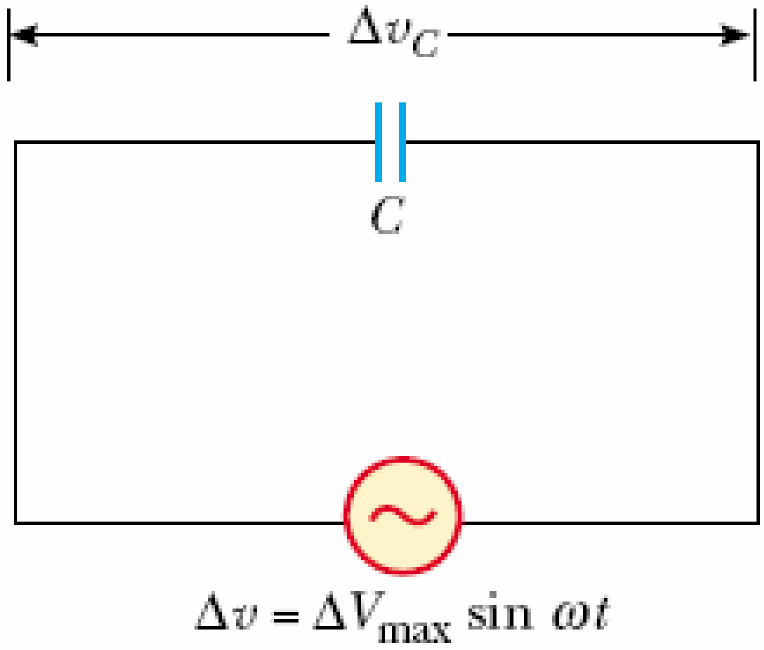
Si la frecuencia de la fuente es variable y la amplitud de V constante. La lámpara brilla más intensamente a:

i) altas frecuencias

ii) bajas frecuencias

iii) igual para todas

Capacitores en un circuito CA



$$\Delta v + \Delta v_C = 0$$

$$\Delta v = \Delta v_C = \Delta V_{\max} \sin \omega t$$

$$C = q/\Delta v_C$$

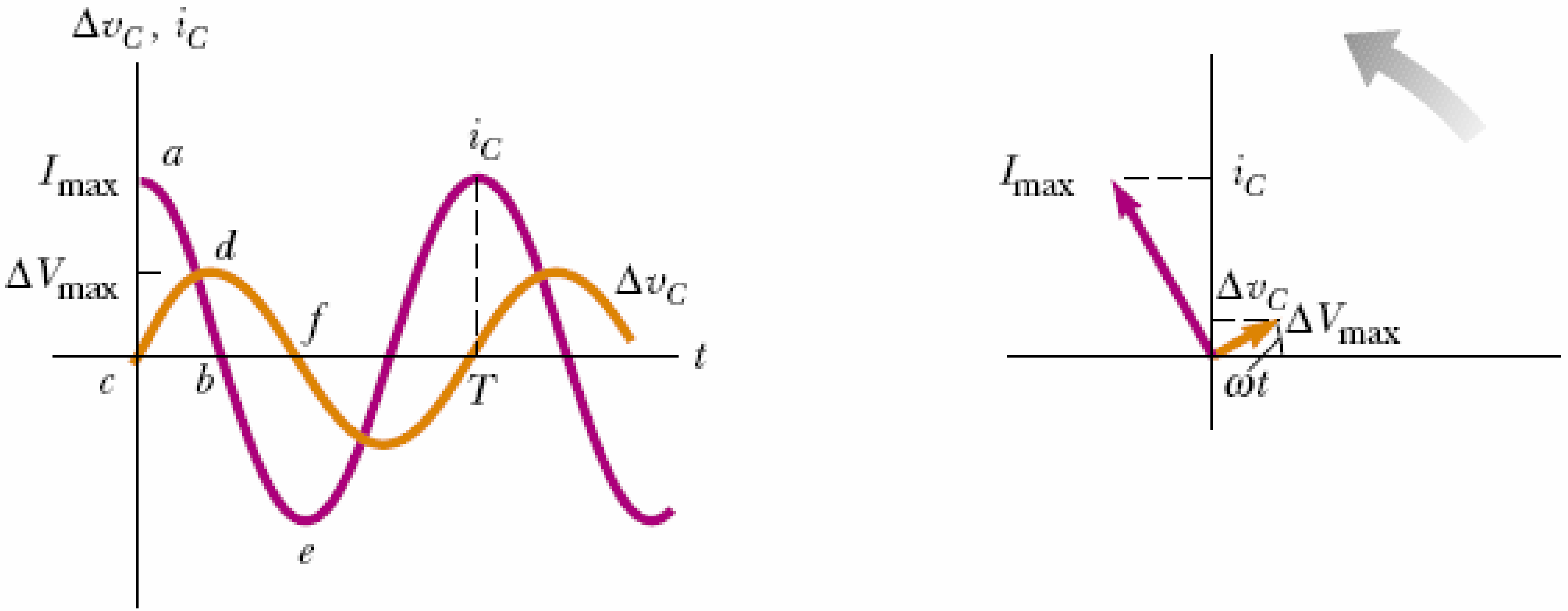
$$q = C\,\Delta V_{\max} \sin \omega t$$

$$i_C = \frac{dq}{dt} = \omega C\,\Delta V_{\max} \cos \omega t$$

Usando:

$$\cos \omega t = \sin\left(\omega t + \frac{\pi}{2}\right)$$

$$\boxed{i_C = \omega C \,\Delta V_{\max} \sin\left(\omega t + \frac{\pi}{2}\right)}$$



La corriente está adelantada respecto al voltaje en π/2

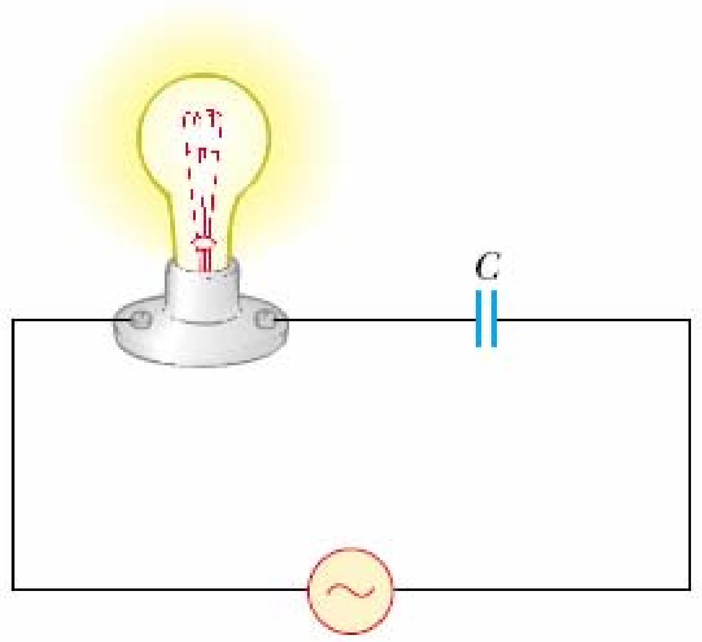
$$I_{max} = \omega C \Delta V_{max} = \frac{\Delta V_{max}}{(1/\omega C)}$$

$$X_C \equiv \frac{1}{\omega C}$$

reactancia capacitiva

$$I_{max} = \frac{\Delta V_{max}}{X_C}$$

$$[X_L] = \Omega$$

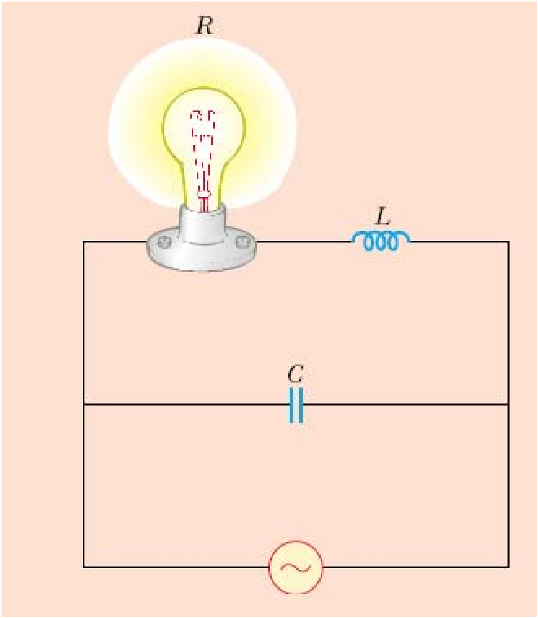


Si la frecuencia de la fuente es variable y la amplitud de V constante. La lámpara brilla más intensamente a:

