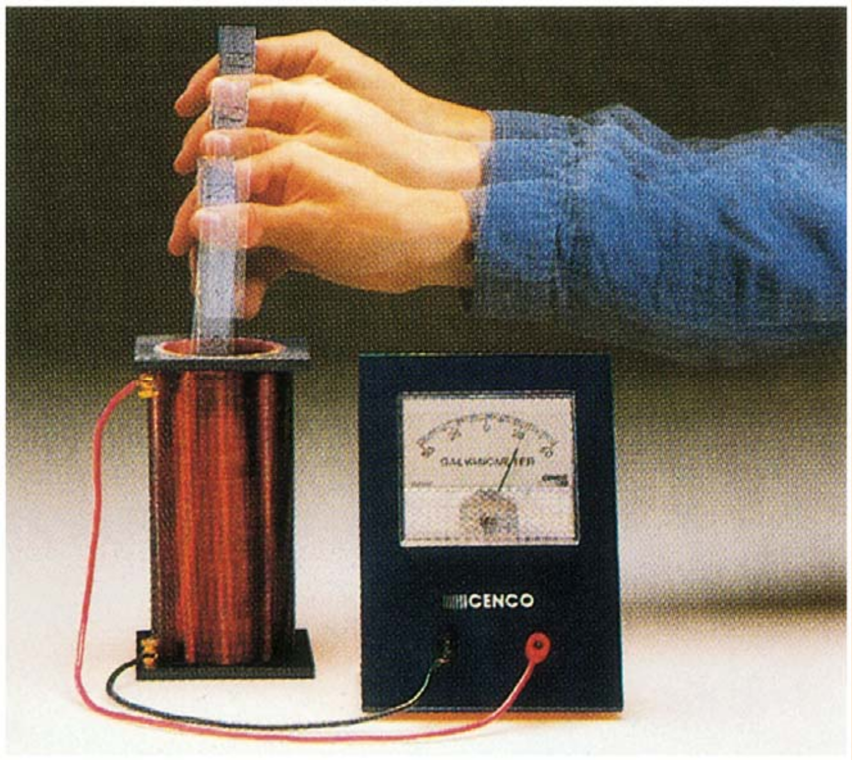
Capítulo 5

Campos electromagnéticos dependientes del tiempo



$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_o}$$

$$\vec{\nabla} \times \vec{E} = 0$$

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{\mathbf{B}} = \mu_0 \vec{\mathbf{J}}$$

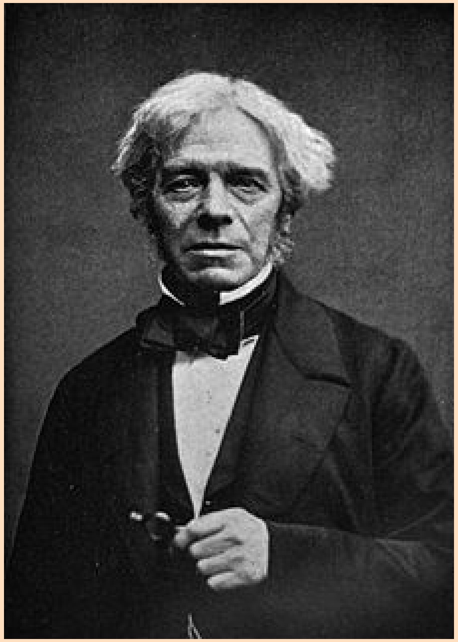
Ecuaciones de Maxwell para campos estáticos

Electricidad

Magnetismo

$\Rightarrow$ **dos fenómenos independientes**

Ley de Faraday-Henry



Michael Faraday (1791-1867)

Joseph Henry (1797-1878)

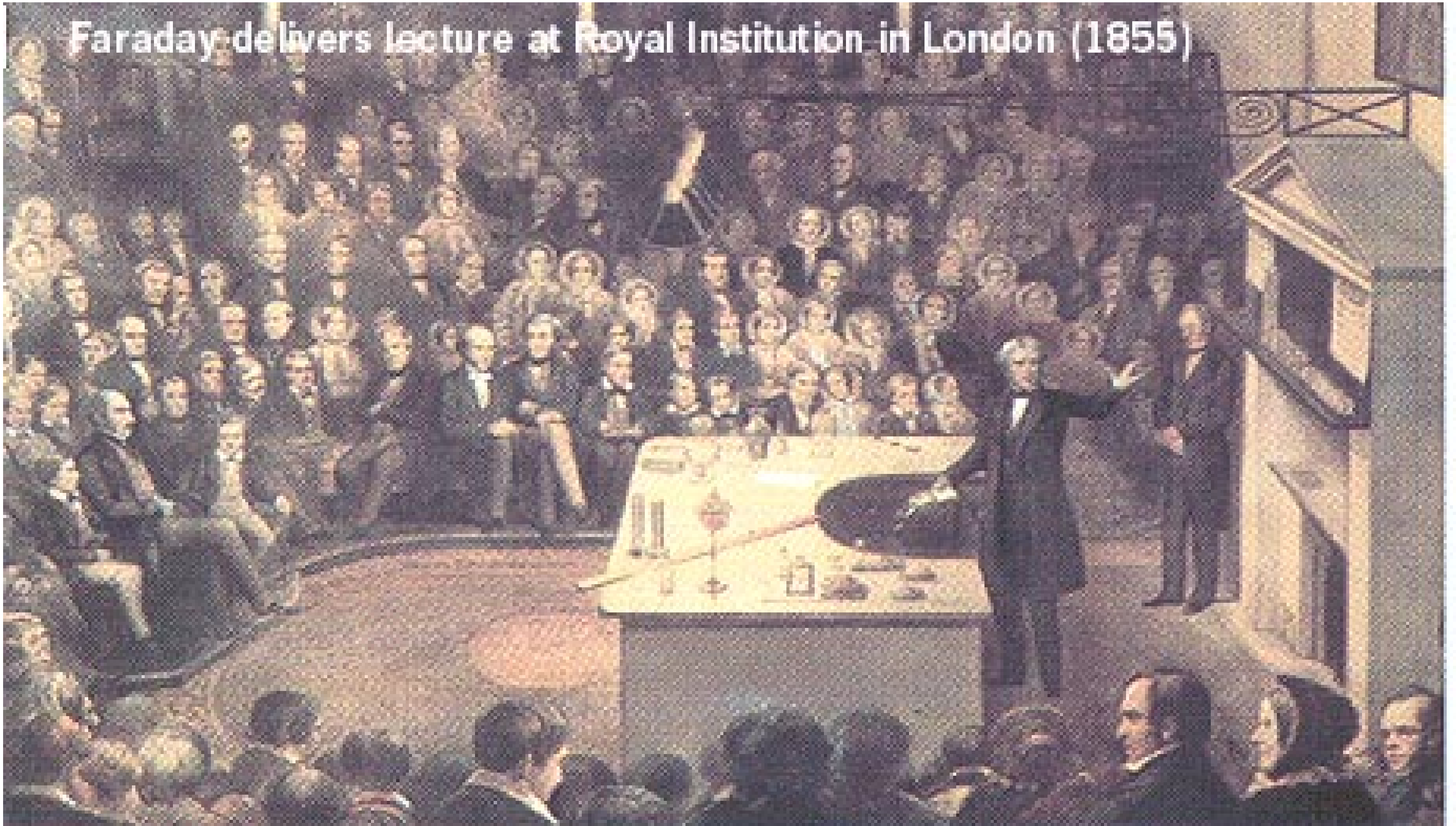

BANK of ENGLAND
TWENTY

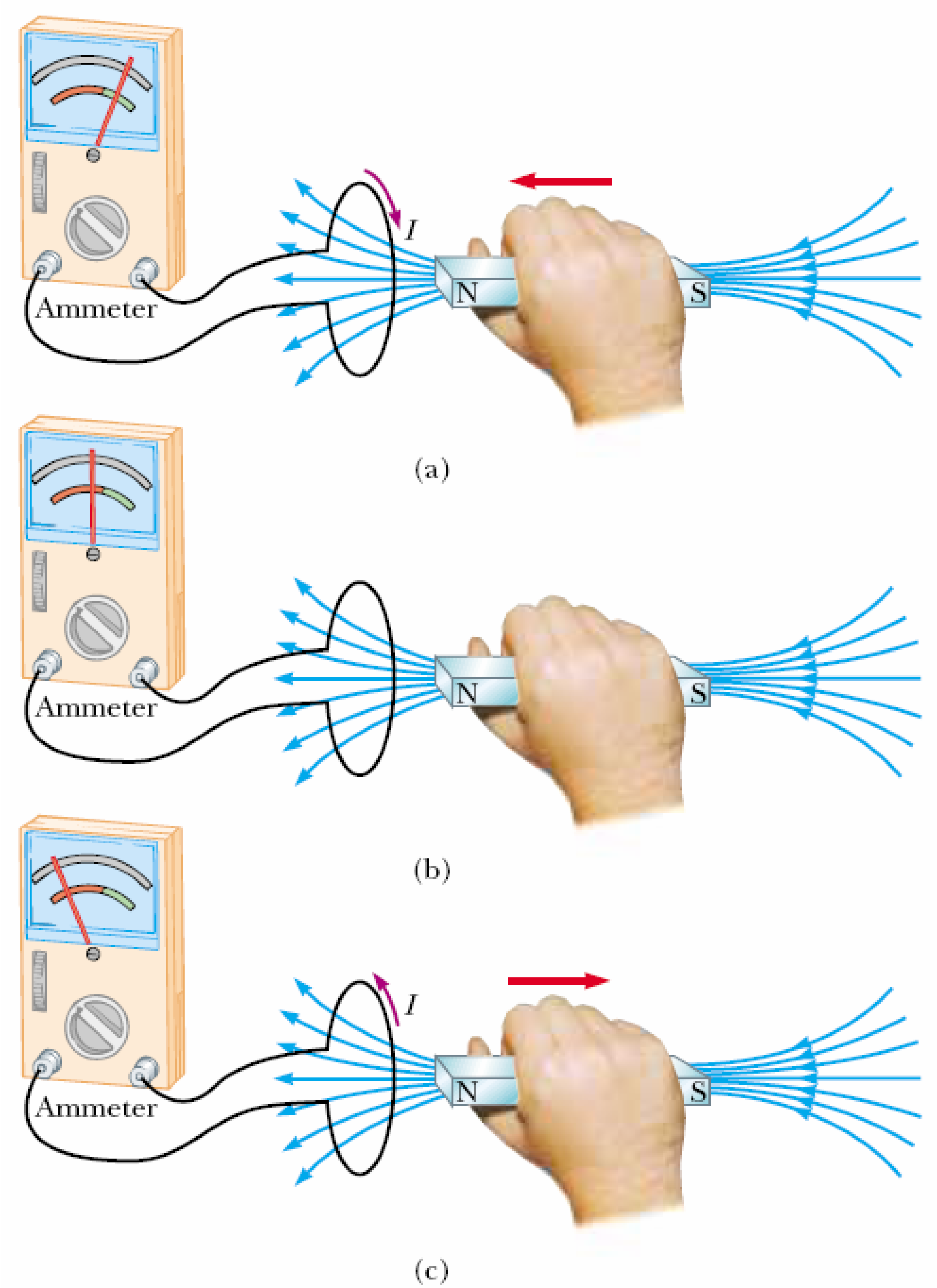

BANCA D'ITALIA
DUEMILA

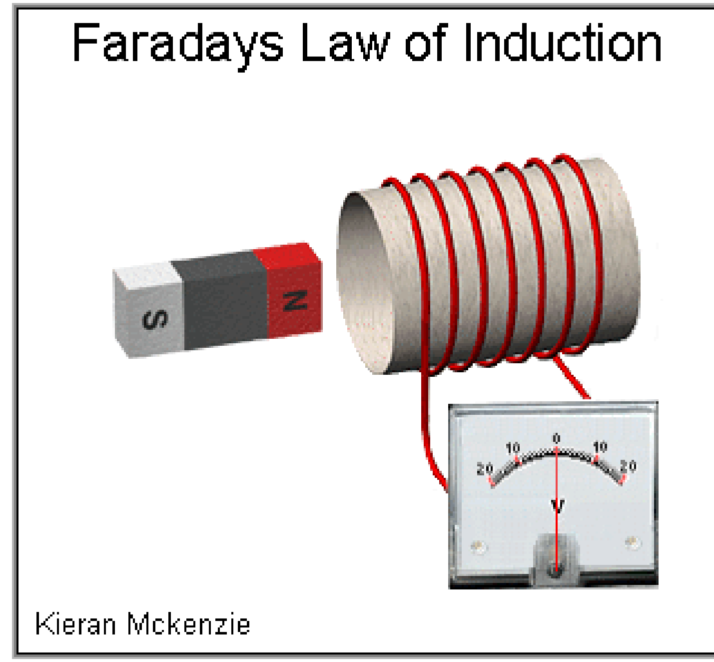

10000
DIECIMILA


TAUSEND SCHILLING
1000
1000
1000


500
FEM HUNDREDE KRONER
DANMARKS NATIONALBANK

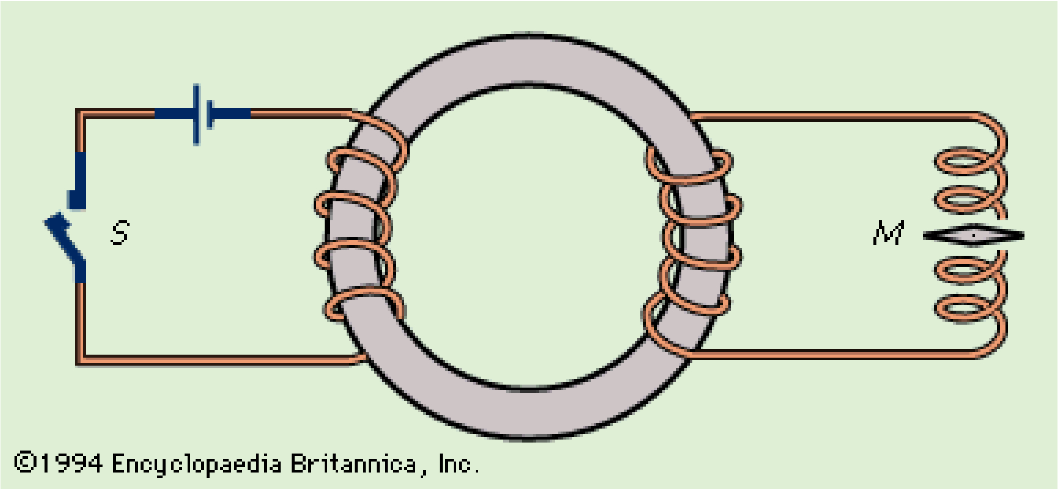

Faraday delivers lecture at Royal Institution in London (1855)



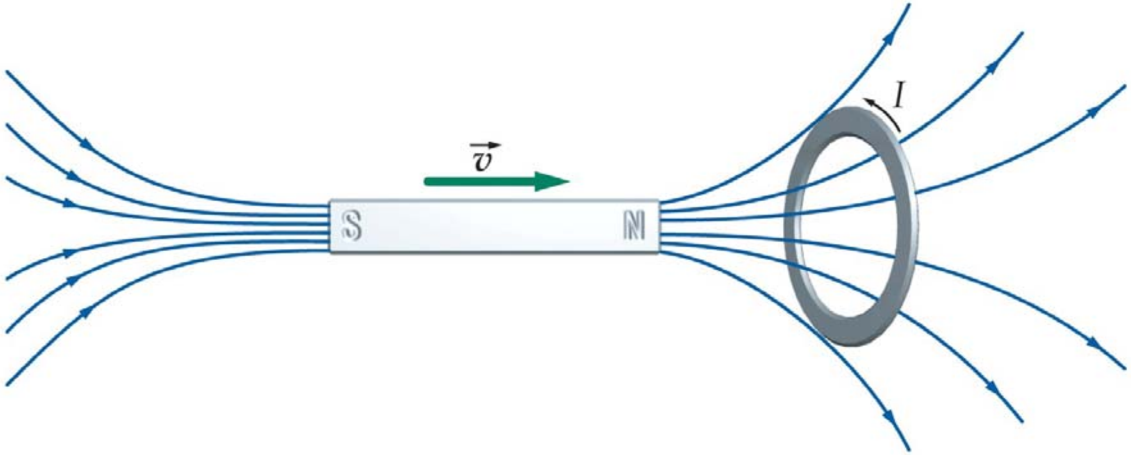


Una corriente eléctrica puede producirse mediante un campo magnético variable

Experimento de Faraday

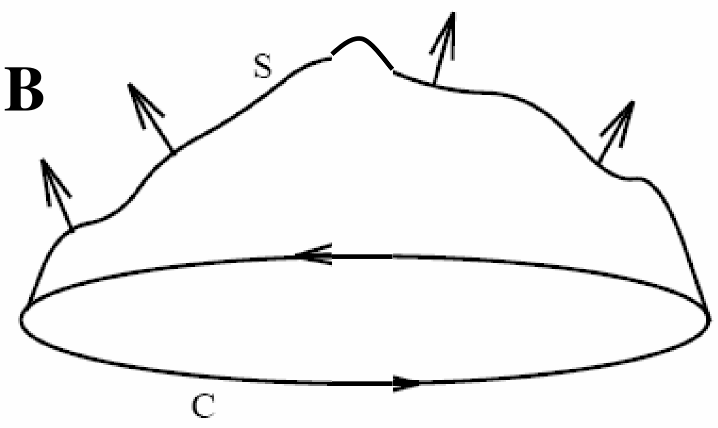


Un campo magnético variable induce una fem en el circuito secundario



$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$

Enunciado general de la ley de Faraday-Henry



$$\mathcal{E} = -\frac{d\phi_{\mathrm{m}}}{dt}$$

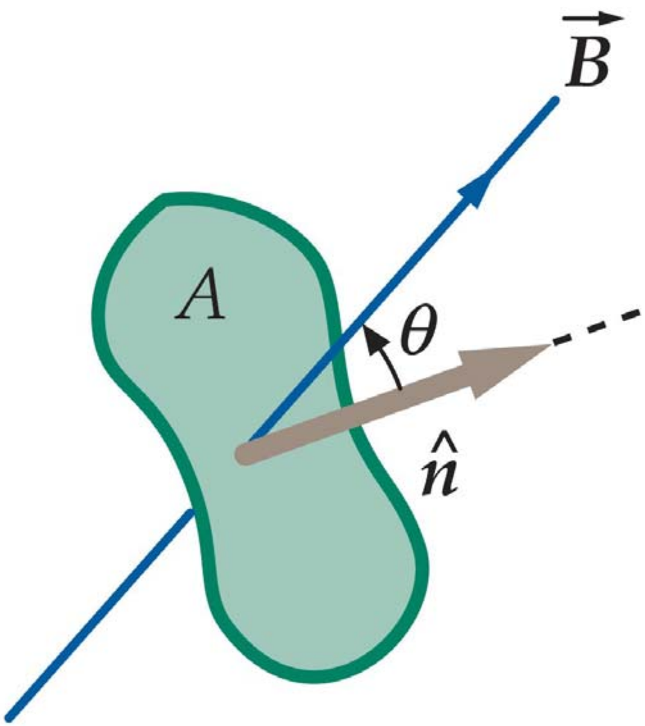
$$\phi_{\mathrm{m}} = \int_S \vec{B} \cdot \hat{\boldsymbol{n}}\, dA = \int_S B_n\, dA$$

$$\boxed{\mathcal{E} = \oint_C \vec{E}_{\mathrm{nc}} \cdot d\vec{\ell} = -\frac{d}{dt}\int_S \vec{B} \cdot \hat{\boldsymbol{n}}\, dA = -\frac{d\phi_{\mathrm{m}}}{dt}}$$

En una bobina con N vueltas, todas de la misma área

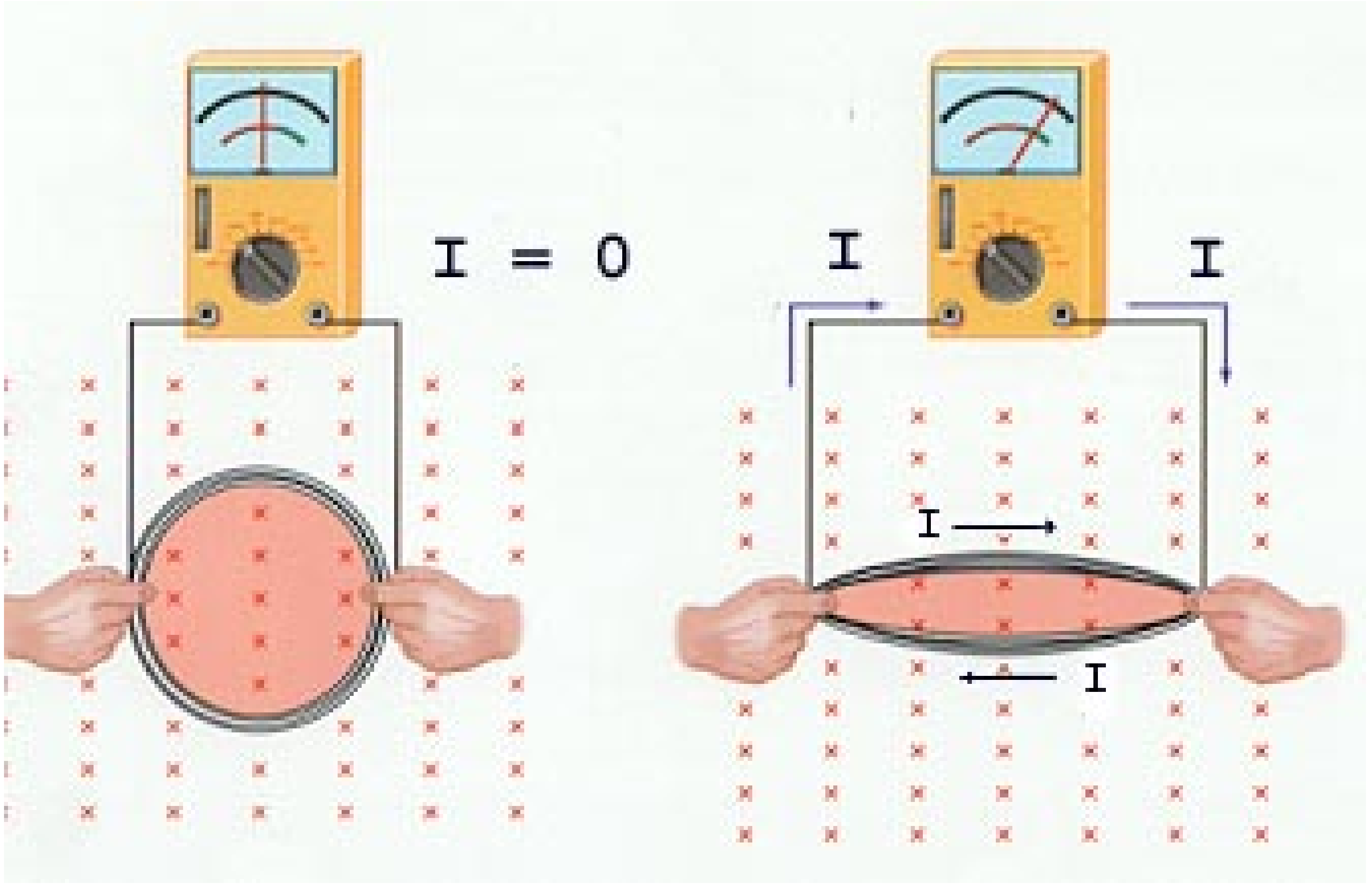
$$\mathcal{E} = -N\frac{d\Phi_B}{dt}$$

$\Phi_B(t)$?

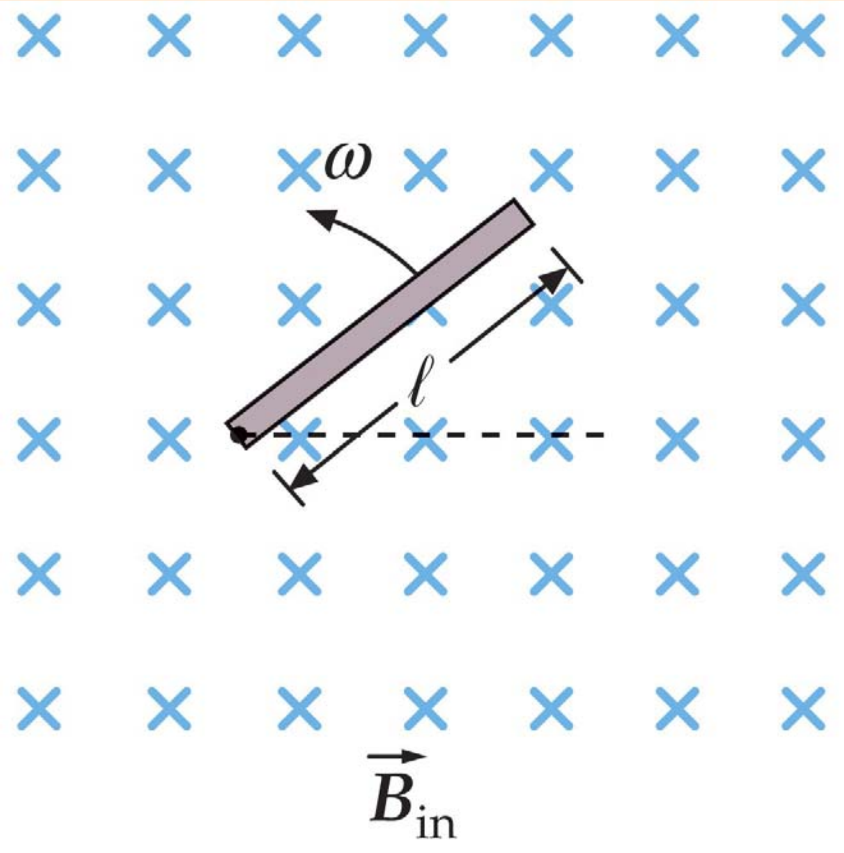


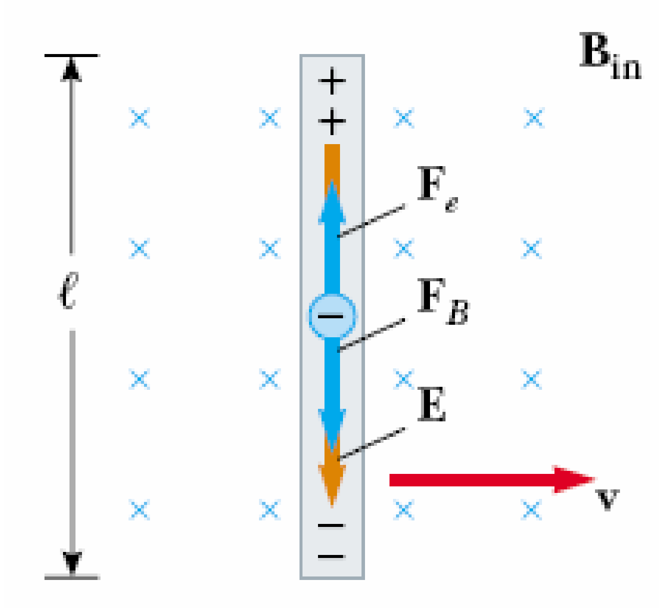
$$\mathcal{E} = -\frac{d}{dt}(BA\cos\theta)$$

- B(t)
- A(t)
- θ(t)


I = 0
I
I
I
I

fem de movimiento



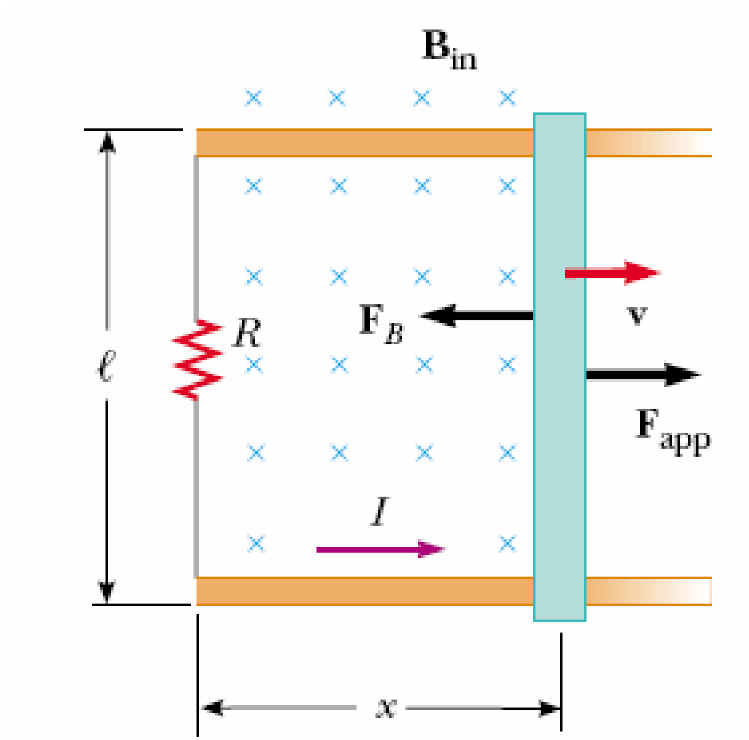


$$qE = qvB$$

$$E = vB$$

$$\Delta V = E\ell = B\ell v$$

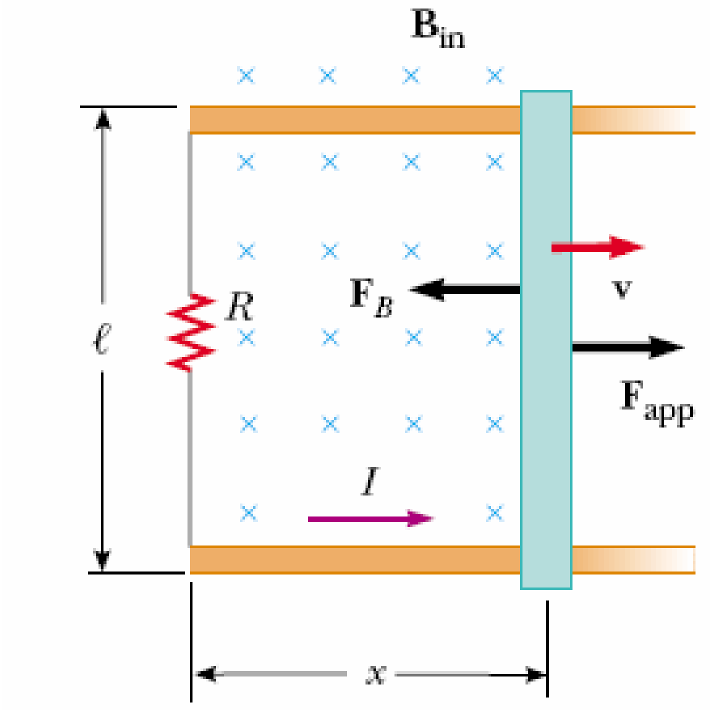
Si se invierte v, se invierte la polaridad de V

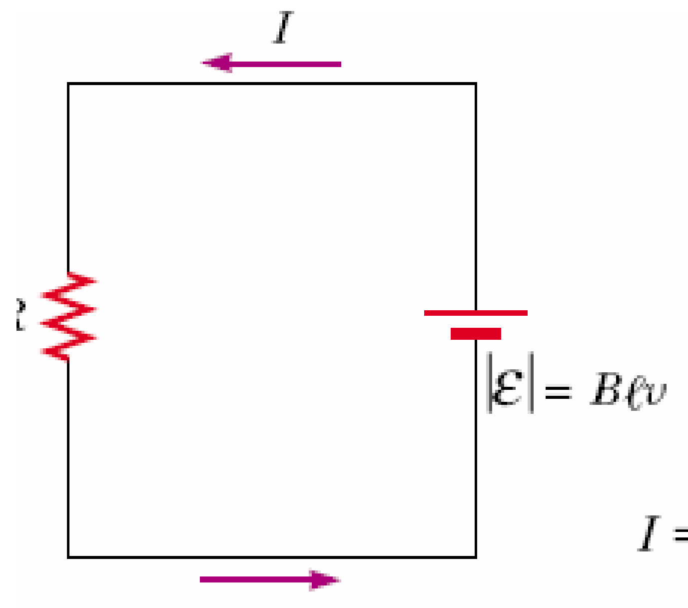


$$\Phi_B = B\ell x$$

$$\mathcal{E} = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}(B\ell x) = -B\ell\frac{dx}{dt}$$

$$\mathcal{E} = -B\ell v$$





$$I = \frac{|\mathcal{E}|}{R} = \frac{B\ell v}{R}$$

Consideración energética: si v = cte $\longrightarrow$ $F_{app} = I\ell B$

La potencia entregada por la fuerza aplicada es:

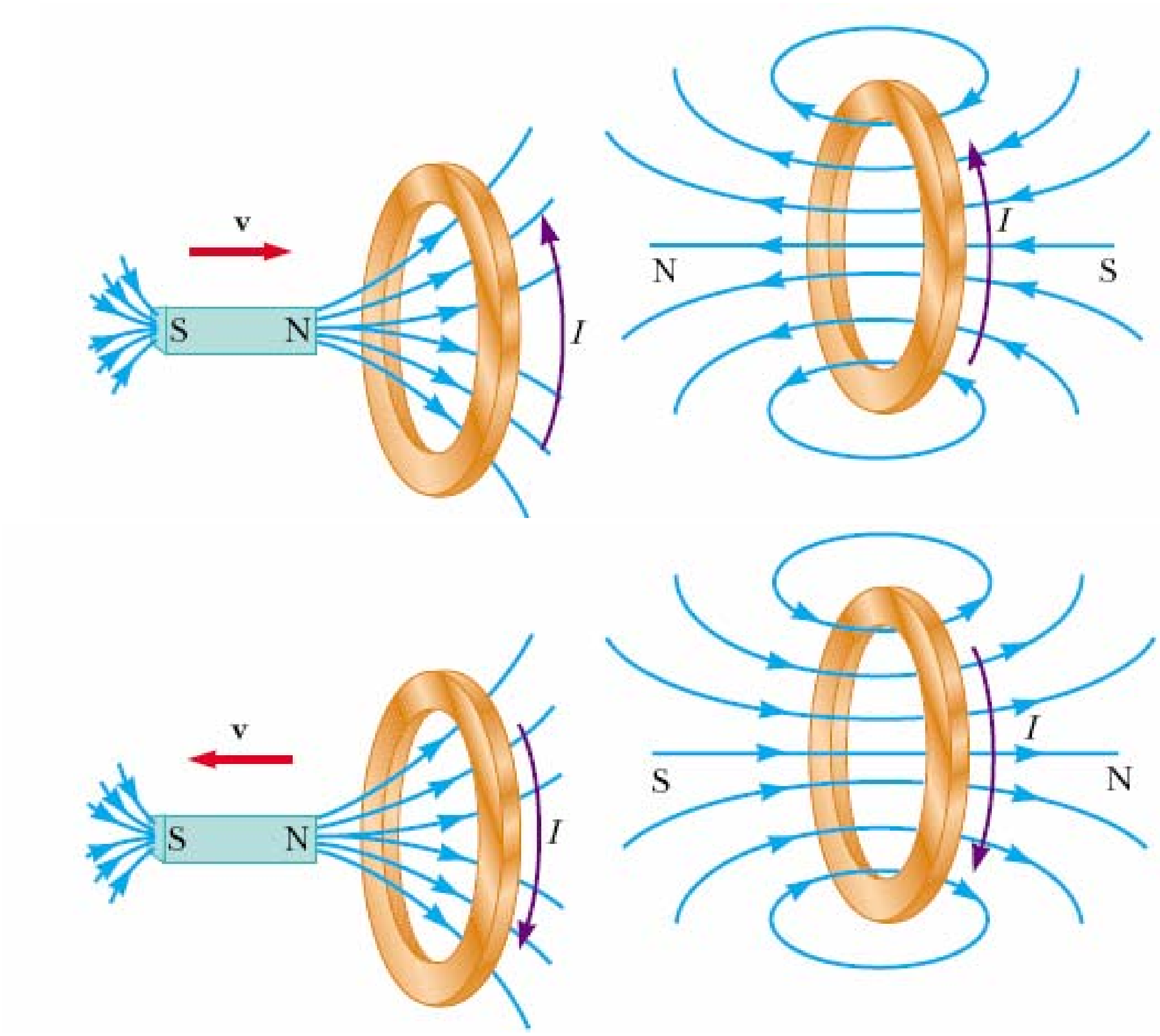
$$\mathscr{P} = F_{app} v = (I\ell B)\, v = \frac{B^2 \ell^2 v^2}{R} = \frac{\mathcal{E}^2}{R}$$

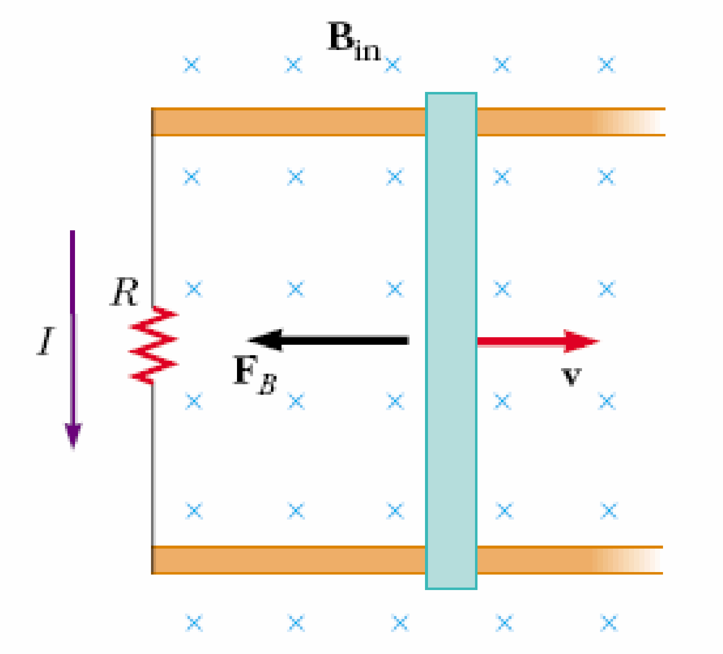
Energía Mecánica → **Energía Eléctrica** → **Energía Térmica**

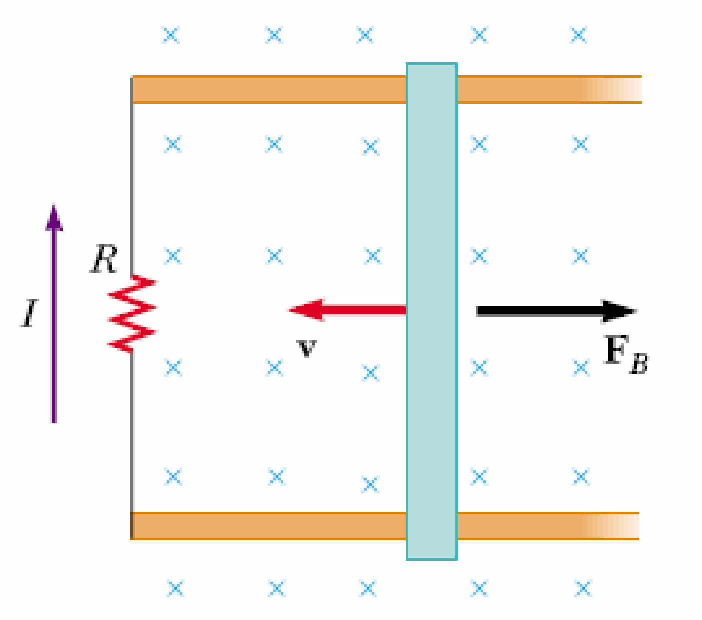
Ley de Lenz

Heinrich Lenz (1804-1865)

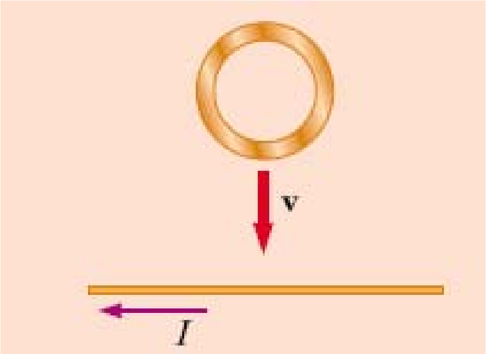
La polaridad de una fem inducida es tal que tiende a producir una corriente eléctrica que creara un flujo magnético que se opone al cambio de Φ_B a través del lazo.