i) altas frecuencias

ii) bajas frecuencias

iii) igual para todas



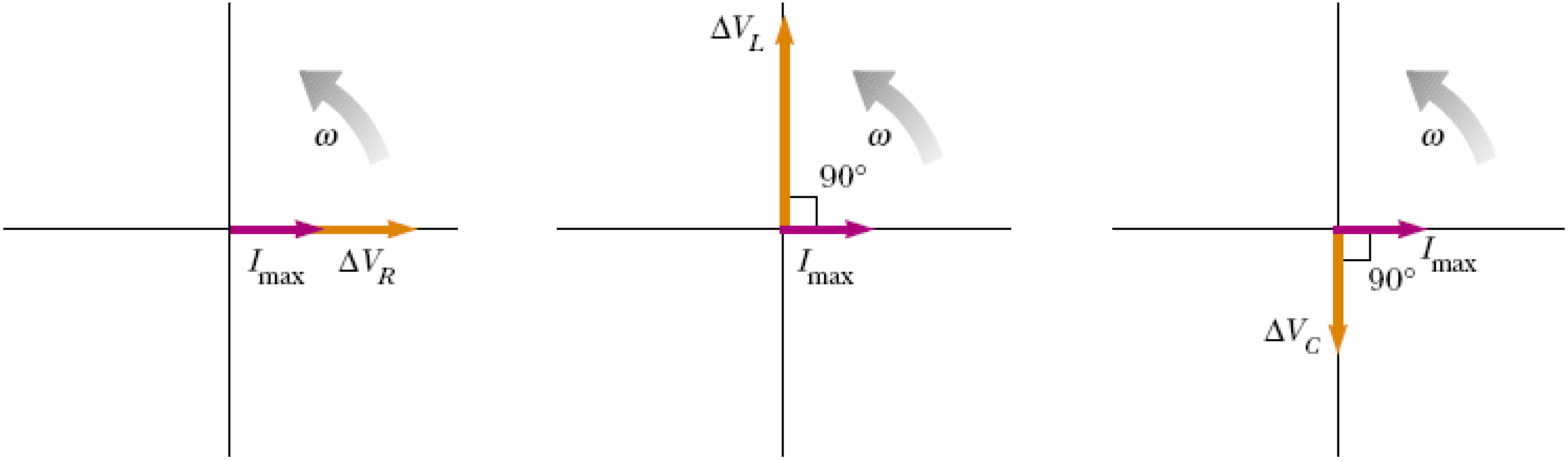
La lámpara brilla más intensamente a:

i) altas frecuencias

ii) bajas frecuencias

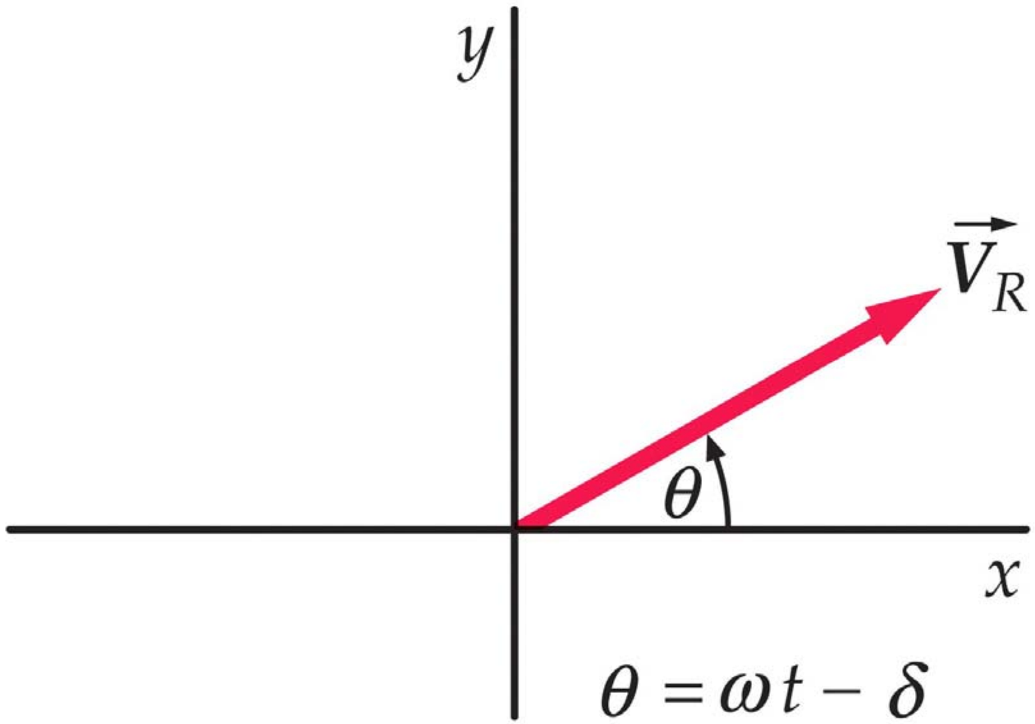
iii) igual para todas

Resumiendo:



(a) Resistor (b) Inductor (c) Capacitor

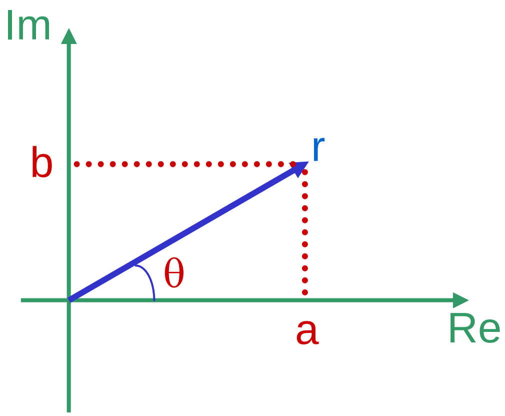
Representación compleja



La representación fasorial, la podemos llevar a cabo en el plano complejo:

$$y \longrightarrow \mathrm{Im}$$

$$x \longrightarrow \mathrm{Re}$$



Coordenadas cartesianas $z = a + jb$

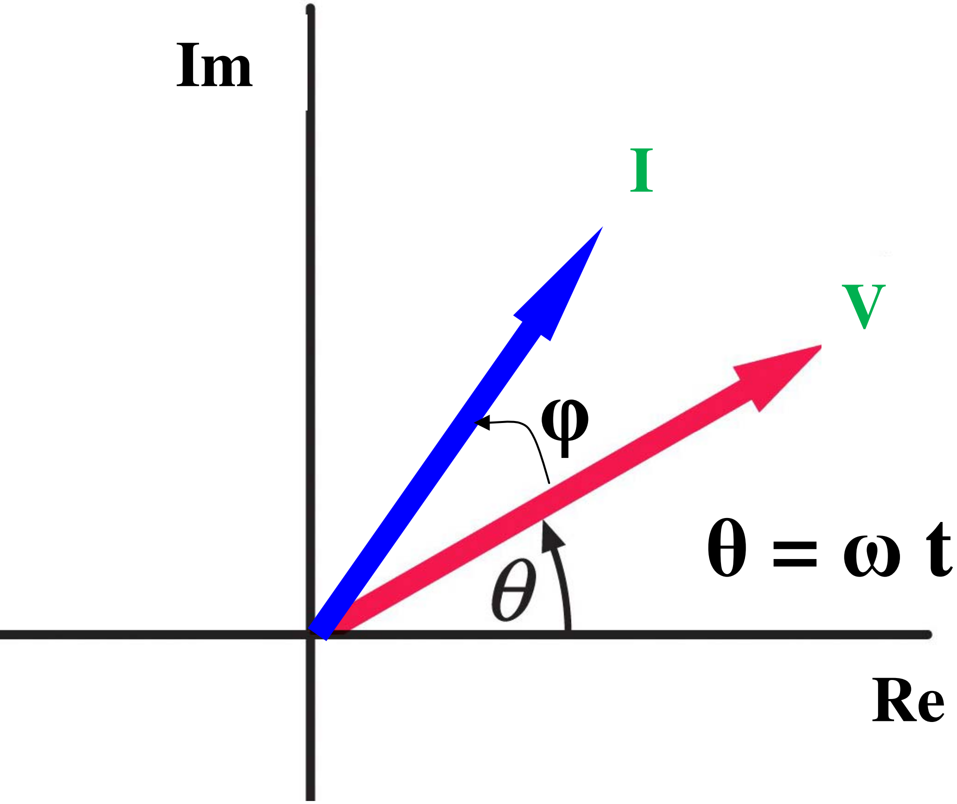
Coordenadas polares $z = r_{\underline{|\theta}}$

Cambio de coordenadas

- Cartesianas a polares
$$\begin{cases} r = \sqrt{a^2 + b^2} \\ \theta = \operatorname{arc\,tg} \dfrac{b}{a} \end{cases}$$

- Polares a cartesianas
$$\begin{cases} a = r\cos\theta \\ b = r\,\mathrm{sen}\theta \end{cases}$$

Fórmula de Euler $\Longrightarrow$ $re^{\pm j\theta} = r(\cos\theta \pm j\,sen\,\theta)$



$$\mathbf{V} = V_0 \, e^{j\omega t}$$

$$\mathbf{I} = I_0 \, e^{j\omega t+\varphi}$$

Sentido físico:

$V(t) = \text{Im}\ \mathbf{V} = V_0\ \text{sen}(\omega t)$

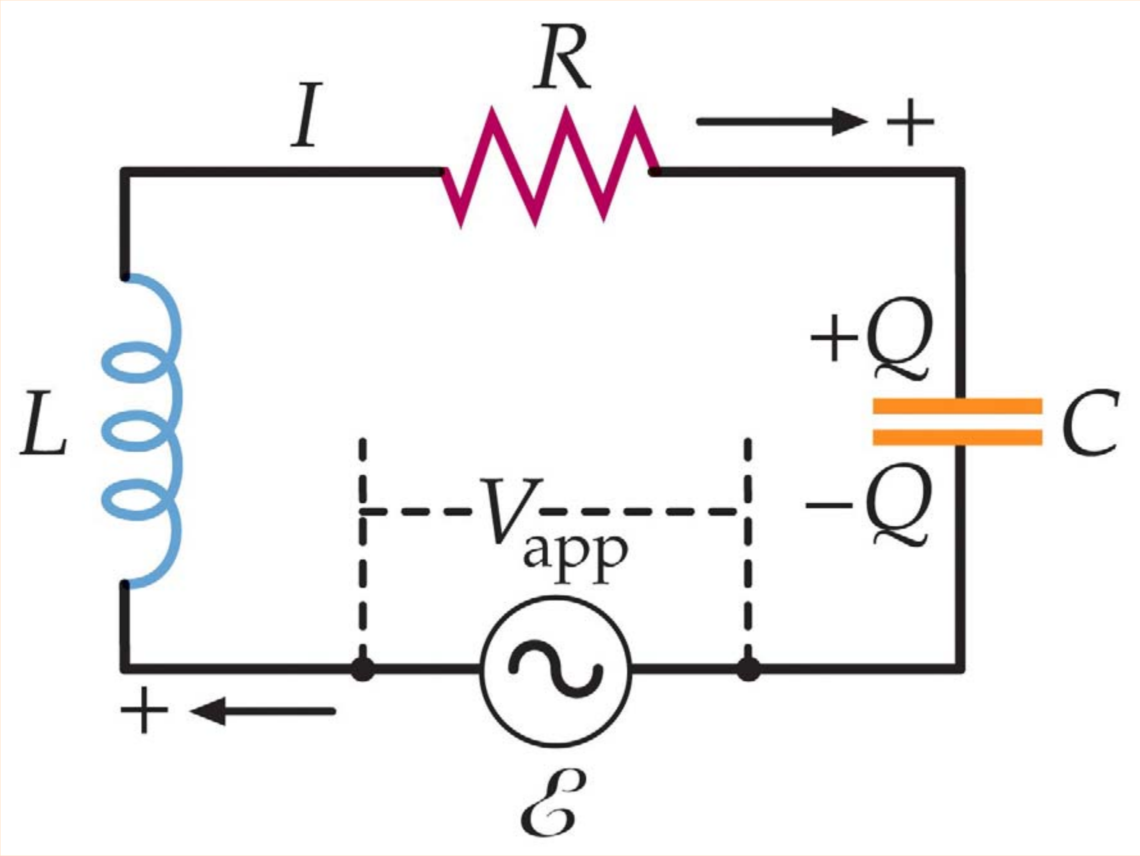
$I(t) = \text{Im}\ \mathbf{I} = I_0\ \text{sen}(\omega t+\varphi)$

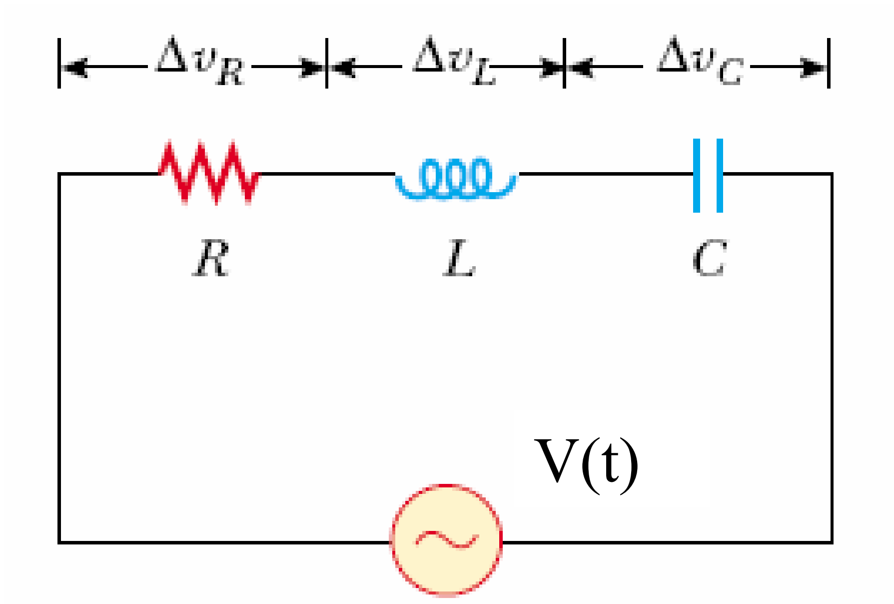
Se opera con números complejos $\longrightarrow$ **la parte Im**

$$d\mathbf{I}/dt = d/dt\,(I_0\, e^{j\omega t+\varphi}) = j\omega\, \mathbf{I}$$

$$\int \mathbf{I}\, dt = \int I_0\, e^{j\omega t+\varphi}\, dt = \mathbf{I} / j\omega$$

Circuito en serie RLC





$$V = \Delta v_R + \Delta v_L + \Delta v_C$$

$$V = I\,R + L\,dI/dt + q/C$$

$$V = I\,R + L\,dI/dt + 1/C \int I\,dt$$

Pasando a complejos $\mathbf{V} = \mathbf{I}\,R + L\,d\mathbf{I}/dt + 1/C \int \mathbf{I}\,dt$

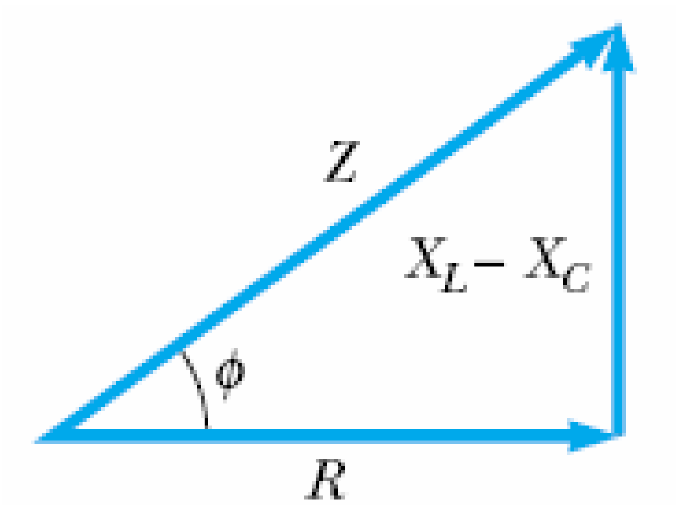
$$\mathbf{V} = \mathbf{I}\,R + j\omega L\,\mathbf{I} + (1/j\omega C)\,\mathbf{I}$$

$$\mathbf{V} = [\,R + j\,(\omega L - 1/\omega C)\,]\,\mathbf{I}$$

$$\mathbf{V} = [\,R + j\,(\omega L - 1/\omega C)\,]\,\mathbf{I}$$

$$\mathbf{V} = [\,R + j\,(XL - XC)\,]\,\mathbf{I}$$

$\boxed{\mathbf{V} = \mathbf{Z}\,\mathbf{I}}$ donde $\mathbf{Z} = \mathbf{R + j\,(XL\text{-}XC)}$ **Impedancia**