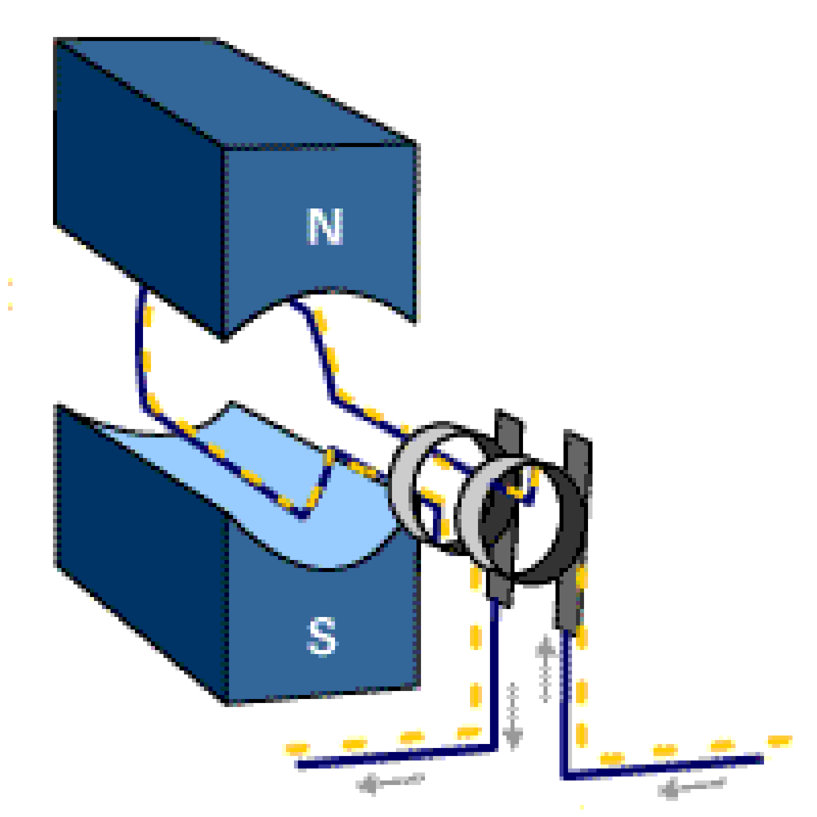


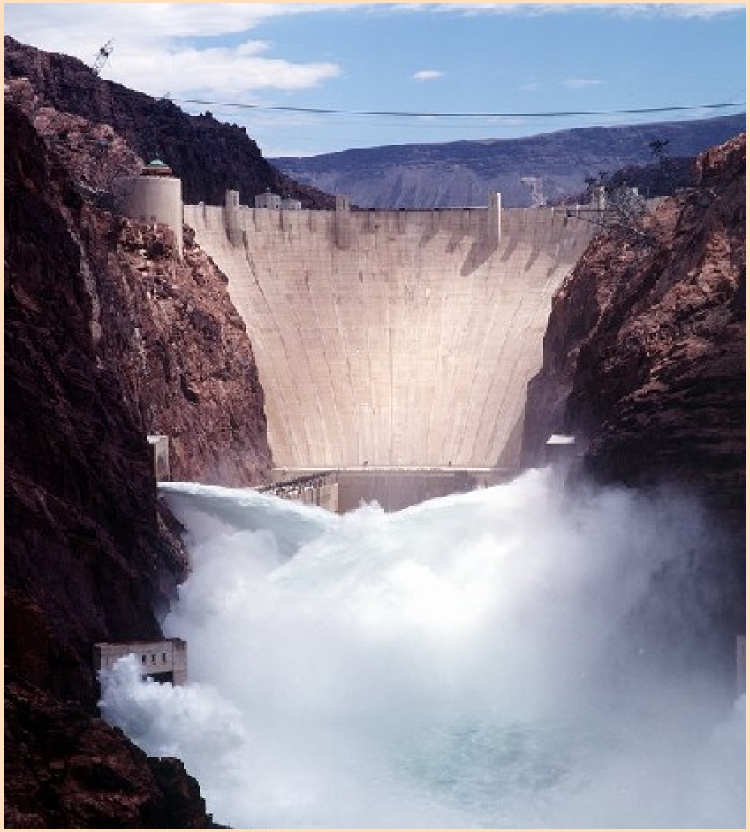
Qué sucede si el sentido de I es contrario al obtenido utilizando la ley de Lenz en los dos casos anteriores ?



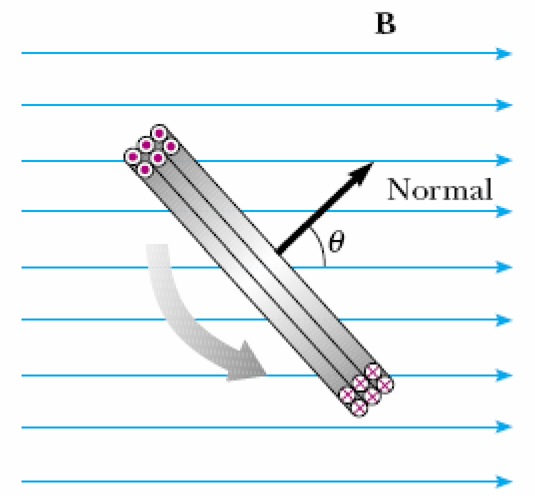
Determinar el sentido de la corriente inducida en la espira circular.

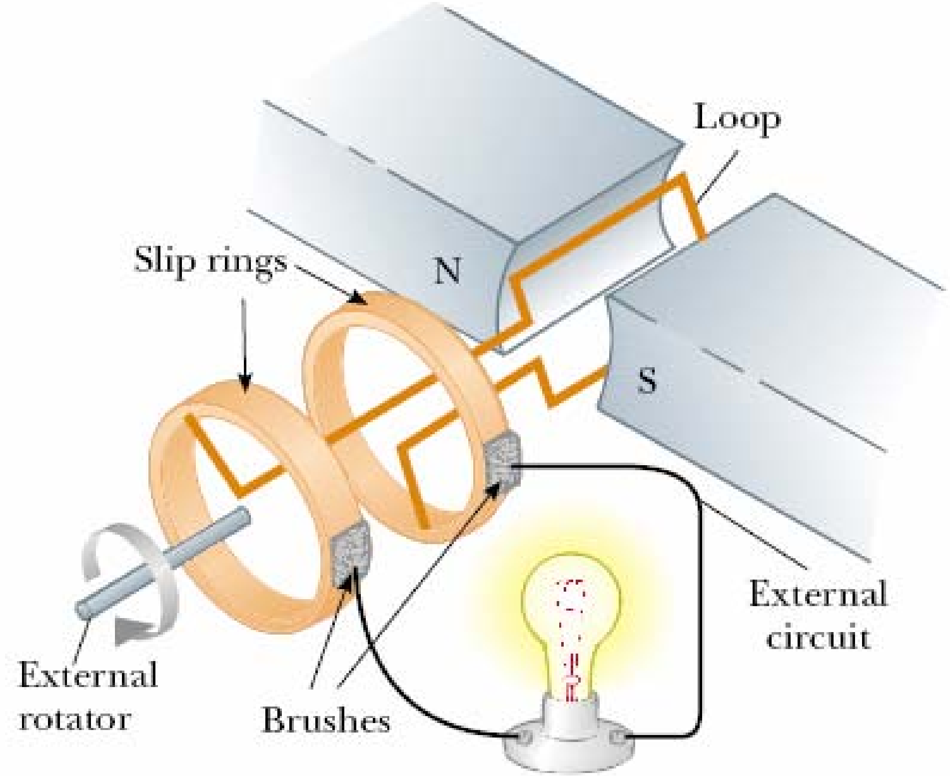
Aplicaciones de la ley de Faraday





Generador eléctrico

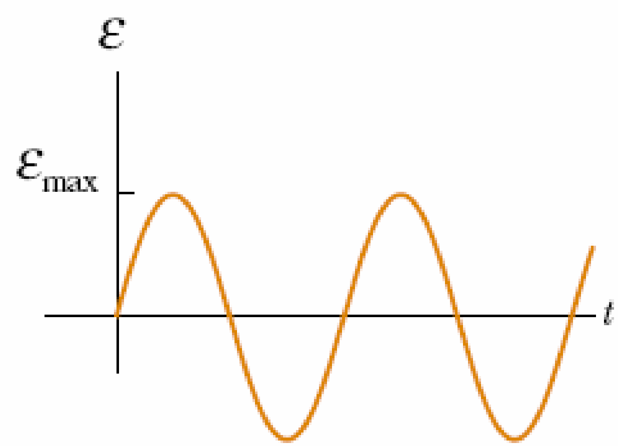


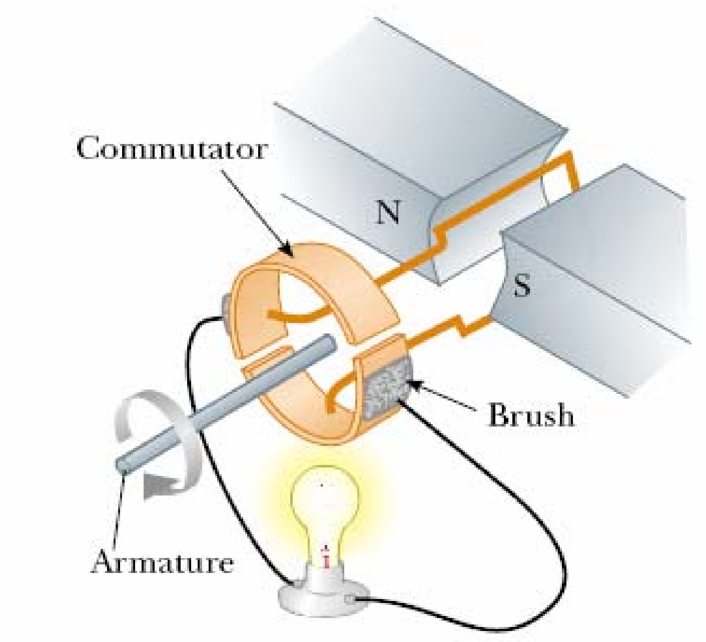


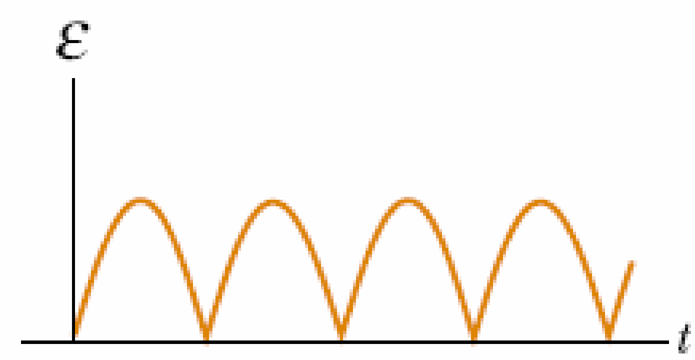
$$\Phi_B = BA\cos\theta = BA\cos\omega t$$

$$\mathcal{E} = -N\frac{d\Phi_B}{dt} = -NAB\frac{d}{dt}(\cos\omega t) = NAB\omega\sin\omega t$$

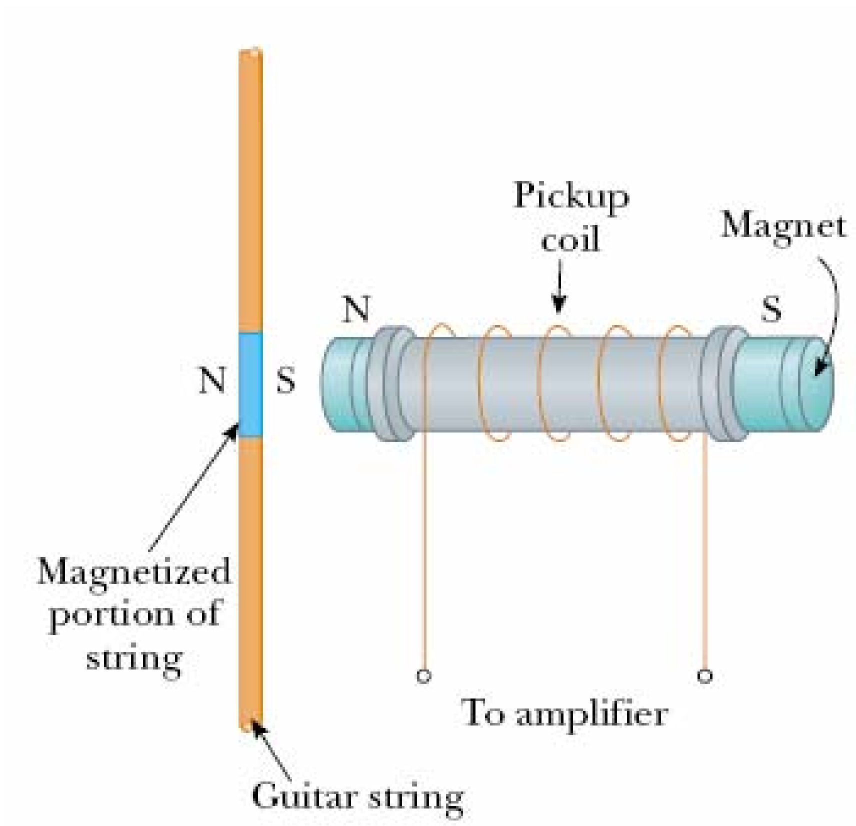
$$\mathcal{E}_{\max} = NAB\omega$$



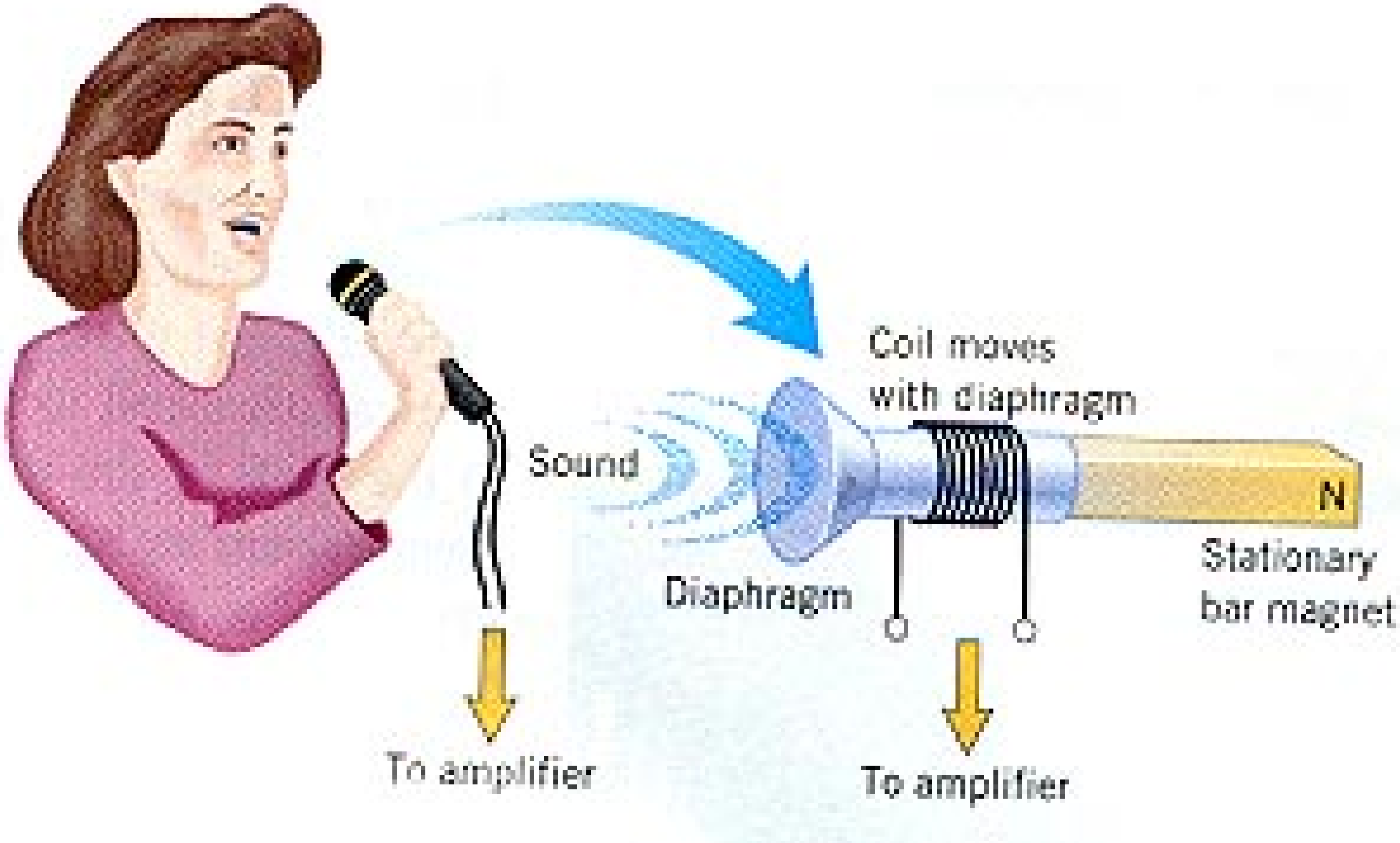

Commutator
N
S
Brush
Armature


ℰ
t

Guitarra eléctrica

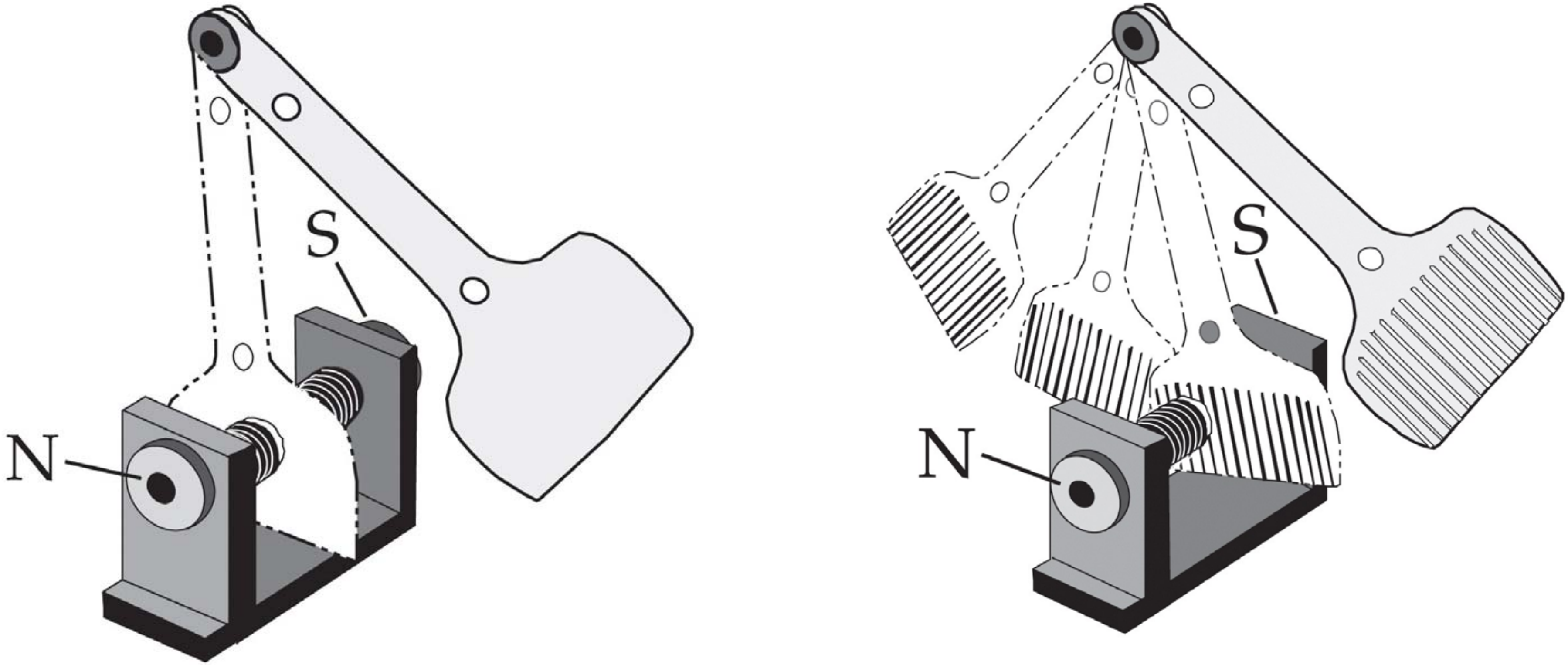


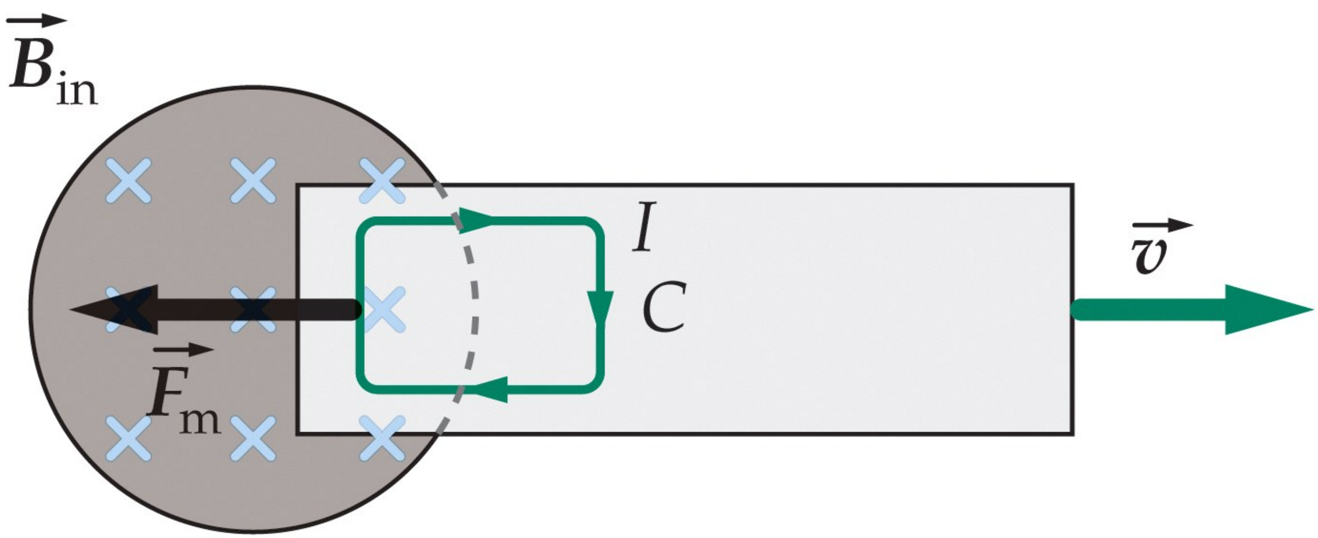
Micrófono de bobina móvil

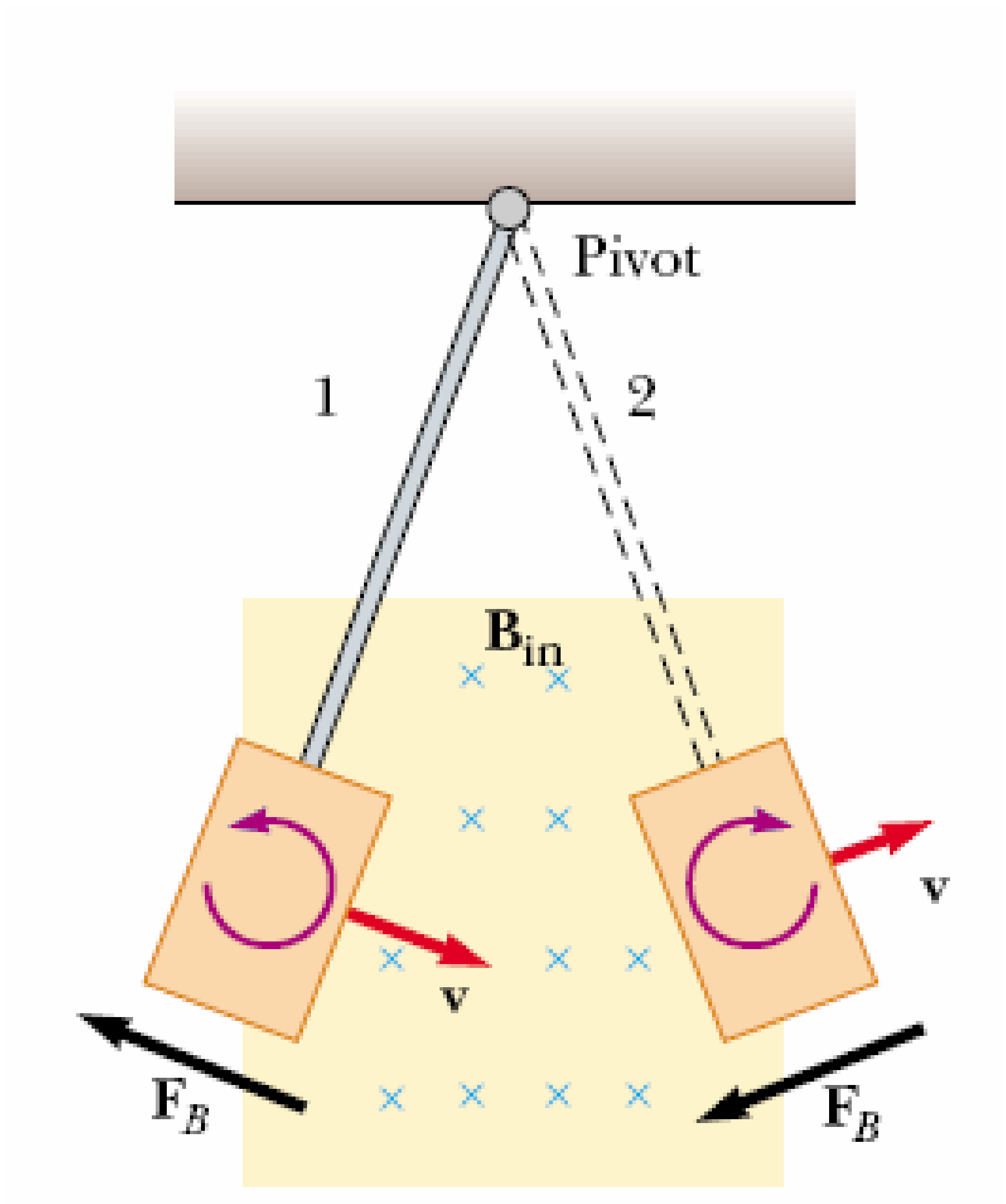


Corrientes de Foucault

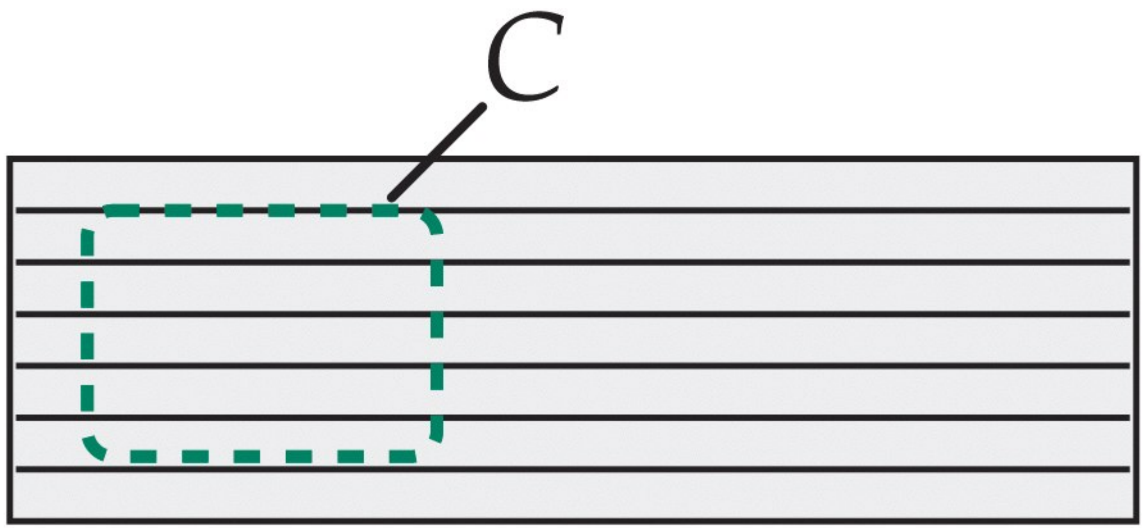
Jean Leon Foucault (1819-1868)

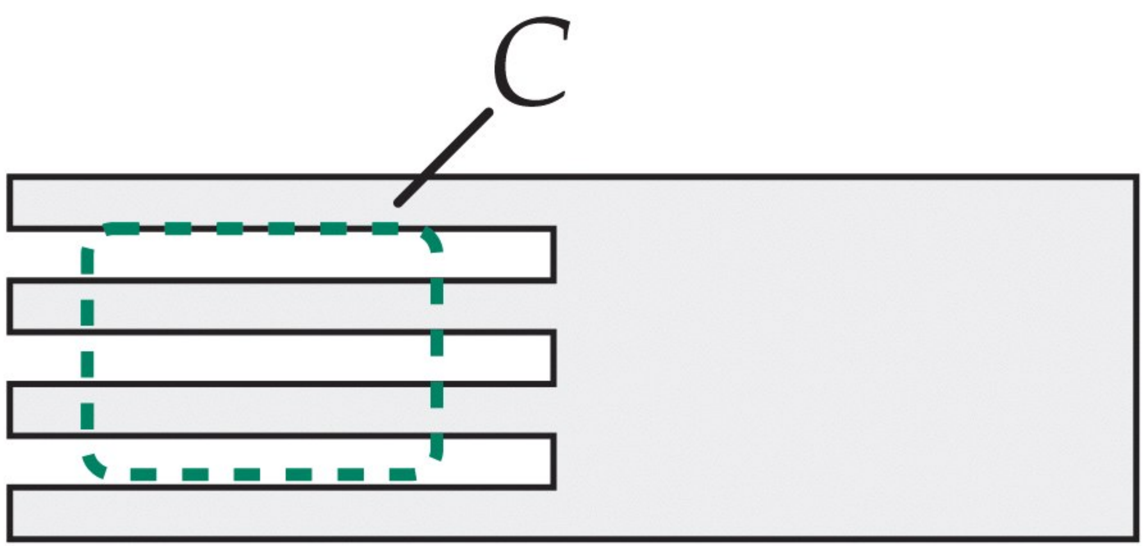

S
N
S
N


$\vec{B}_{\mathrm{in}}$
I
C
$\vec{v}$
$\vec{F}_{\mathrm{m}}$



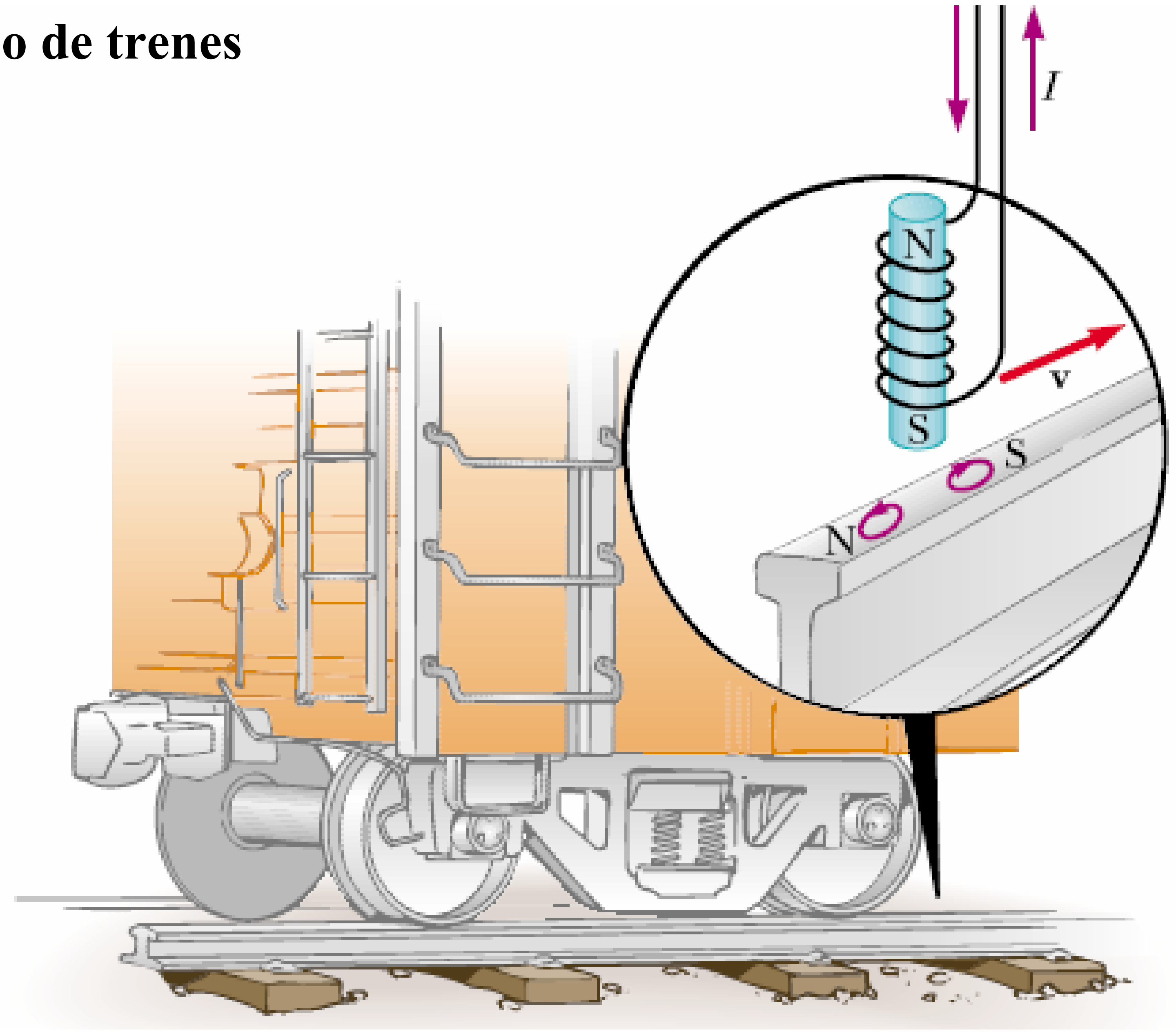
Pivot
1
2
$\mathbf{B}_{in}$
v
v
$\mathbf{F}_B$
$\mathbf{F}_B$