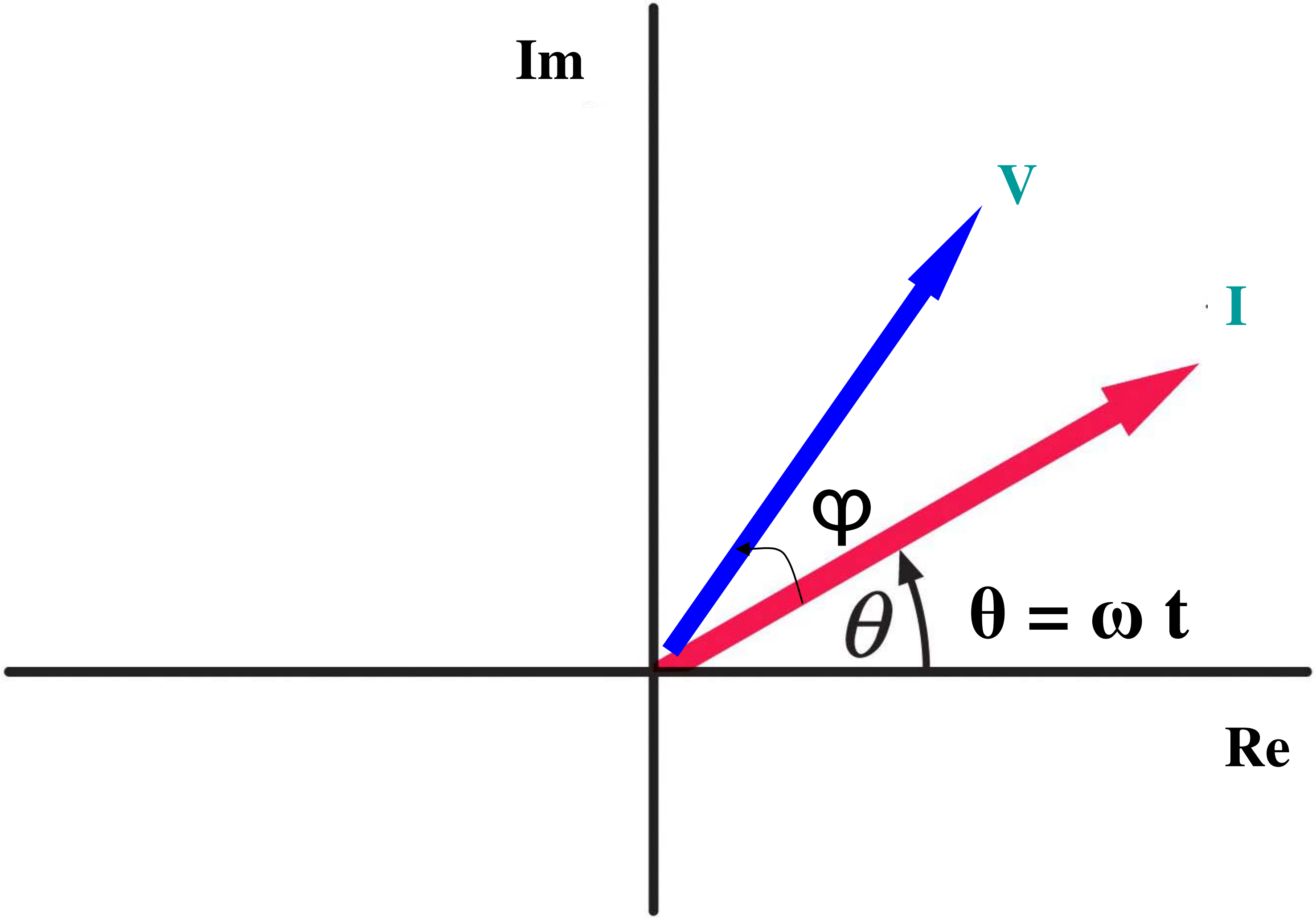
$$Z \equiv \sqrt{R^2 + (X_L - X_C)^2}$$

$$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$$

$$\mathbf{V} = \mathbf{I}\ \mathbf{Z} = (I_0\ e^{j\omega t})\ (Z\ e^{j\varphi}) = I_0\ Z\ e^{j(\omega t + \varphi)}$$

$$V = V_0 \operatorname{sen}(\omega t + \varphi) \qquad \text{donde } V_0 = I_0\ Z$$

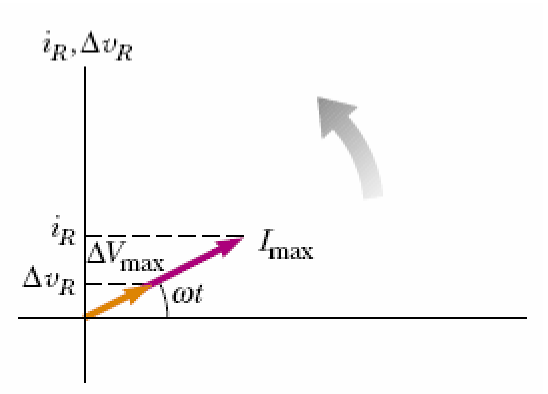
φ : Desfasaje entre V e I



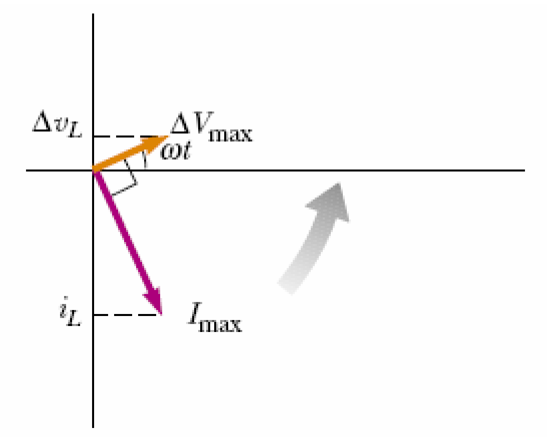
$$I_{\max} = \frac{\Delta V_{\max}}{\sqrt{R^2 + (X_L - X_C)^2}}$$

Casos anteriores:

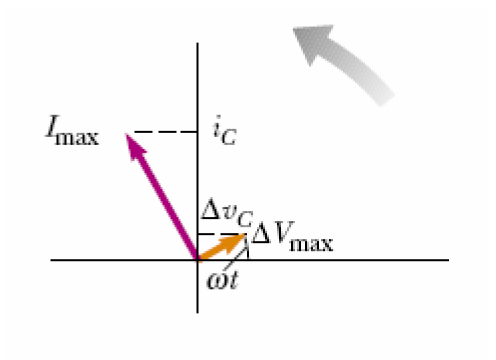
a) R puro $\longrightarrow$ $Z = R$ $\varphi = 0$



b) L puro $\longrightarrow$ $Z = \omega L$ $\varphi = \pi/2$



c) C puro $\longrightarrow$ $Z = 1/\omega C$ $\varphi = -\pi/2$



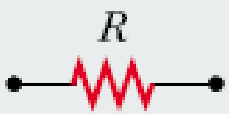
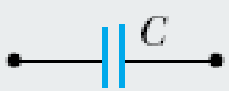
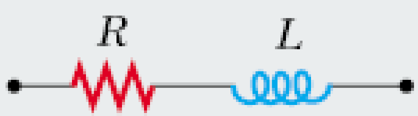
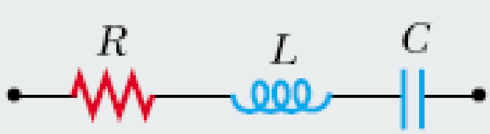
$$Z \equiv \sqrt{R^2 + (X_L - X_C)^2}$$

$$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$$

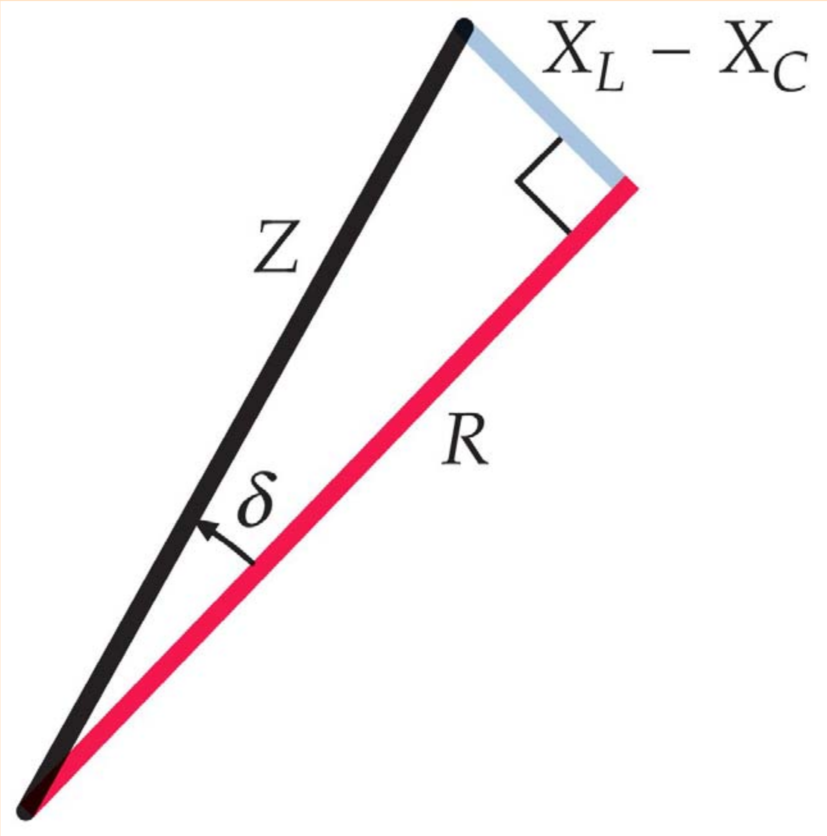
Notemos que Z(ω) y φ(ω)

Alta ω	$X_L > X_C$	$\varphi > 0$	**I**	**retrasada**
Baja ω	$X_L < X_C$	$\varphi < 0$	**I**	**adelantada**

Impedance Values and Phase Angles for Various Circuit-Element Combinations[a]

Circuit Elements	Impedance Z	Phase Angle ϕ
	R	$0°$
	X_C	$-90°$
	X_L	$+90°$
	$\sqrt{R^2 + X_C{}^2}$	Negative, between $-90°$ and $0°$
	$\sqrt{R^2 + X_L{}^2}$	Positive, between $0°$ and $90°$
	$\sqrt{R^2 + (X_L - X_C)^2}$	Negative if $X_C > X_L$ Positive if $X_C < X_L$

Potencia en un circuito CA



La potencia instantánea entregada por el generador es:

$$\begin{aligned}\mathcal{P} &= i\,\Delta v = I_{\max}\sin(\omega t-\phi)\,\Delta V_{\max}\sin\omega t\\ &= I_{\max}\,\Delta V_{\max}\sin\omega t\sin(\omega t-\phi)\end{aligned}$$

usando: $\sin(\omega t-\phi) = \sin\omega t\cos\phi - \cos\omega t\sin\phi.$

$$\mathcal{P} = I_{\max}\,\Delta V_{\max}\sin^2\omega t\cos\phi - I_{\max}\,\Delta V_{\max}\sin\omega t\cos\omega t\sin\phi$$