C

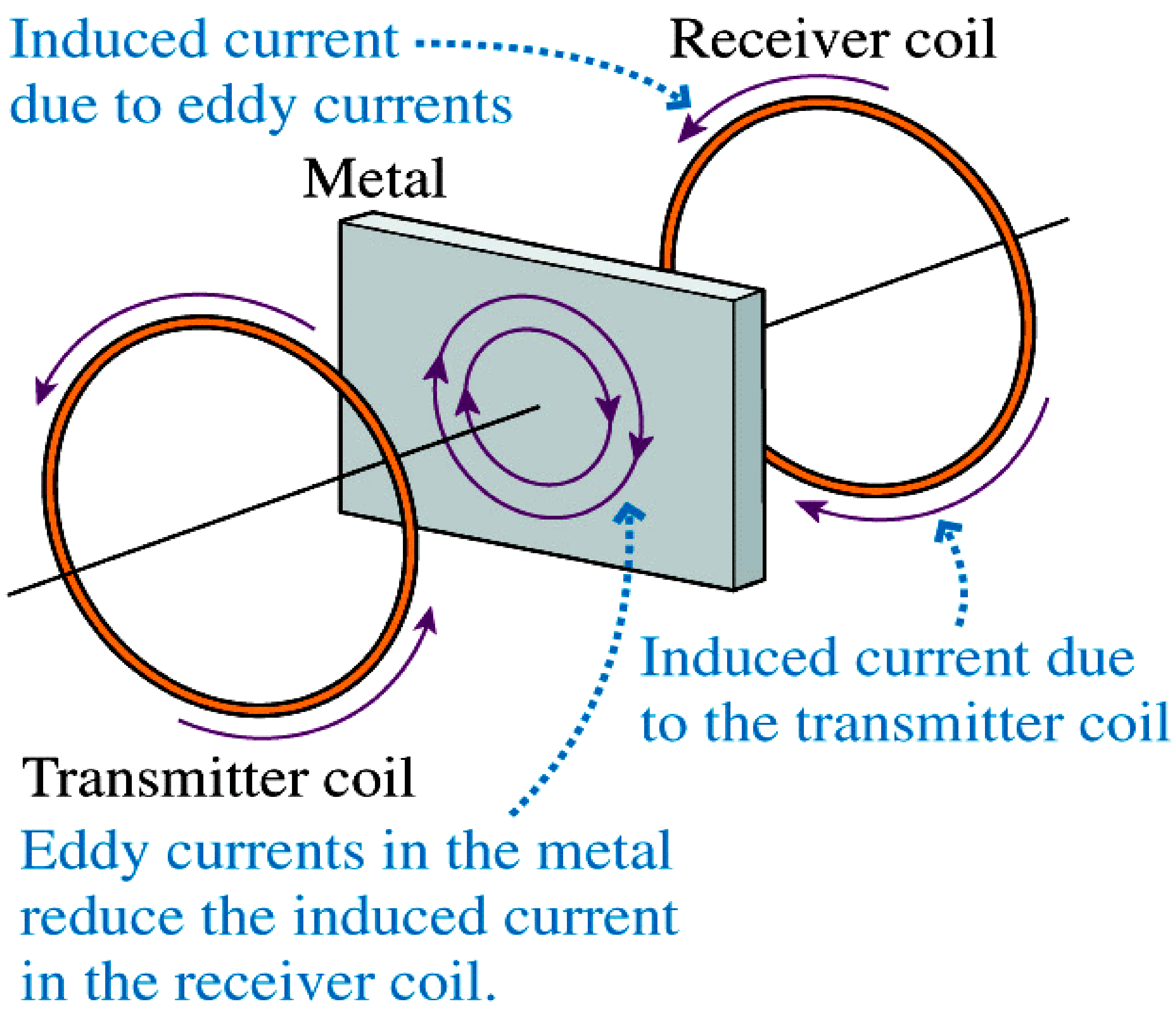

C

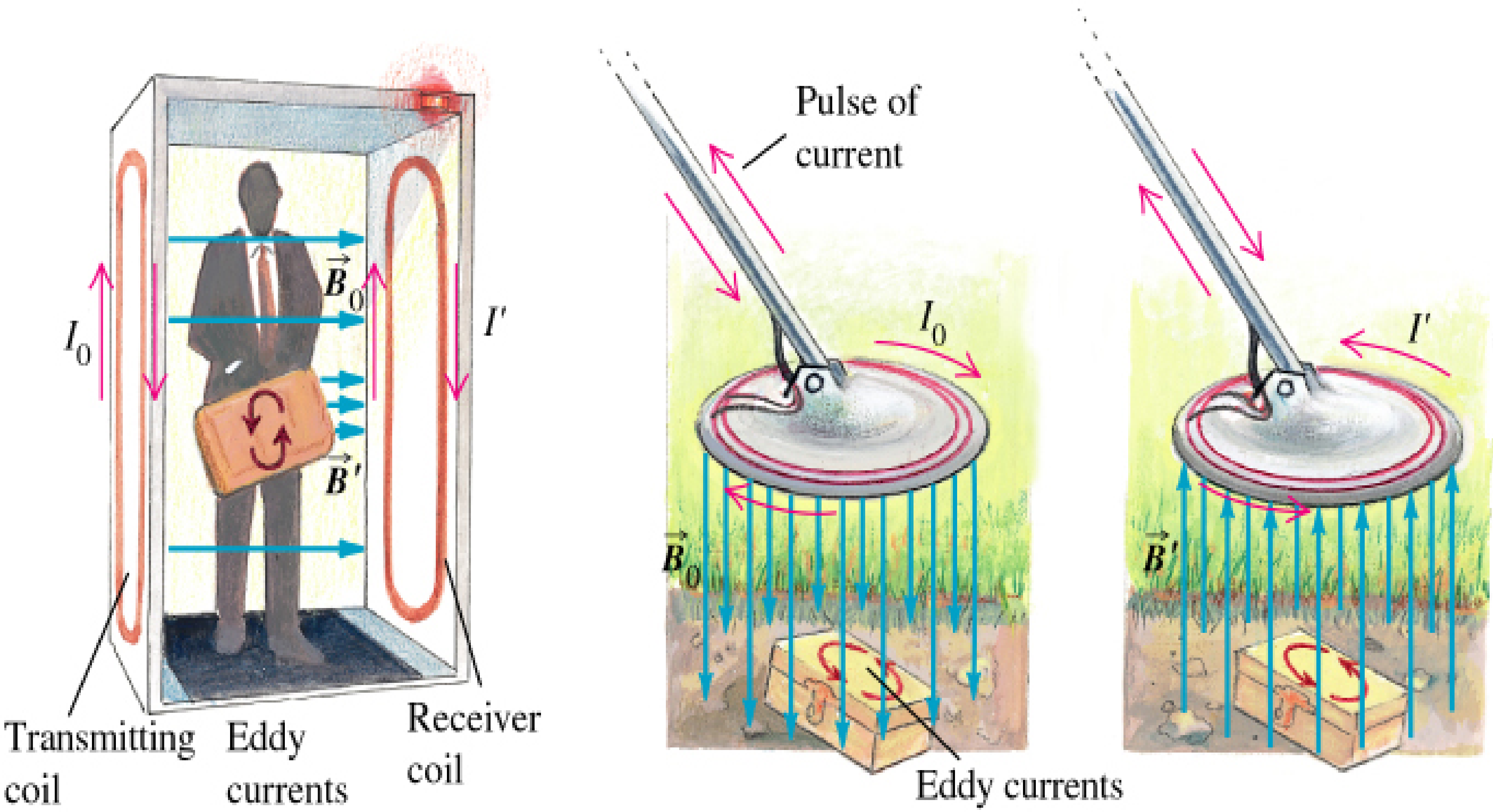
Son útiles para amortiguar oscilaciones mecánicas

Freno de trenes



Detector de metales



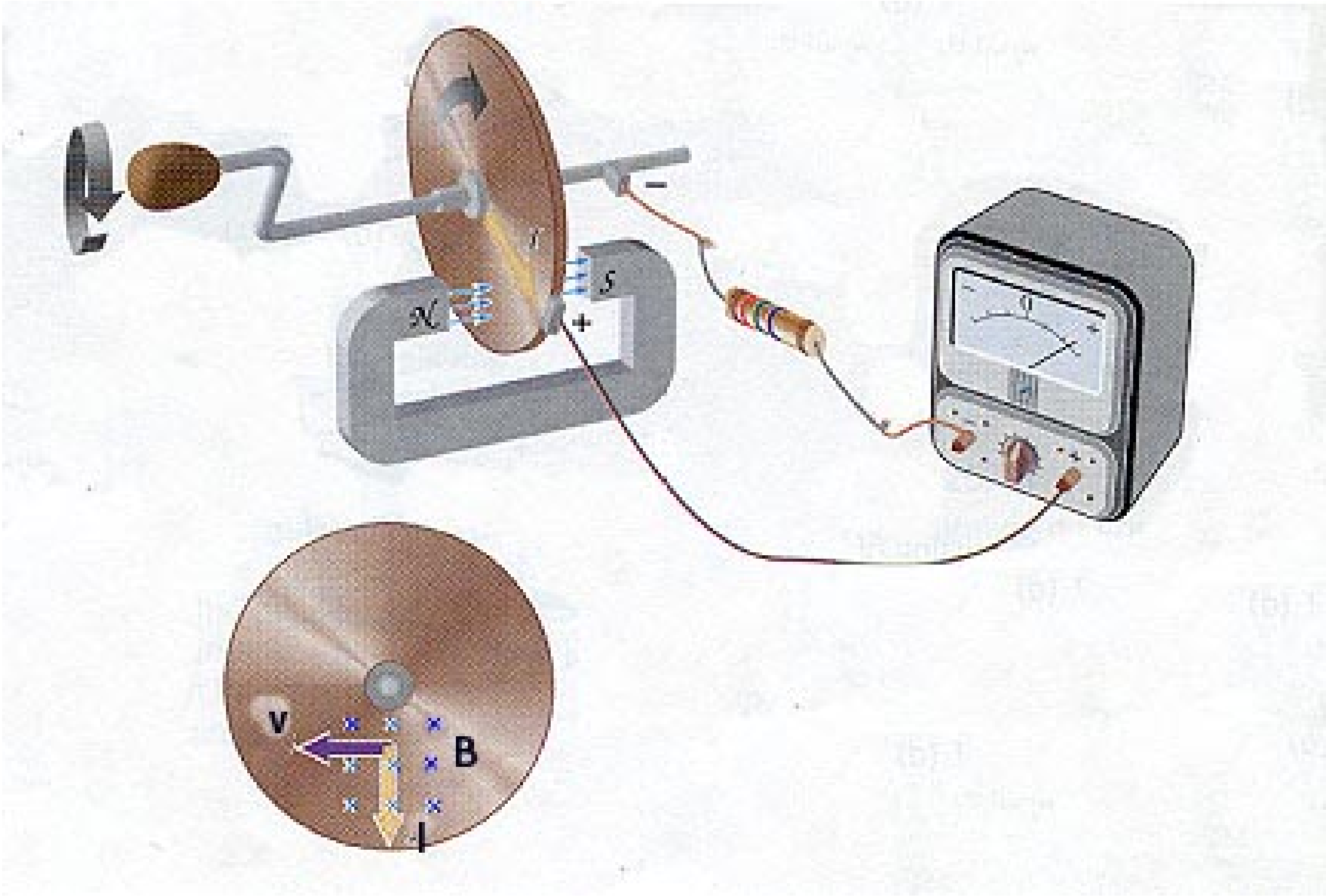


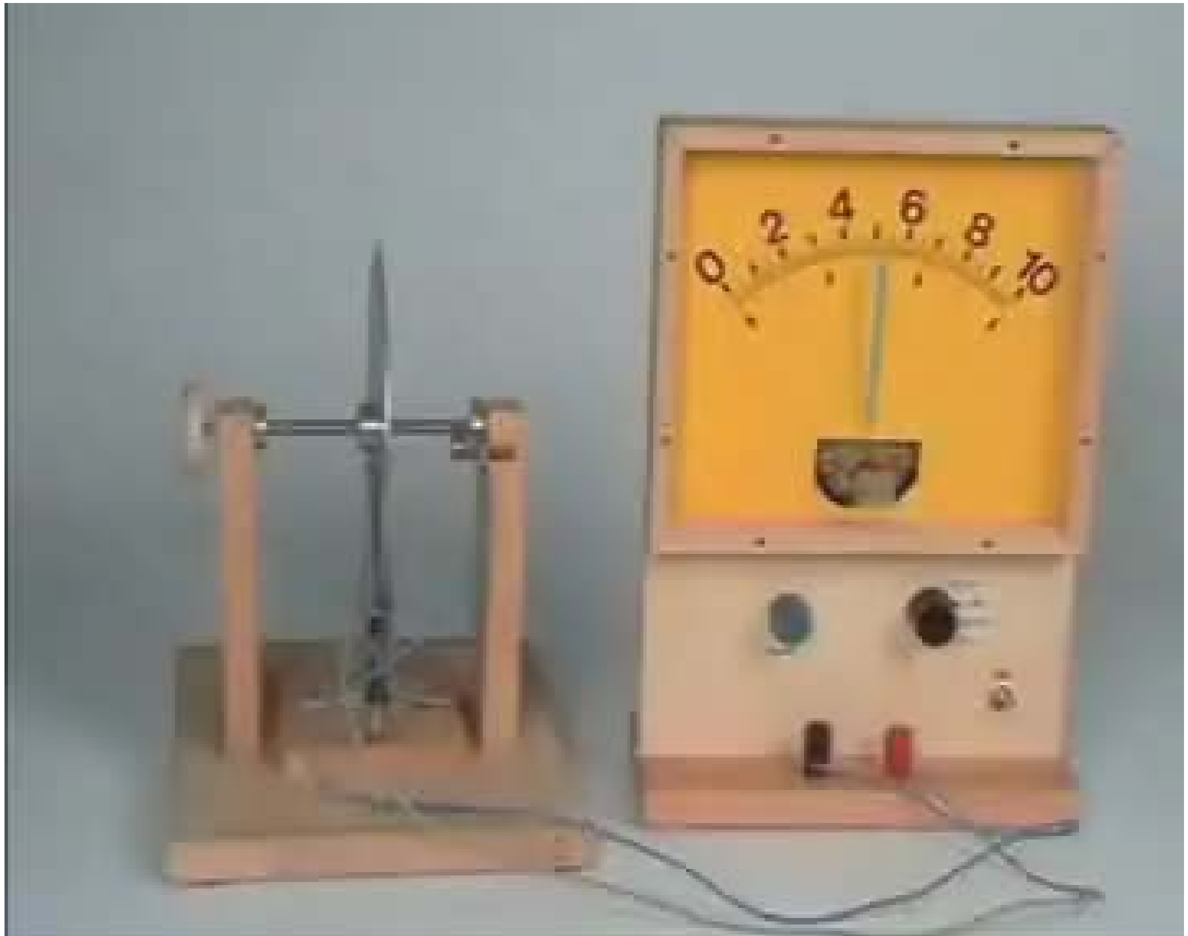
Pulse of current
$\vec{B}_0$
I_0
I'
$\vec{B}'$
Transmitting coil
Eddy currents
Receiver coil
Eddy currents

(a)

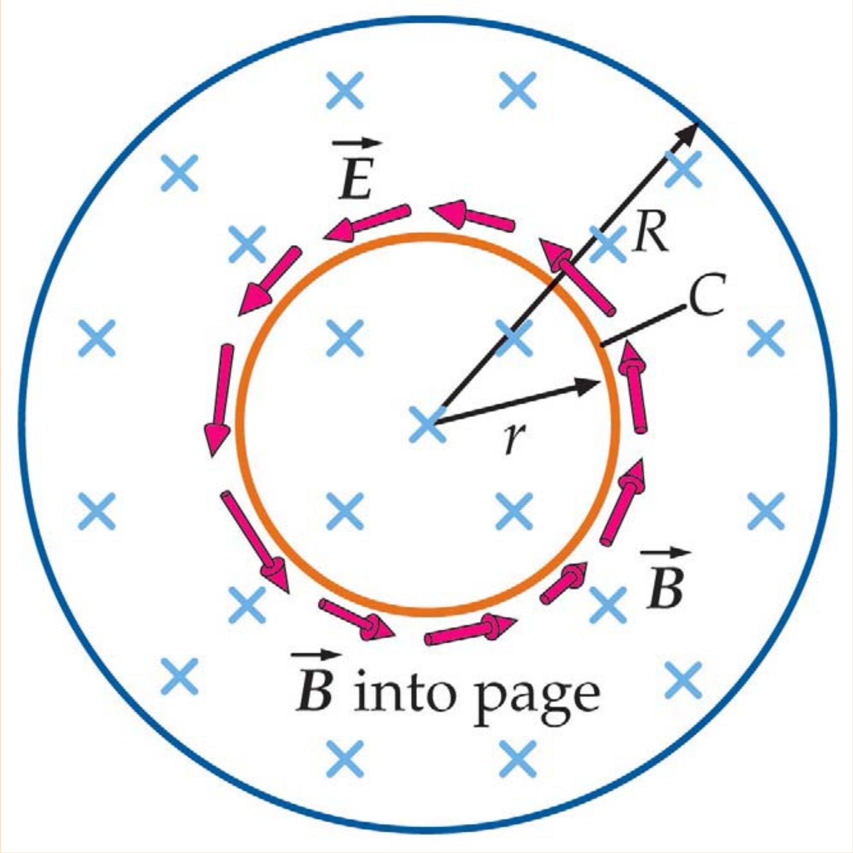
(b)

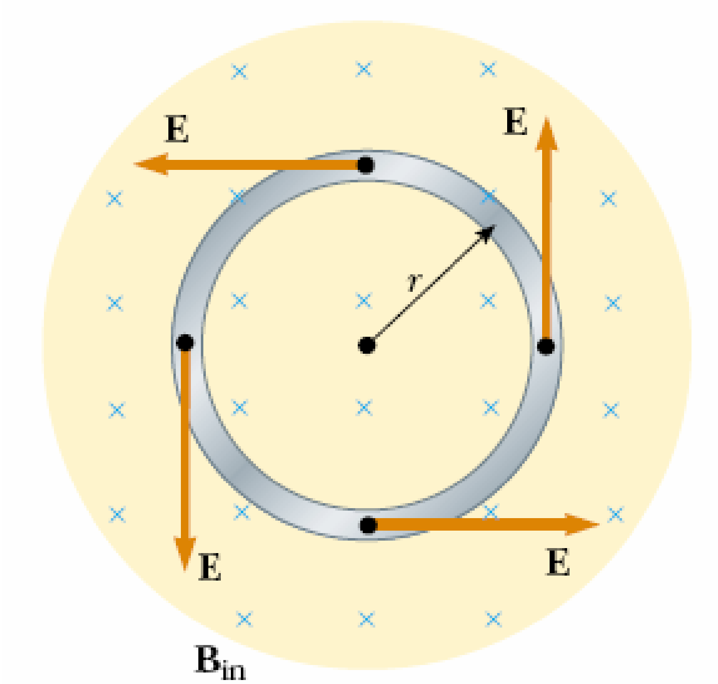
El primer generador




0 2 4 6 8 10

fem inducidas y campos eléctricos





$$\mathcal{E} = -\frac{d\phi_m}{dt}$$

$$\phi_m = \int_S \vec{B} \cdot \hat{n}\, dA = \int_S B_n\, dA$$

$$\mathcal{E} = \oint_C \vec{E}_{nc} \cdot d\vec{\ell} = -\frac{d}{dt}\int_S \vec{B} \cdot \hat{n}\, dA = -\frac{d\phi_m}{dt}$$