La potencia media entregada por el generador es:

$$\mathcal{P}_{\text{av}} = \frac{1}{2}I_{\max}\,\Delta V_{\max}\cos\phi \longrightarrow \boxed{\mathcal{P}_{\text{av}} = I_{\text{rms}}\,\Delta V_{\text{rms}}\cos\phi}$$

$$\cos\phi = \frac{R}{Z}$$ **factor de potencia**

R puro $\longrightarrow$ $\cos\phi = 1$ $\mathcal{P}_{av} = I_{rms}\,\Delta V_{rms}$

L puro

$\longrightarrow$ $\cos\phi = 0$ $\mathcal{P}_{av} = 0$

C puro

$$\cos\phi = \frac{R}{Z}$$

$$I_{max} = \frac{\Delta V_{max}}{\sqrt{R^2 + (X_L - X_C)^2}}$$

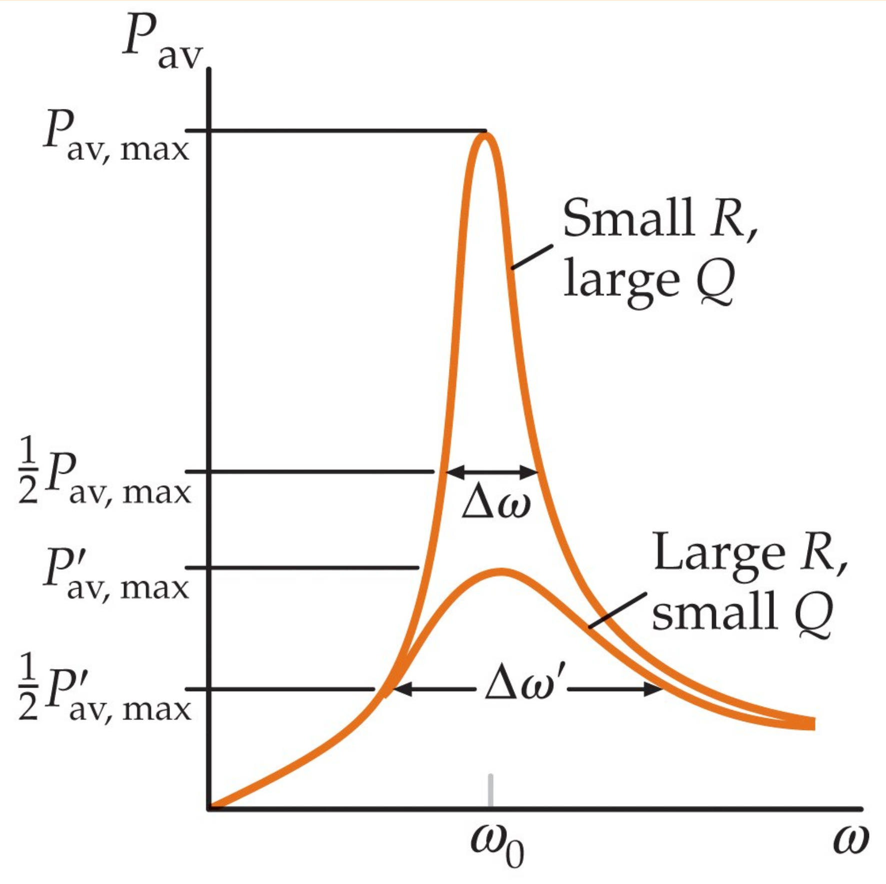
$\longrightarrow$ $\cos\phi = I_{max}\,R/\Delta V_{max}$

$$\mathcal{P}_{av} = I_{rms}\,\Delta V_{rms}\cos\phi = I_{rms}\left(\frac{\Delta V_{max}}{\sqrt{2}}\right)\frac{I_{max}R}{\Delta V_{max}} = I_{rms}\frac{I_{max}R}{\sqrt{2}}$$

$$\boxed{\mathcal{P}_{av} = I_{rms}^2\,R}$$

La potencia media entregada por el generador se disipa como calor en el resistor

Resonancia en un circuito RLC



Un circuito RLC se dice que esta en resonancia cuando la corriente es máxima.

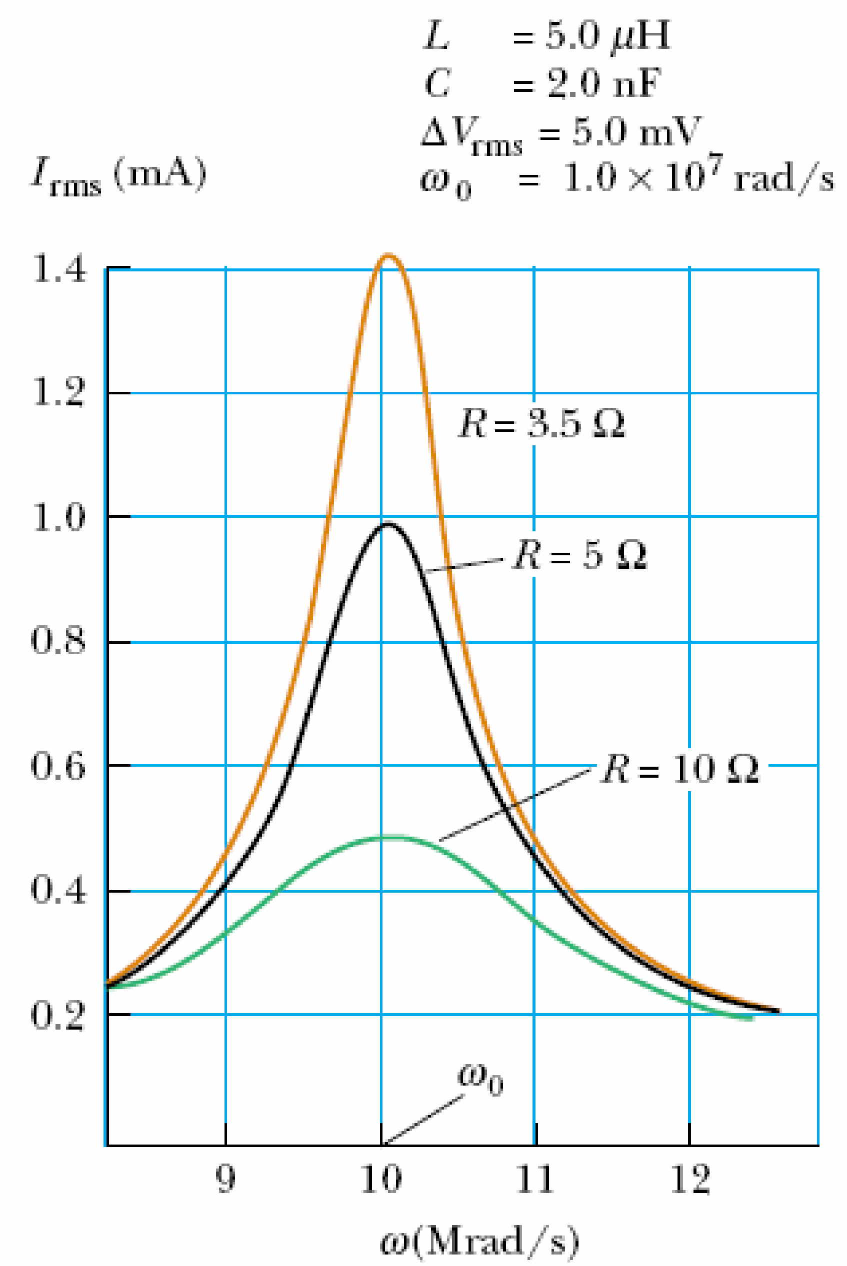
$$I_{\text{rms}} = \frac{\Delta V_{\text{rms}}}{Z}$$

$$I_{\text{rms}} = \frac{\Delta V_{\text{rms}}}{\sqrt{R^2 + (X_L - X_C)^2}}$$

$$X_L = \omega L, \; X_C = 1/\omega C,$$

$$(X_L - X_C)^2 = \left(\omega L - \frac{1}{\omega C}\right)^2 = \frac{L^2}{\omega^2}(\omega^2 - \omega_0^2)^2$$

donde $$\omega_0 = \frac{1}{\sqrt{LC}}$$ **frecuencia de resonancia del circuito**



L = 5.0 μH
C = 2.0 nF
ΔV_{rms} = 5.0 mV
ω_0 = 1.0×10^7 rad/s
I_{rms} (mA)
1.4
1.2
1.0
0.8
0.6
0.4
0.2
R = 3.5 Ω
R = 5 Ω
R = 10 Ω
ω_0
9
10
11
12
ω(Mrad/s)

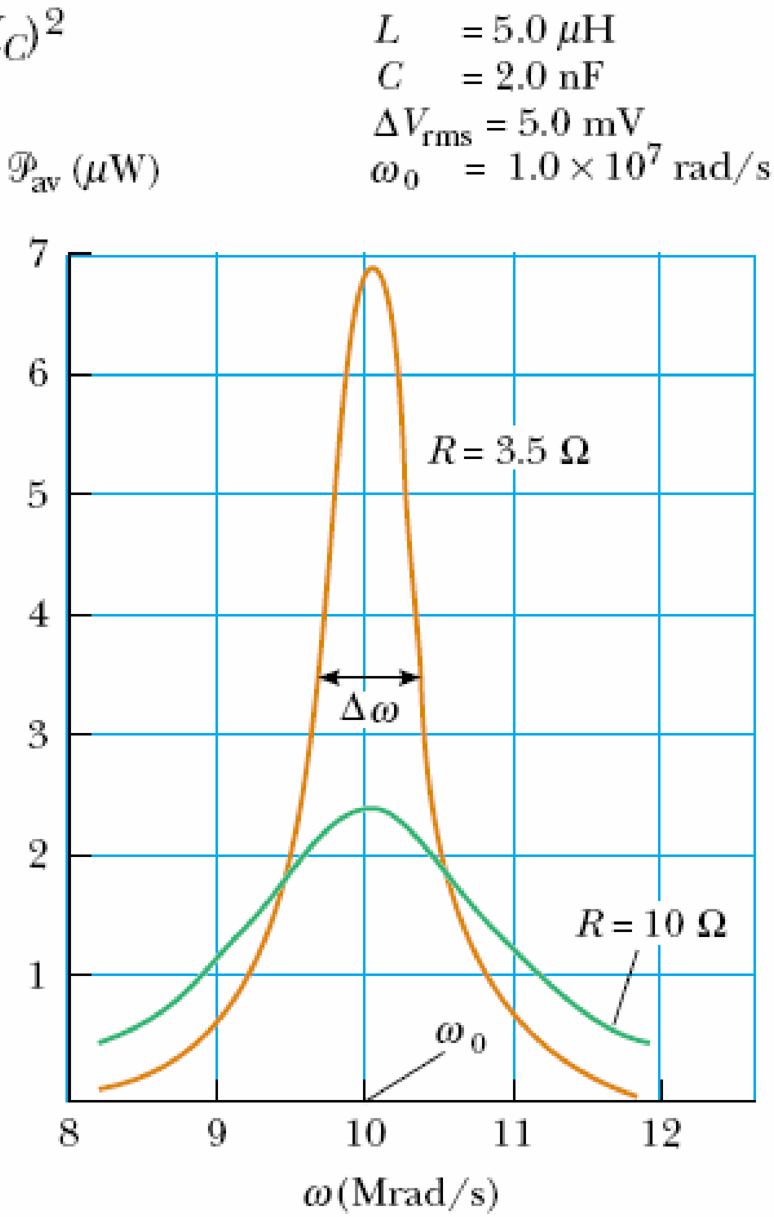
Aplicación

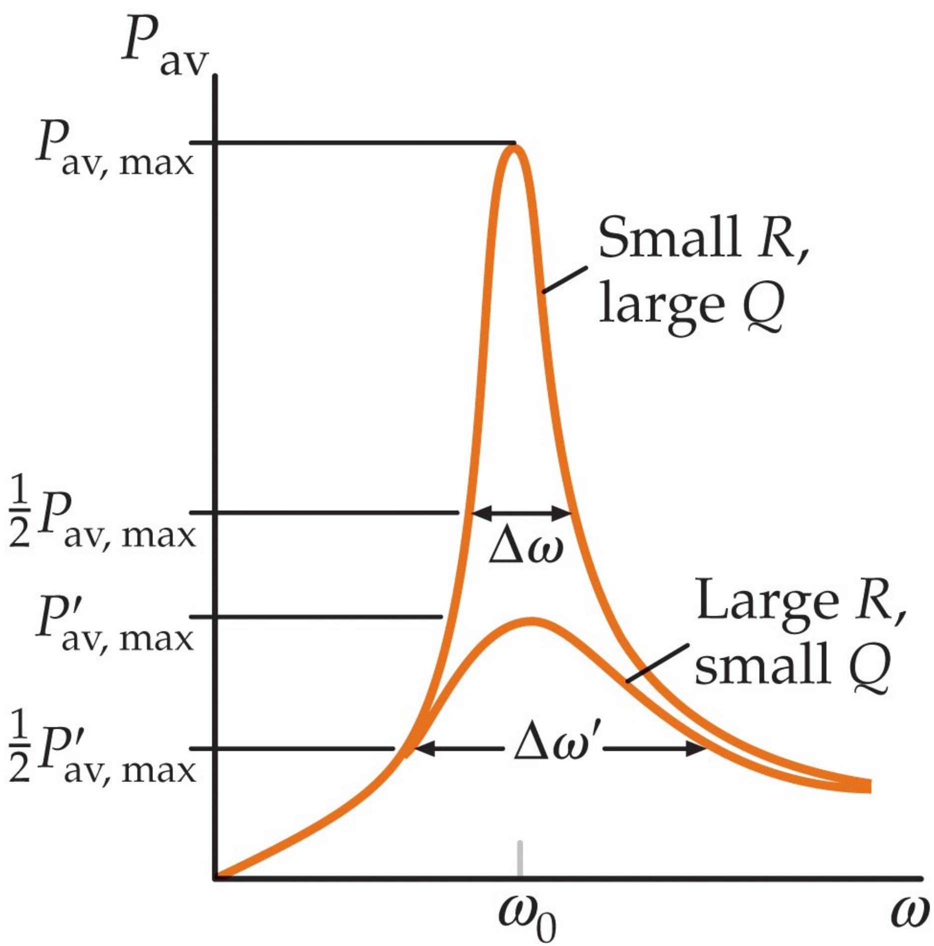
$$\mathcal{P}_{av} = I_{rms}^2 R = \frac{(\Delta V_{rms})^2}{Z^2} R = \frac{(\Delta V_{rms})^2 R}{R^2 + (X_L - X_C)^2}$$

$$(X_L - X_C)^2 = \left(\omega L - \frac{1}{\omega C}\right)^2 = \frac{L^2}{\omega^2}(\omega^2 - \omega_0^2)^2$$

$$\boxed{\mathcal{P}_{av} = \frac{(\Delta V_{rms})^2 R\omega^2}{R^2\omega^2 + L^2(\omega^2 - \omega_0^2)^2}}$$

La potencia es máxima para $\omega=\omega_0$





$$Q = \frac{\omega_0}{\Delta\omega}$$

Factor de calidad o de mérito

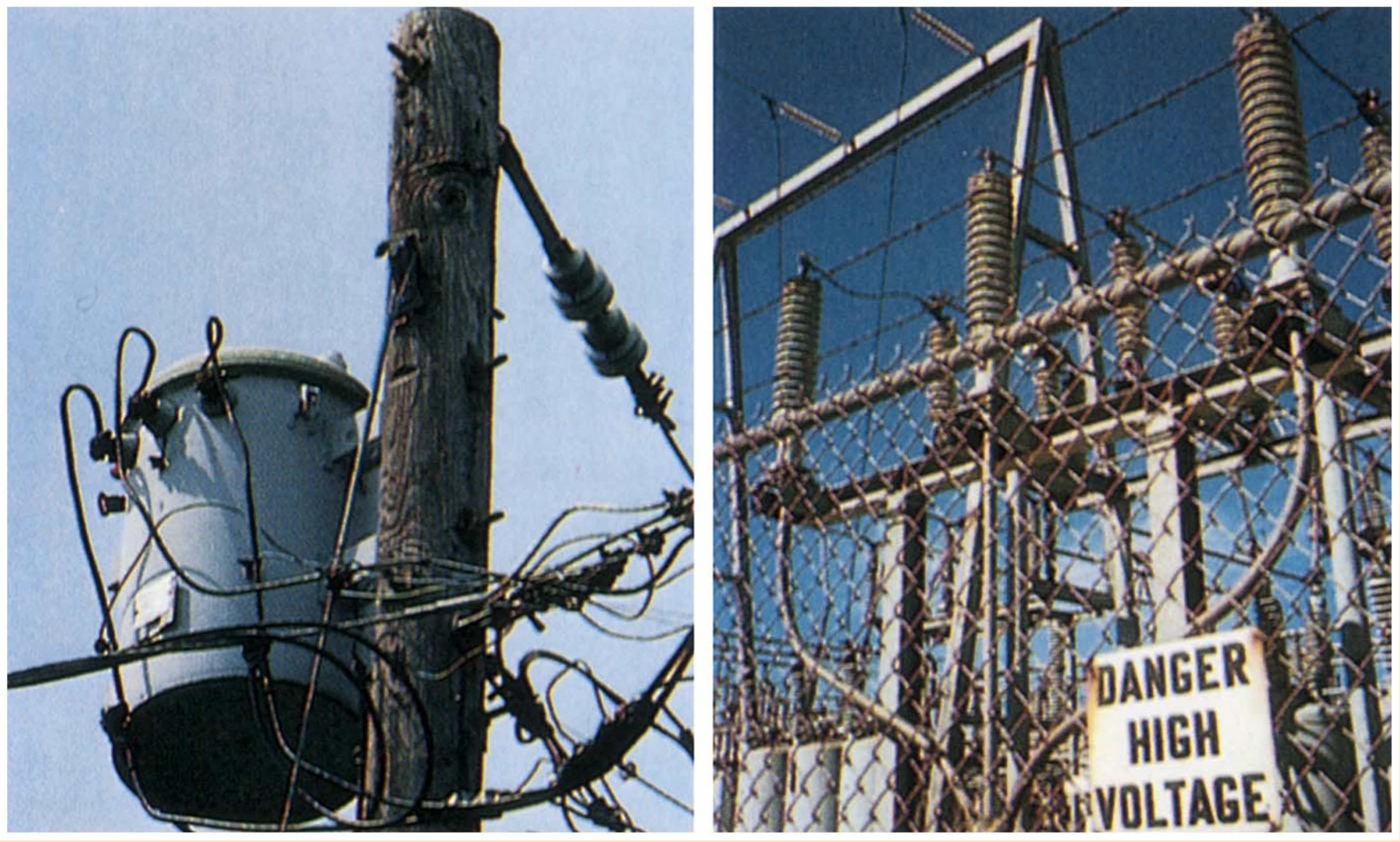
Se puede probar que en un circuito RLC:

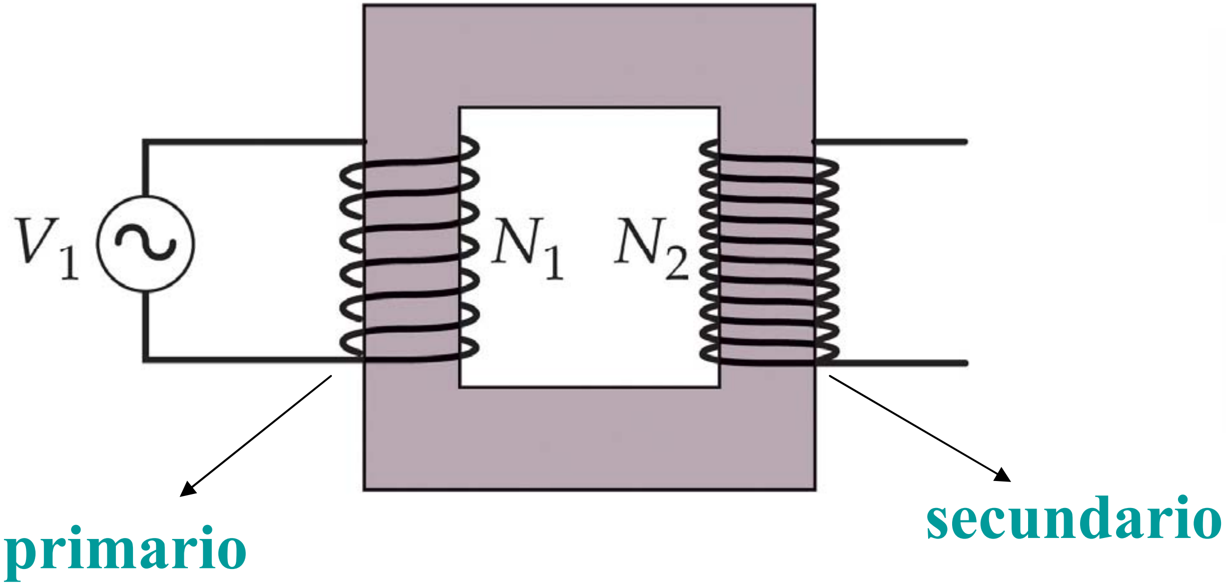
$$\Delta\omega = R/L$$

$$Q = \frac{\omega_0 L}{R}$$

Valores típicos de Q: 10-100

Transformador





$$\Delta V_1 = -N_1 \frac{d\Phi_B}{dt}$$

$$\Delta V_2 = -N_2 \frac{d\Phi_B}{dt}$$

Si suponemos que no hay pérdidas de flujo fuera del núcleo de hierro $\longrightarrow$

$$\Delta V_2 = \frac{N_2}{N_1} \Delta V_1$$

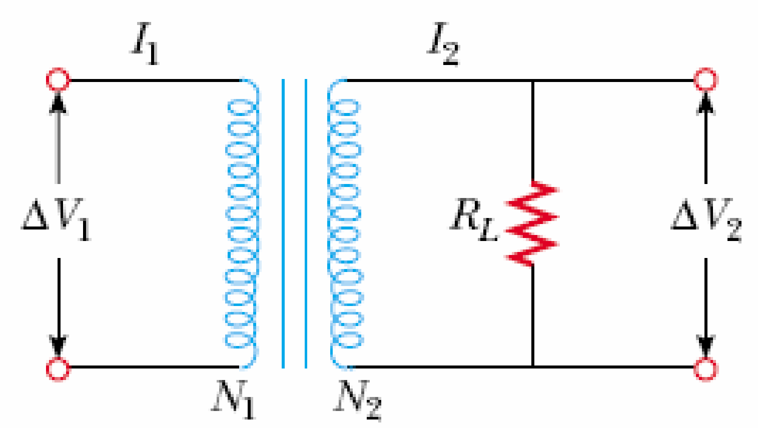
Dependiendo de N_1 y N_2, podemos tener un elevador o un reductor de voltaje

Cerrando el circuito secundario y admitiendo pérdidas de energía por unidad de tiempo pequeñas, la potencia entregada por el primario será igual a la del secundario

$$I_1\,\Delta V_1 = I_2\,\Delta V_2 \longrightarrow I_1 = \left(\frac{V_2}{V_1}\right) I_2 = \left(\frac{N_2}{N_1}\right) I_2$$

Transformador reductor V ↓

I ↑

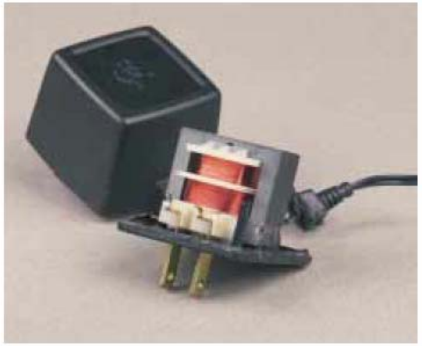


Transformador elevador V ↑

I ↓

símbolo

Los núcleos de hierro se laminan para evitar perdidas por corrientes parasitas



Guerra de las corrientes : CA vs. CC

Vs.

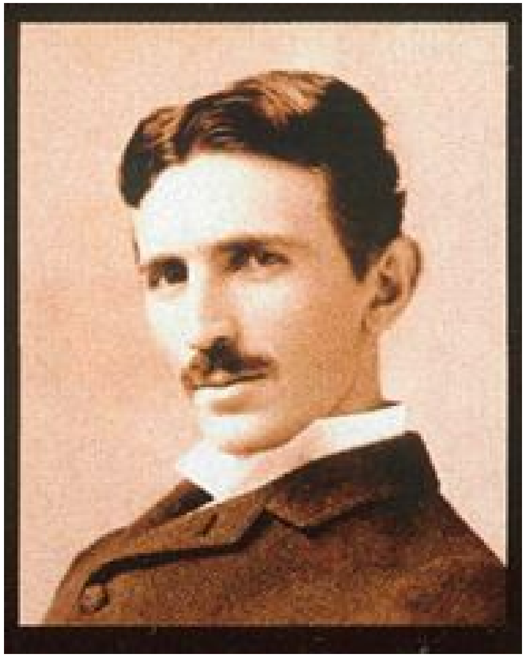
George Westinghouse (1846-1914)

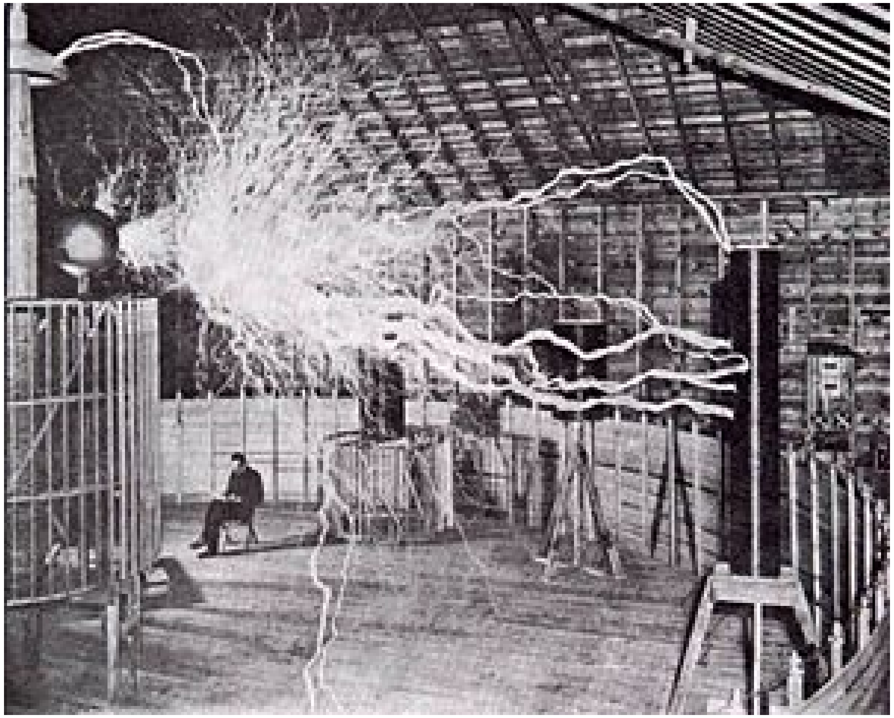
Thomas Alva Edison (1847-1931)

En 1886 fundó Westinghouse Electric

En 1880 se asocia con J.P. Morgan para fundar la General Electric

Nikola Tesla (1856-1943)