El campo eléctrico inducido no es conservativo

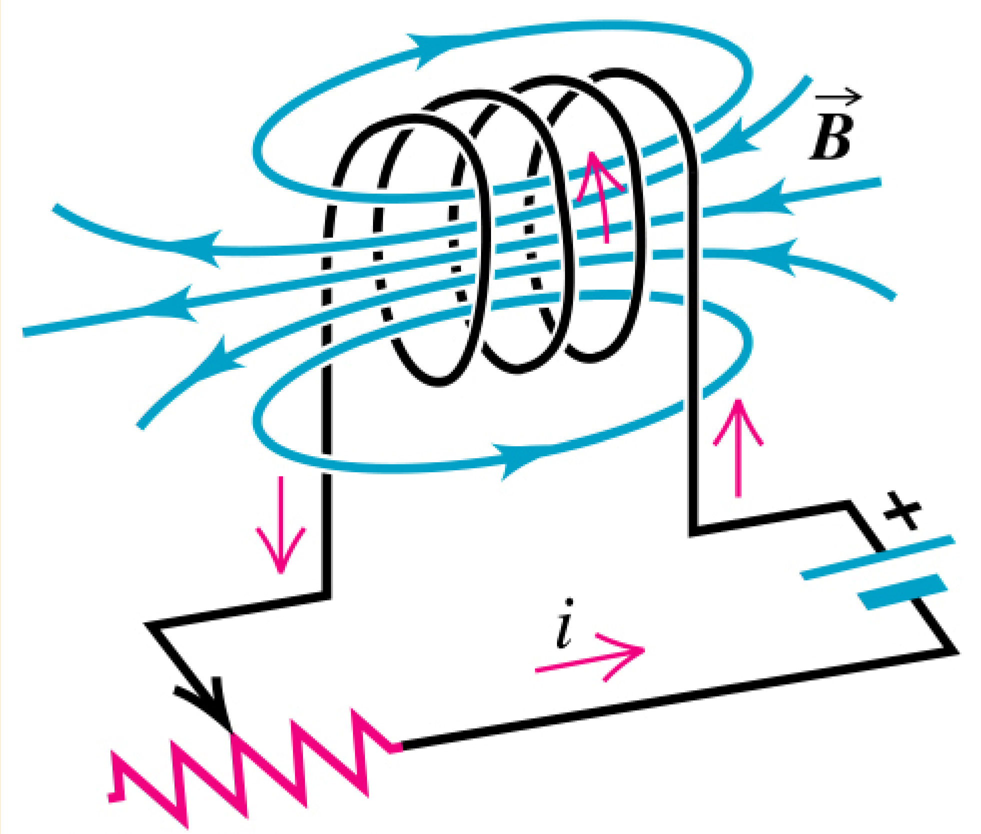
$$\oint_C \vec{E}\cdot d\vec{l} = -\frac{d}{dt}\int_S \vec{B}\cdot d\vec{A}$$

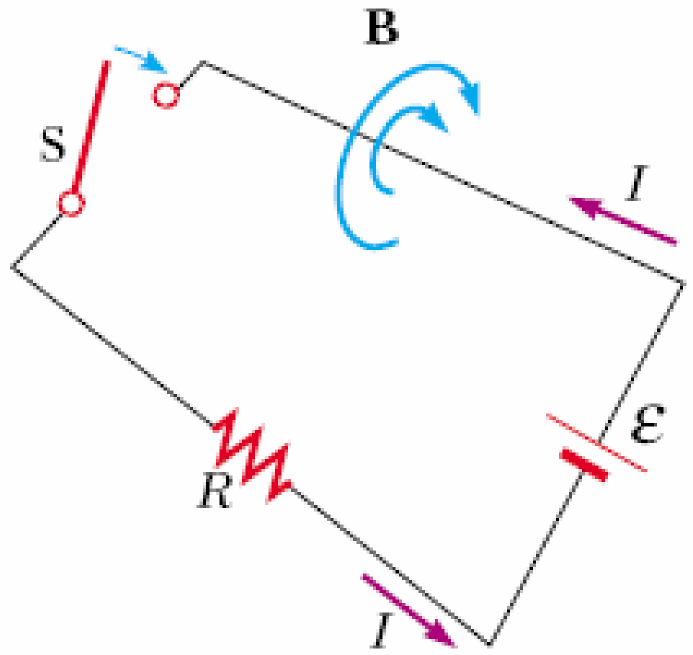
$$\left.\begin{aligned} \oint_L \vec{E}\cdot d\vec{l} &= \int_S \nabla \wedge \vec{E}\cdot d\vec{s} \\ \frac{d}{dt}\int_S \vec{B}\cdot d\vec{s} &= \int_S \frac{\partial \vec{B}}{\partial t}\cdot d\vec{s} \end{aligned}\right\} \rightarrow \nabla \wedge \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\boxed{\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}}$$

La ley de Faraday-Henry en forma diferencial

Autoinducción





S se cierra $\Rightarrow$ I (t)

$\Downarrow$

B(t)

Variación temporal de Φ_B a través del área del circuito

fem autoinducida

$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$

B α I

Φ_B α I

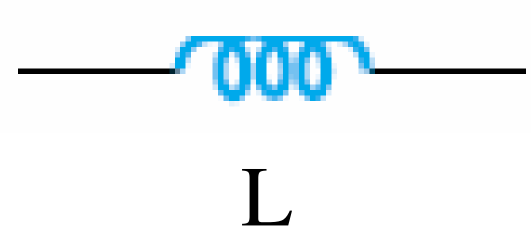
Definimos:

$$\Phi_B = L\ I$$

L: coeficiente de autoinductancia o inductancia $\Rightarrow$

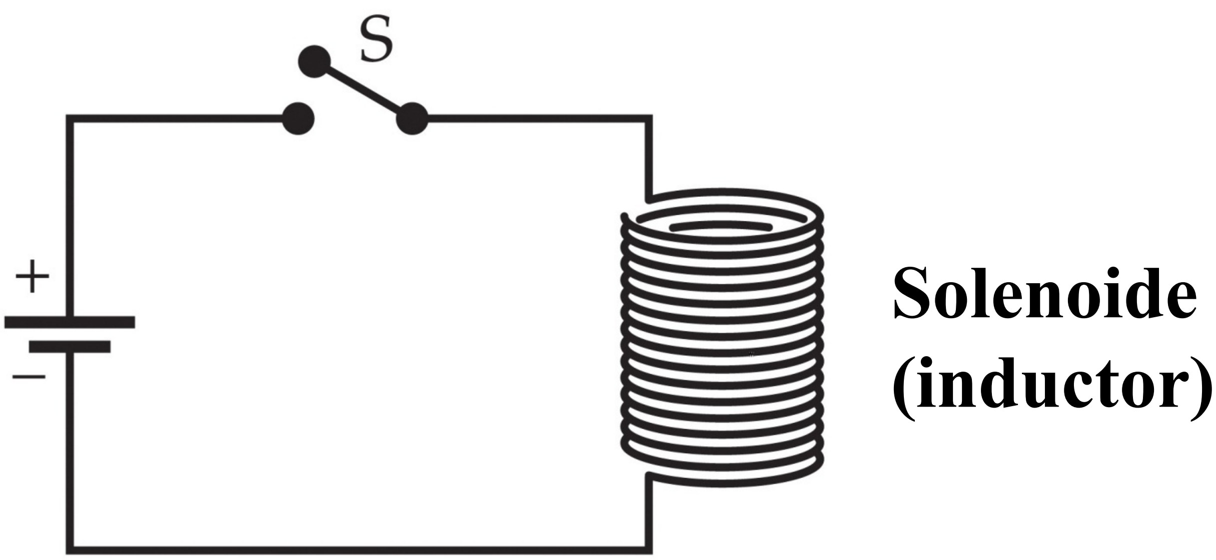
$$\mathcal{E}_L = -L\frac{dI}{dt}$$

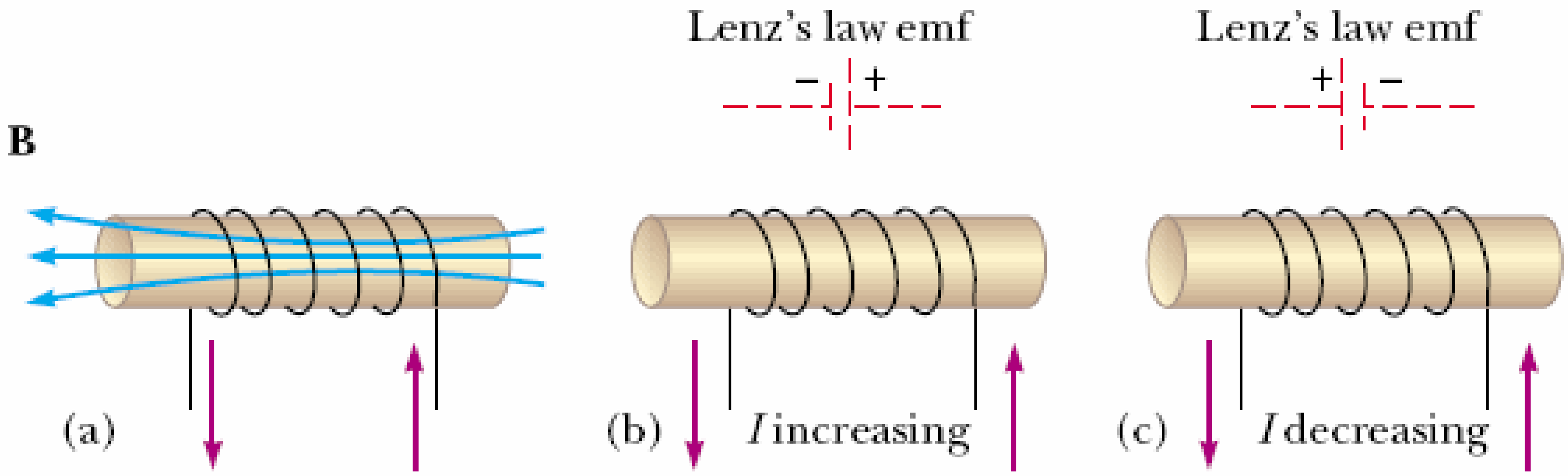
$$L = \Phi_B \ / \ I = -\varepsilon_L \ / \ (dI/dt)$$



$[L] = T\ m^2 / A = Wb / A = V\ s / A = H$ (Henry)

La inductancia de un circuito depende de su geometría





$$\boxed{L = \frac{N\Phi_B}{I}}$$

$$B = \mu_0 n I = \mu_0 \frac{N}{\ell} I$$

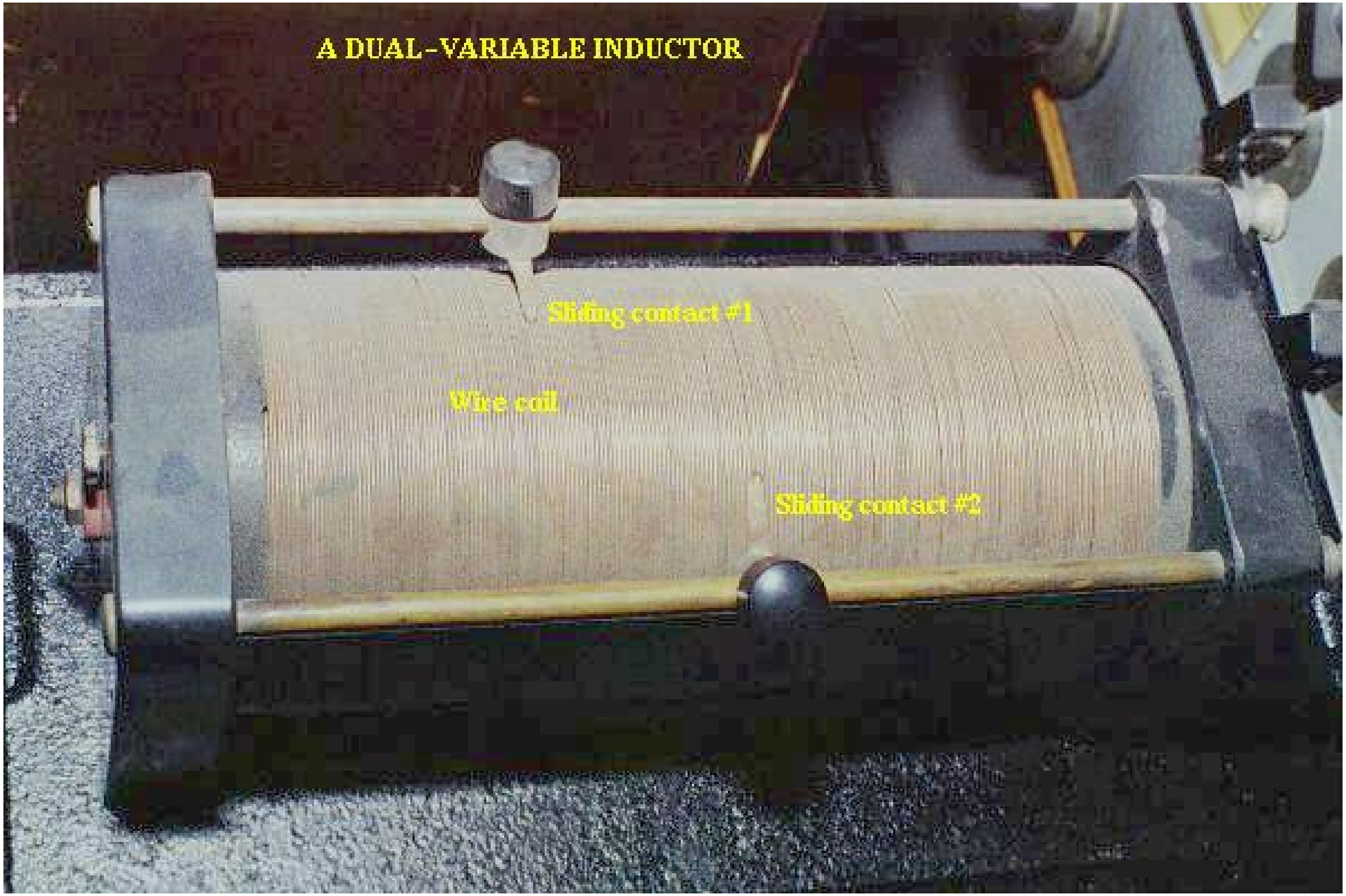
$$n = N/\ell$$

$$\Phi_B = BA = \mu_0 \frac{NA}{\ell} I$$

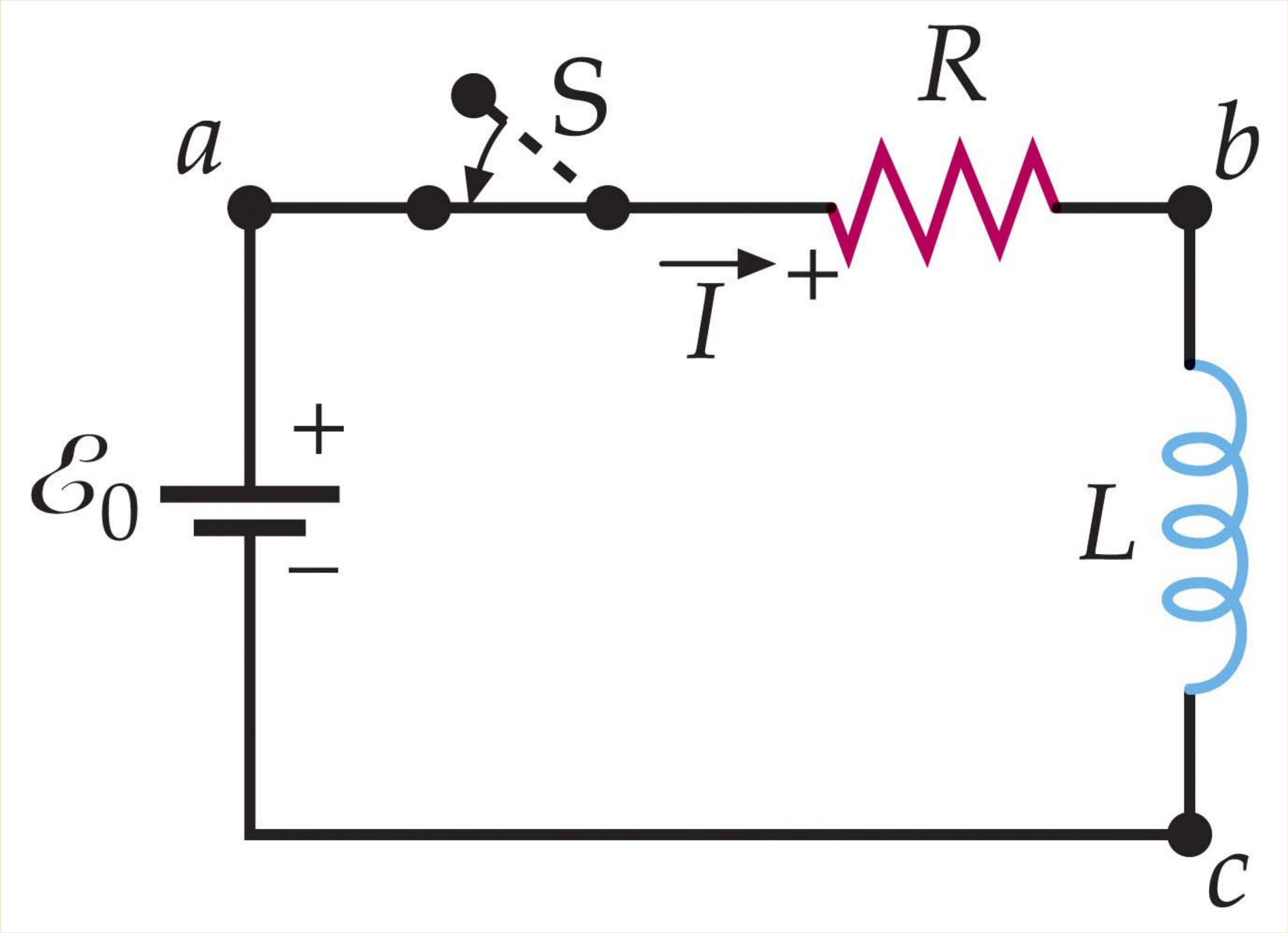
$$L = \frac{N\Phi_B}{I} = \boxed{\frac{\mu_0 N^2 A}{\ell}}$$

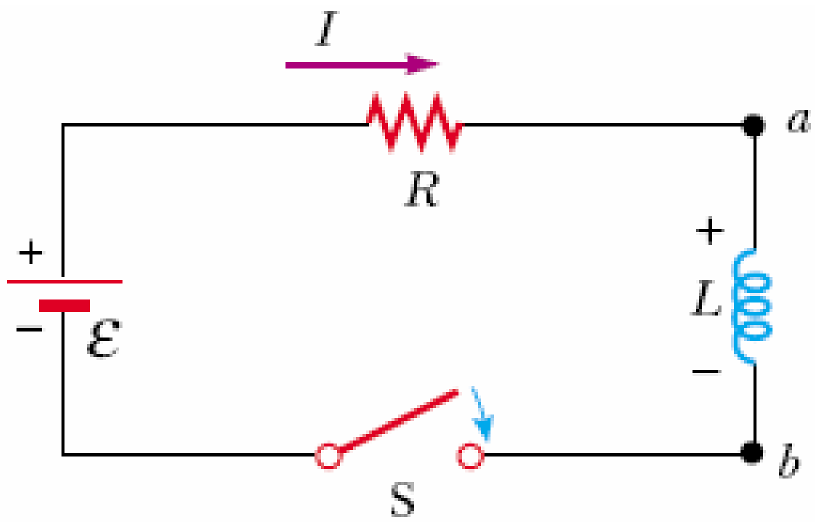
$$L = \mu_0 \frac{(n\ell)^2}{\ell} A = \mu_0 n^2 A\ell = \mu_0 n^2 V$$

$$V = A\ell$$


A DUAL-VARIABLE INDUCTOR
Sliding contact #1
Wire coil
Sliding contact #2

Circuito *RL*





$$\mathcal{E} - IR - L\frac{dI}{dt} = 0$$

$$x = (\mathcal{E}/R) - I, \quad dx = -\,dI.$$

$$x + \frac{L}{R}\frac{dx}{dt} = 0 \qquad \frac{dx}{x} = -\frac{R}{L}\,dt \qquad \int_{x_0}^{x}\frac{dx}{x} = -\frac{R}{L}\int_0^t dt$$

$$\ln\frac{x}{x_0} = -\frac{R}{L}\,t \quad \Longrightarrow \quad x = x_0 e^{-Rt/L}$$