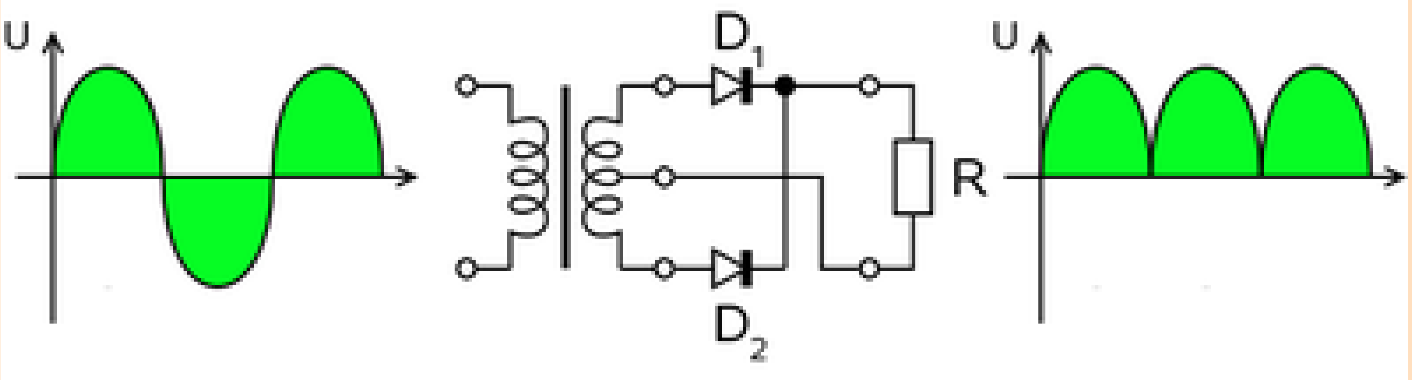


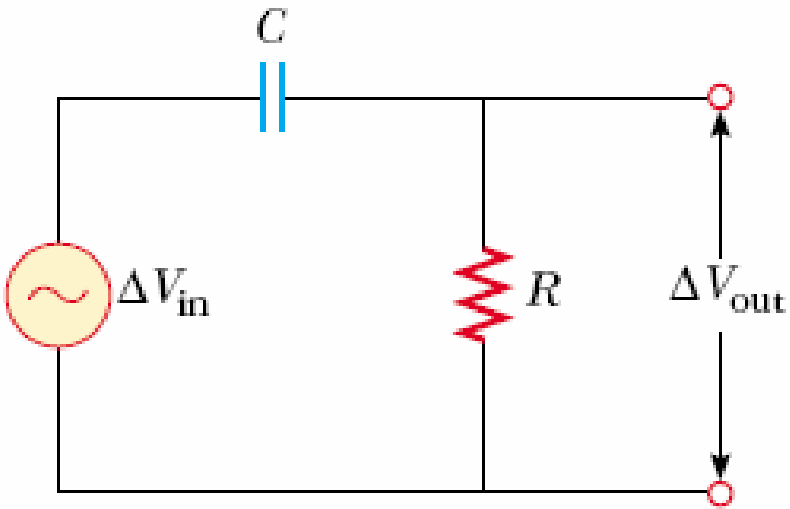
Con el apoyo financiero de George Westinghouse, la corriente alterna sustituyó a la continua. Tesla fue considerado desde entonces el fundador de la industria eléctrica.

Bobina de Tesla: están compuestas por una serie de circuitos eléctricos resonantes acoplados. Crean descargas eléctricas de largo alcance.

Torre Tesla: torre-antena de telecomunicaciones inalámbricas pionera diseñada para demostrar la transmisión de energía sin cables conectores entre los años 1901 y 1917.

Rectificadores y Filtros

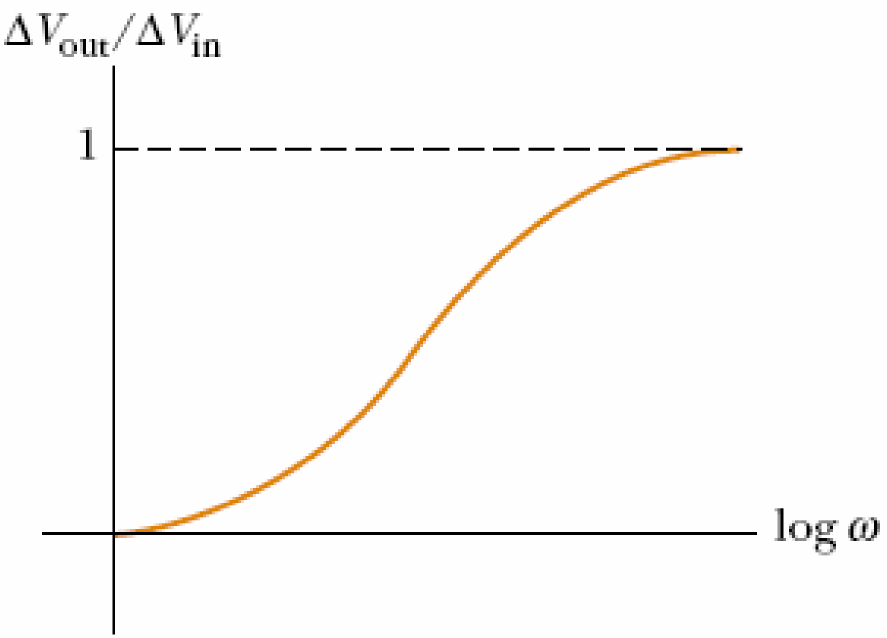




$$V_{in} = V^0_{in} \text{ sen } (\omega\, t)$$

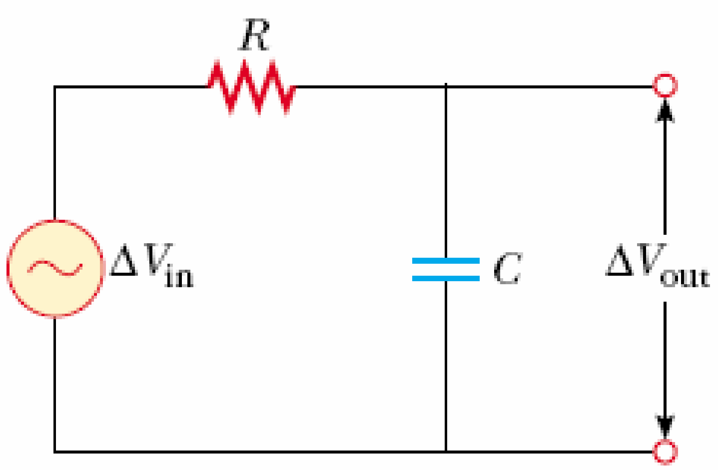
$$V^0_{in} = I^0\, Z = I^o \ (R^2 + (1/\omega C)\ )^{1/2}$$

$$V^0_{out} = I^0\, R$$



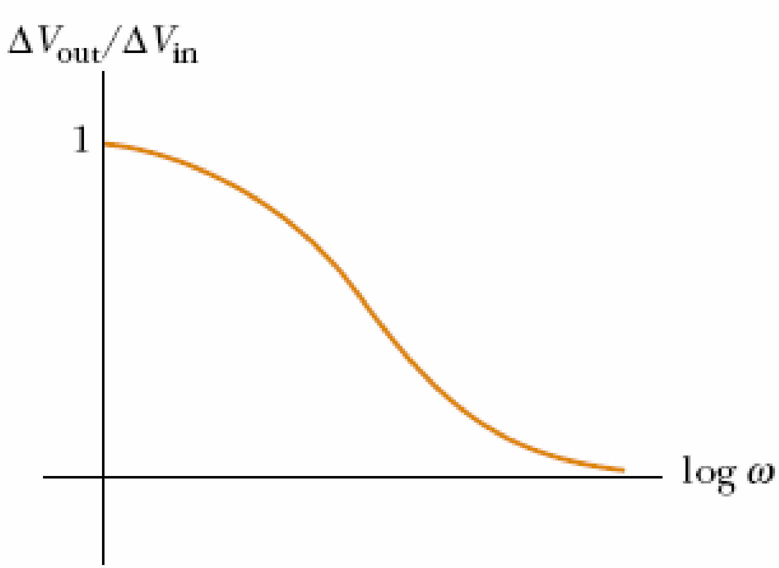
$$\frac{V^0_{out}}{V^0_{in}} = \frac{R}{(R^2 + (1/\omega C)\ )^{1/2}}$$

Filtro pasa altos



$$V^0_{in} = I^0\,Z = I^o\,(R^2 + (1/\omega C)\,)^{1/2}$$

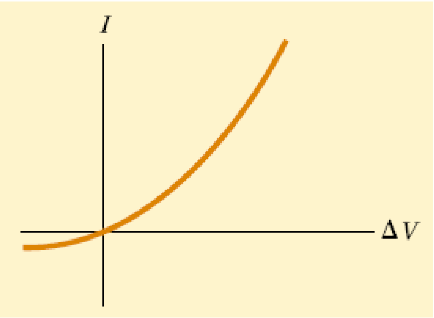
$$V^0_{out} = I^0\,X_c = I^0 / \omega C$$

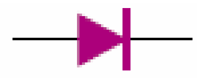


$$\frac{V^0_{out}}{V^0_{in}} = \frac{1/\,\omega C}{(R^2 + (1/\omega C)\,)^{1/2}}$$

Filtro pasa bajos

Rectificador





Símbolo del iodo