$x_0 = x\,(t=0) = \mathcal{E}/R$

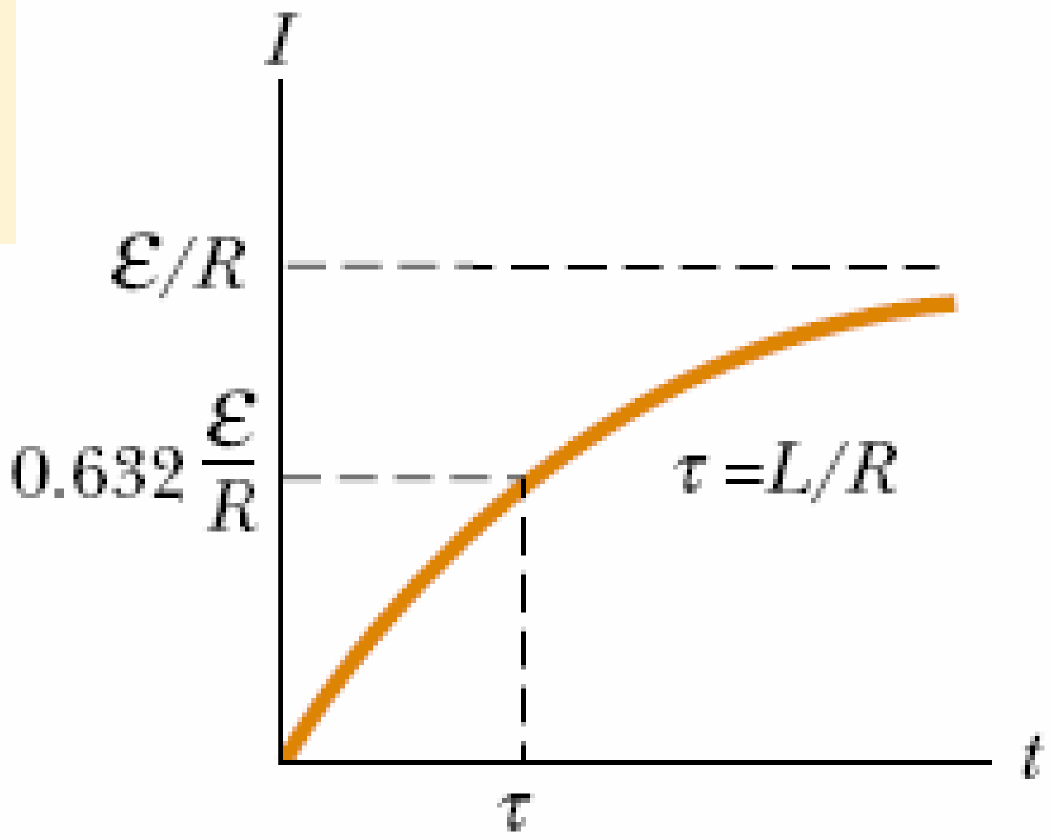
$$\frac{\mathcal{E}}{R} - I = \frac{\mathcal{E}}{R} e^{-Rt/L}$$

$$I = \frac{\mathcal{E}}{R}\left(1 - e^{-Rt/L}\right)$$

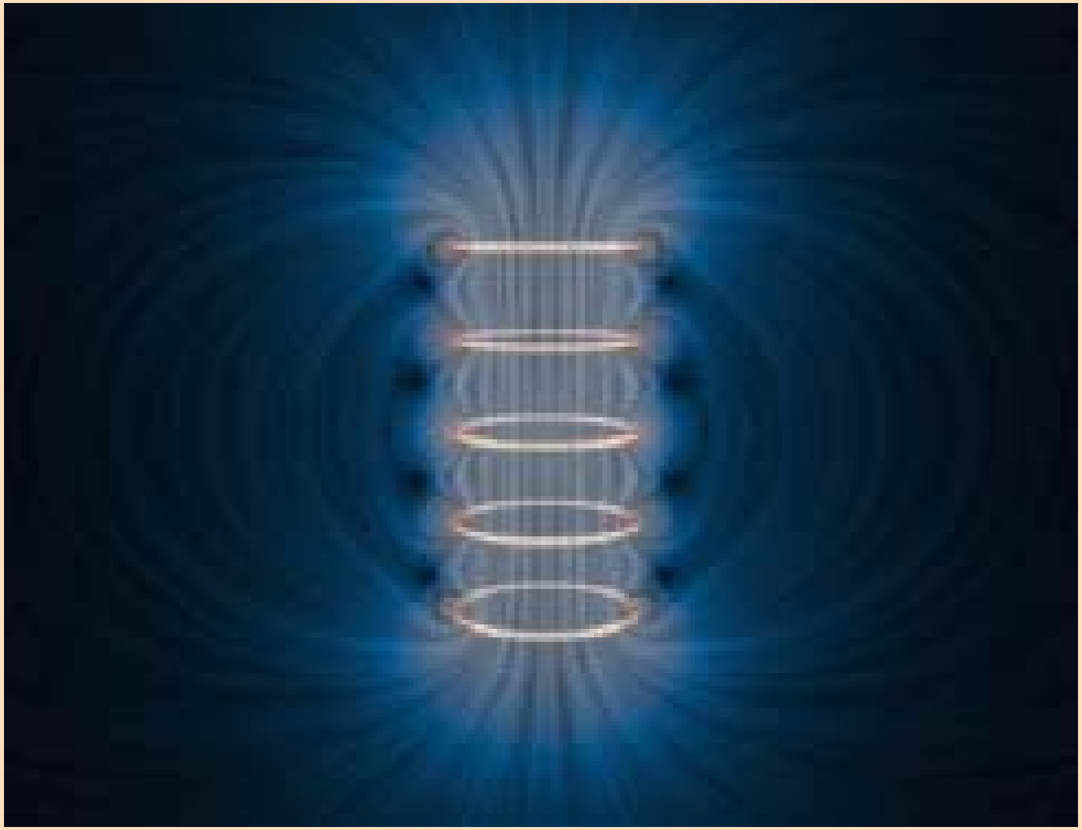
$$I = \frac{\mathcal{E}}{R}\left(1 - e^{-t/\tau}\right)$$

$$\tau = \frac{L}{R}$$

Constante de tiempo del circuito RL



Energía del Campo Magnético



La fem inducida evita que la batería establezca una corriente: la batería efectúa trabajo contra el inductor

Energía suministrada por la batería $\Longrightarrow$ **Calor Joule** + **Energía almacenada en el inductor**

$$\mathcal{E} - IR - L\,\frac{dI}{dt} = 0$$

$$I\mathcal{E} = I^2R + LI\,\frac{dI}{dt}$$

U: energía almacenada en el inductor

$$\frac{dU}{dt} = LI\,\frac{dI}{dt}$$

$$U = \int dU = \int_0^I LI\,dI = L\int_0^I I\,dI$$

$$\boxed{U = \tfrac{1}{2}\,LI^2}$$

$$U = \tfrac{1}{2}C(\Delta V)^2.$$

Análogo eléctrico

Para calcular la densidad de energía almacenada en el campo magnético, consideramos un solenoide:

$$L = \mu_0 n^2 A\ell$$

El campo magnético dentro del solenoide es $B = \mu_0 n I$

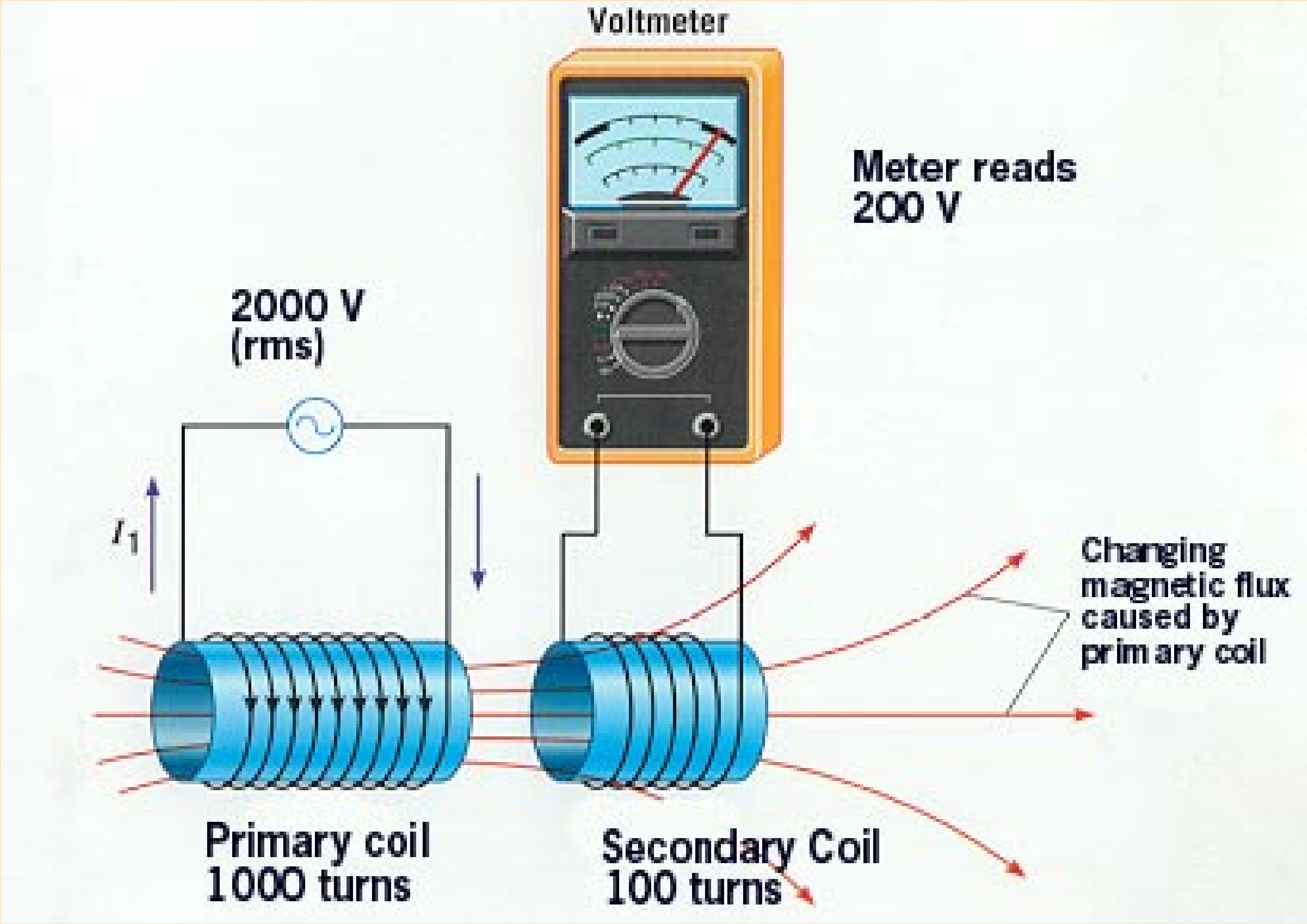
$$U = \tfrac{1}{2} L I^2 = \tfrac{1}{2}\,\mu_0 n^2 A\ell \left(\frac{B}{\mu_0 n}\right)^2 = \frac{B^2}{2\mu_0}\,A\ell$$

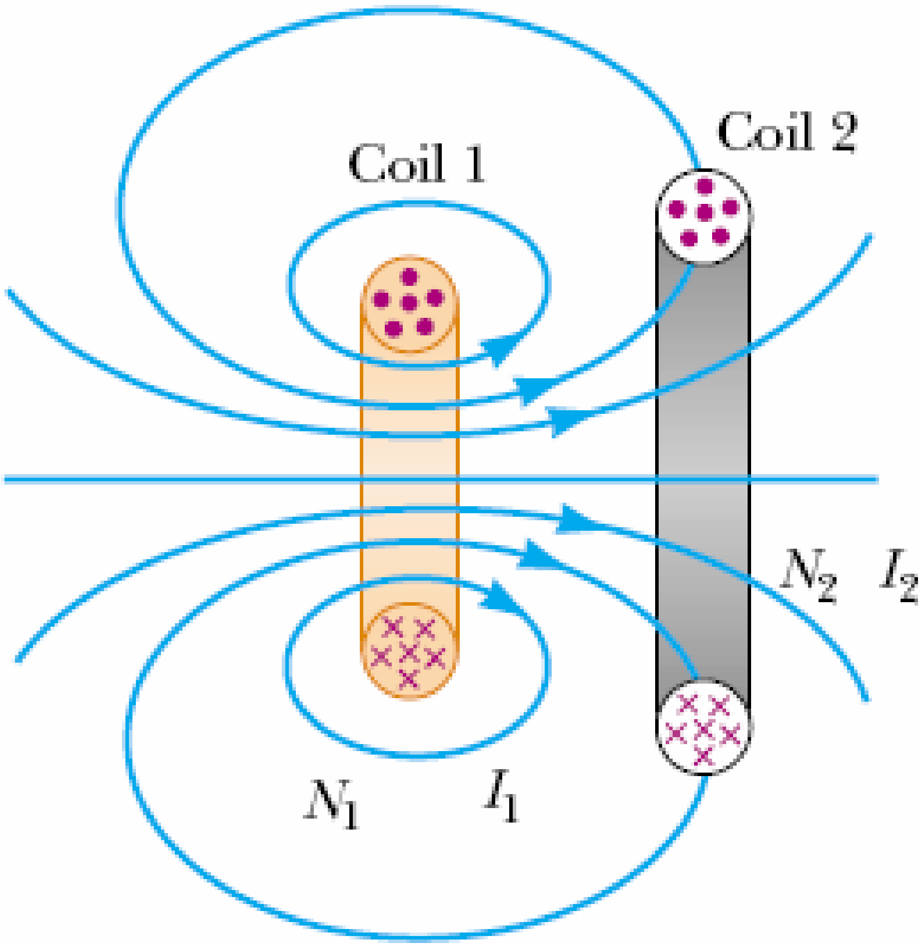
$A\ell$ **volumen del solenoide donde se encuentra confinado B**

$$u_B = \frac{U}{A\ell} = \frac{B^2}{2\mu_0}$$

Es válida para cualquier región del espacio donde haya B

Inductancia mutua





Φ_{12}

Flujo magnético a través de la bobina 2 producido por la bobina 1.

Definimos inductancia mutua de la bobina 2 respecto a la 1

$$M_{12} \equiv \frac{N_2 \Phi_{12}}{I_1}$$

M12 depende de la geometría de ambos circuitos y de la orientación de uno respecto del otro.

La fem inducida en la bobina 2 es:

$$\varepsilon_2 = -N_2 \frac{d\Phi_{12}}{dt} = -N_2 \frac{d}{dt}\left(\frac{M_{12} I_1}{N_2}\right) = -M_{12} \frac{dI_1}{dt}$$

De la misma manera:

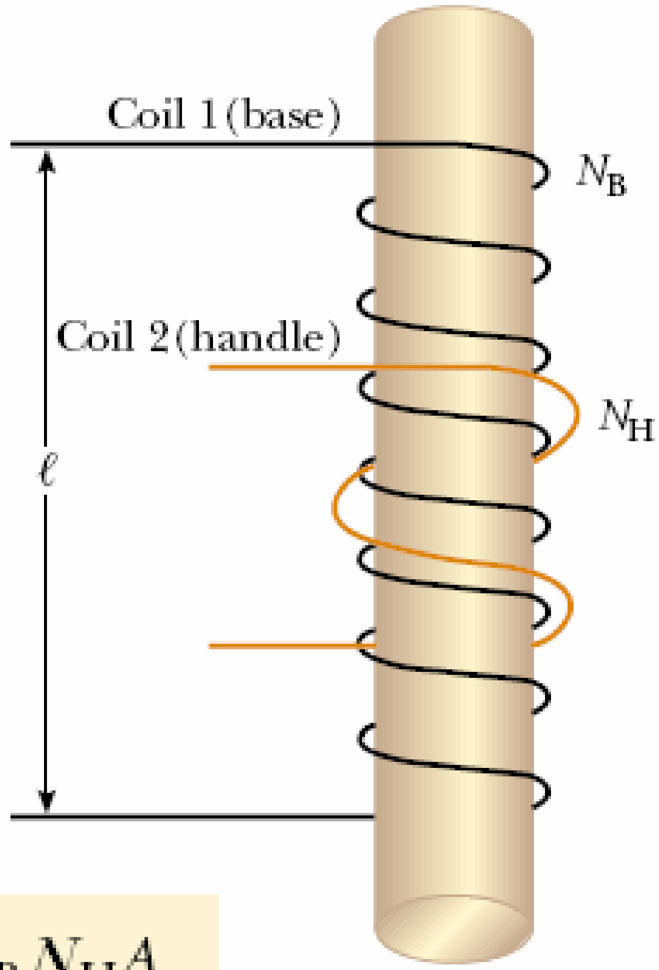
$$\varepsilon_1 = -M_{21} \frac{dI_2}{dt}$$

Se puede demostrar que $M_{12} = M_{21} = M$

$$\varepsilon_2 = -M \frac{dI_1}{dt} \qquad \varepsilon_1 = -M \frac{dI_2}{dt}$$

[M] = Hy

Ejemplo:

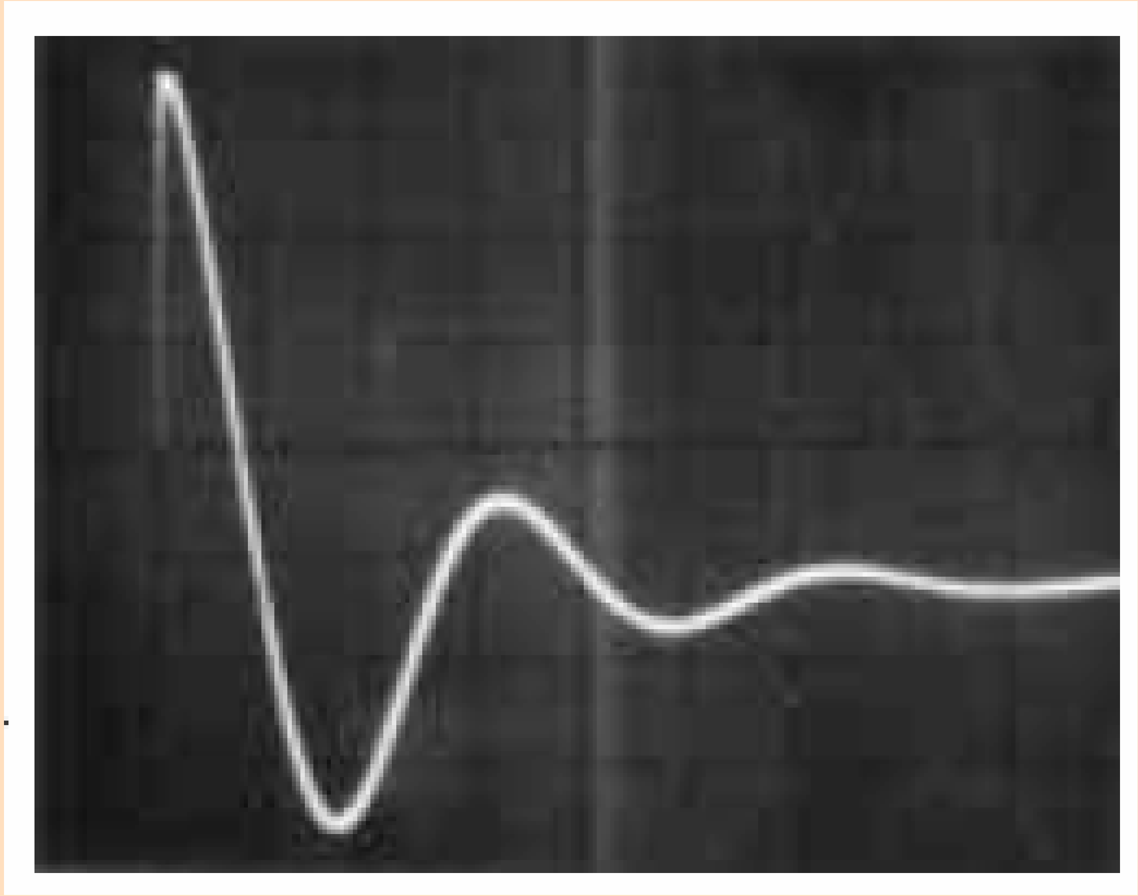


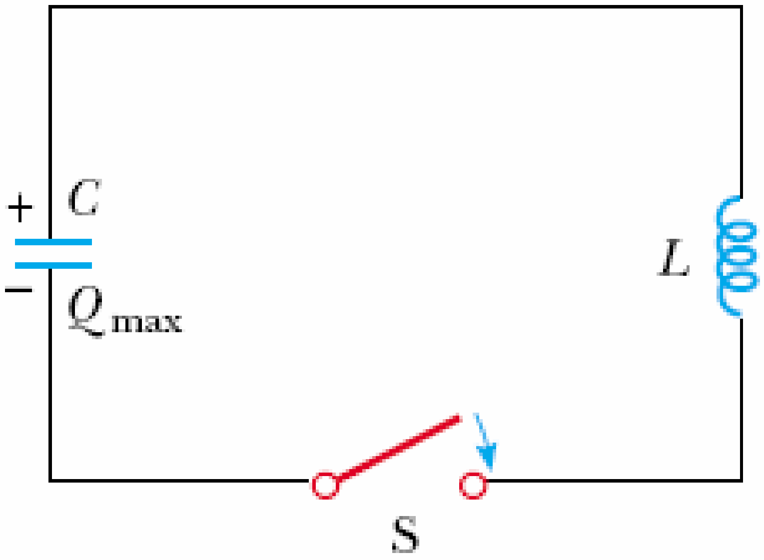
$$B = \frac{\mu_0 N_B I}{\ell}$$

$$M = \frac{N_H \Phi_{BH}}{I} = \frac{N_H B A}{I} = \boxed{\mu_0 \frac{N_B N_H A}{\ell}}$$

Usualmente M se determina experimentalmente.

Oscilaciones eléctricas
Circuitos LC y RLC





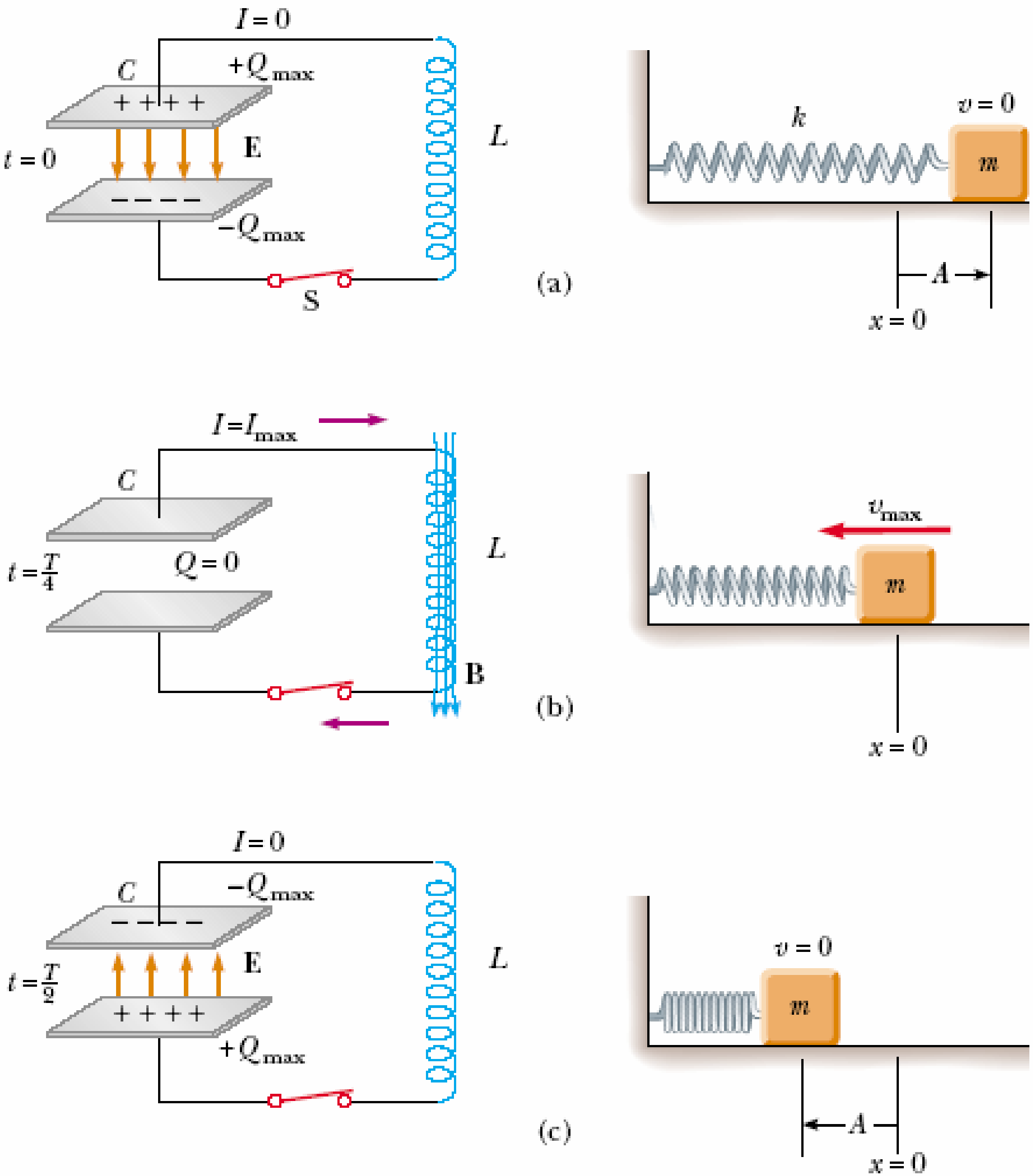
$$U = U_C + U_L = \frac{Q^2}{2C} + \tfrac{1}{2} L I^2$$

$$\frac{dU}{dt} = \frac{d}{dt}\left(\frac{Q^2}{2C} + \tfrac{1}{2} L I^2\right) = \frac{Q}{C}\frac{dQ}{dt} + LI\frac{dI}{dt} = 0$$

$I = dQ/dt.$ $\longrightarrow$ $\dfrac{Q}{C} + L\dfrac{d^2Q}{dt^2} = 0$

$$\frac{d^2Q}{dt^2} = -\frac{1}{LC}Q$$

Ecuación diferencial del oscilador armónico



$$\frac{d^2x}{dt^2} = -\frac{k}{m}x = -\omega^2 x$$

$$\omega = \sqrt{k/m}.$$

$$x = A\cos(\omega t + \phi)$$

$$Q = Q_{\max}\cos(\omega t + \phi)$$

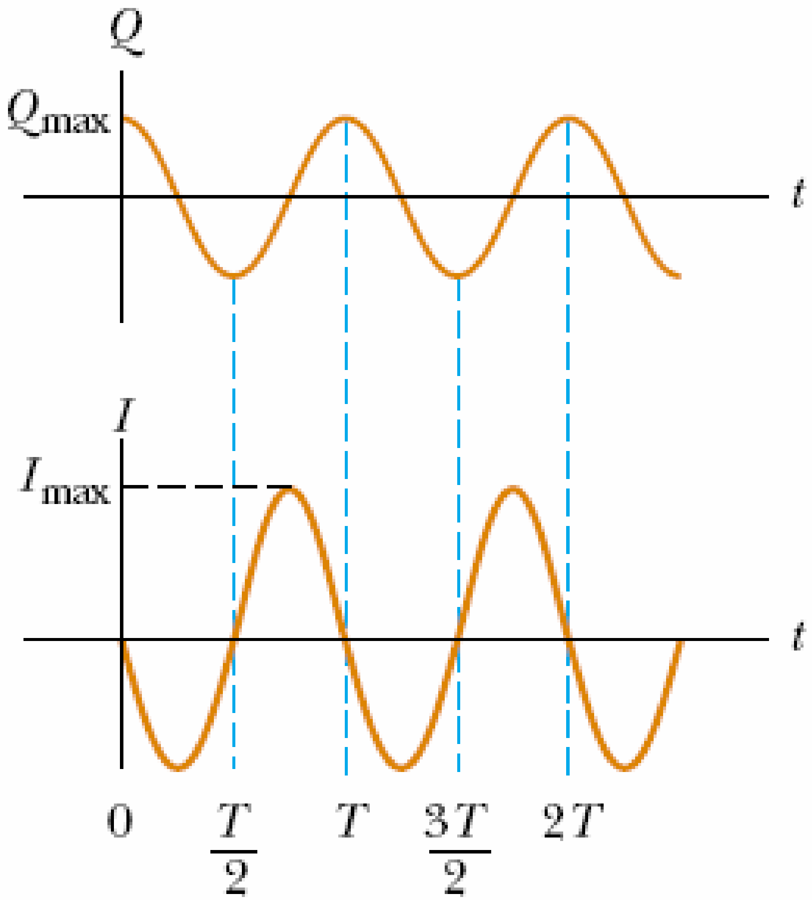
$$\boxed{\omega = \frac{1}{\sqrt{LC}}}$$

$$Q = Q_{\max} \cos(\omega t + \phi) \qquad I = \frac{dQ}{dt} = -\omega Q_{\max} \sin(\omega t + \phi)$$

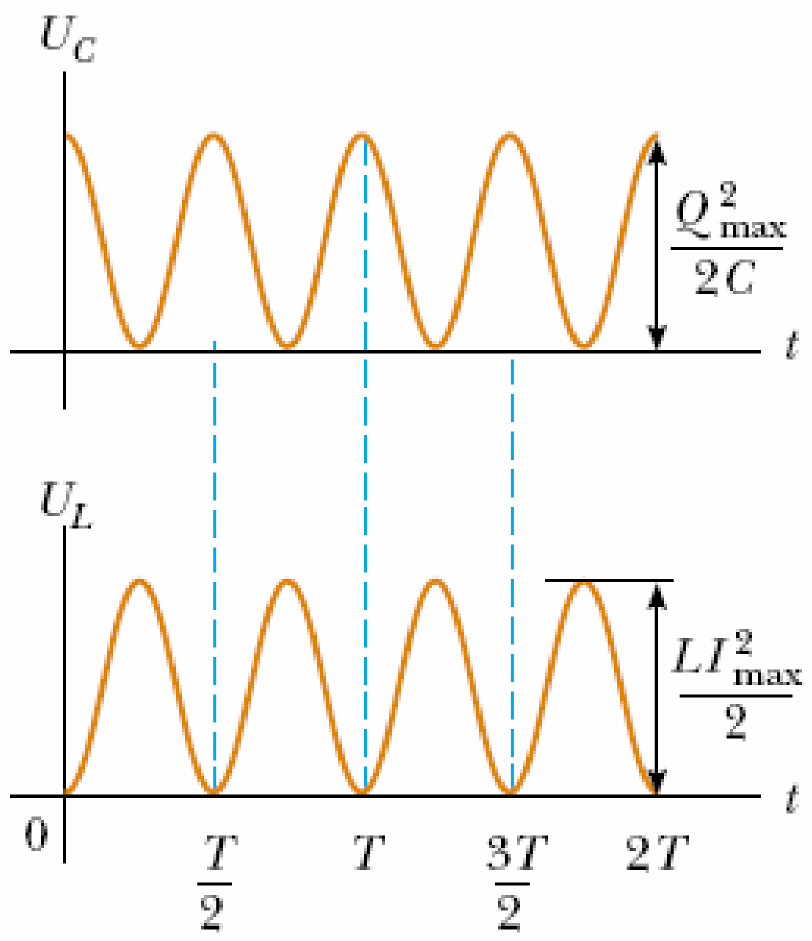
$$I = 0 \text{ at } t = 0 \qquad 0 = -\omega Q_{\max} \sin \phi$$

$$Q = Q_{\max} \cos \omega t$$

$$I = -\omega Q_{\max} \sin \omega t = -I_{\max} \sin \omega t$$



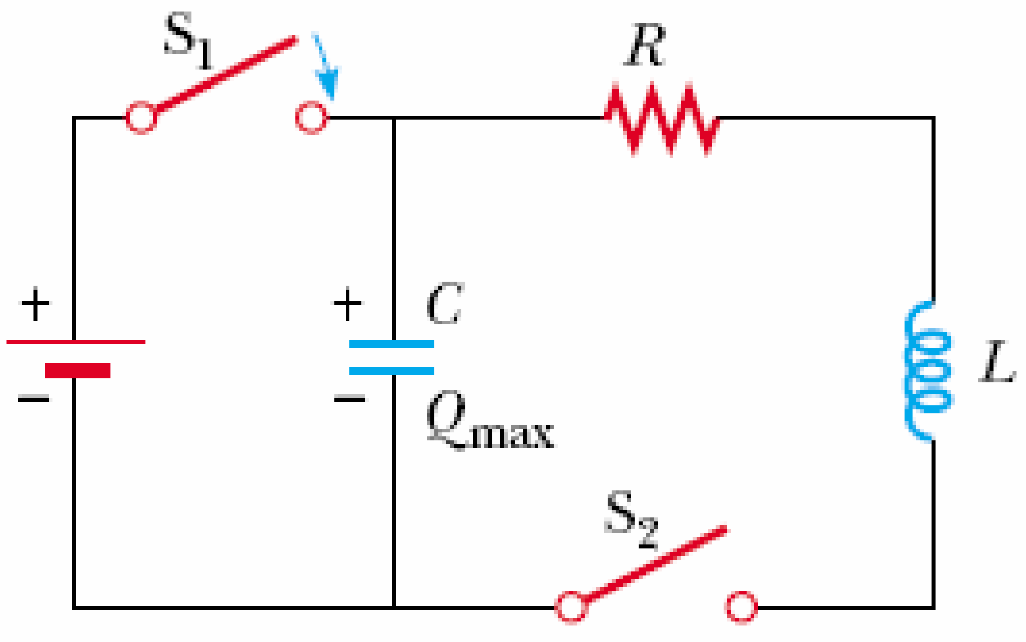
$$U = U_C + U_L = \frac{Q_{max}^2}{2C}\cos^2 \omega t + \frac{1}{2} L I_{max}^2 \sin^2 \omega t$$



$$\frac{Q_{max}^2}{2C} = \frac{L I_{max}^2}{2}$$

$$U = \frac{Q_{max}^2}{2C}(\cos^2 \omega t + \sin^2 \omega t) = \frac{Q_{max}^2}{2C}$$

Que ocurre si agrego R??



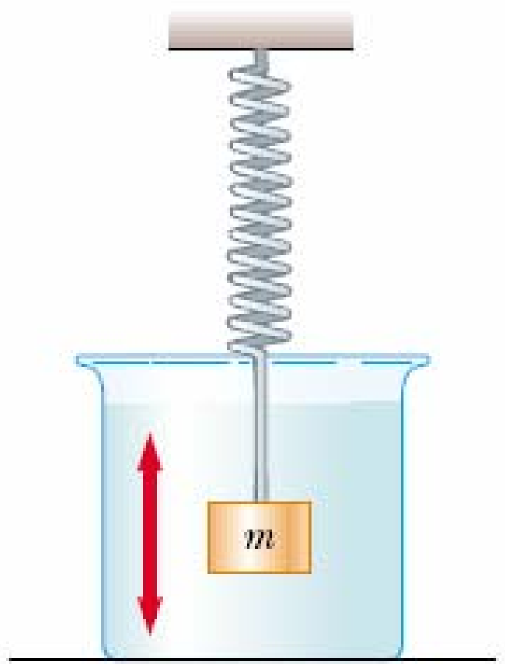
$$\frac{dU}{dt} = -I^2R$$

$$LI\,\frac{dI}{dt} + \frac{Q}{C}\,\frac{dQ}{dt} = -I^2R$$

$$LI\,\frac{d^2Q}{dt^2} + I^2R + \frac{Q}{C}\,I = 0$$

$$L\,\frac{d^2Q}{dt^2} + IR + \frac{Q}{C} = 0$$

$$L\,\frac{d^2Q}{dt^2} + R\,\frac{dQ}{dt} + \frac{Q}{C} = 0$$



$$L\,\frac{d^2Q}{dt^2} + R\,\frac{dQ}{dt} + \frac{Q}{C} = 0 \;\leftrightarrow\; m\,\frac{d^2x}{dt^2} + b\,\frac{dx}{dt} + kx = 0$$

$$\begin{array}{ccc} Q & \leftrightarrow & x \\ I & \leftrightarrow & v_x \\ \Delta V & \leftrightarrow & F_x \\ R & \leftrightarrow & b \\ & & \\ C & \leftrightarrow & 1/k \\ L & \leftrightarrow & m \end{array}$$

$$m\,\frac{d^2x}{dt^2} + b\,\frac{dx}{dt} + kx = 0$$

$$\boxed{Q = Q_{\max}\, e^{-Rt/2L} \cos \omega_d t}$$

donde

$$\omega_d = \left[\frac{1}{LC} - \left(\frac{R}{2L}\right)^2\right]^{1/2}$$

$$I = \frac{dQ}{dt} \;\leftrightarrow\; v_x = \frac{dx}{dt}$$

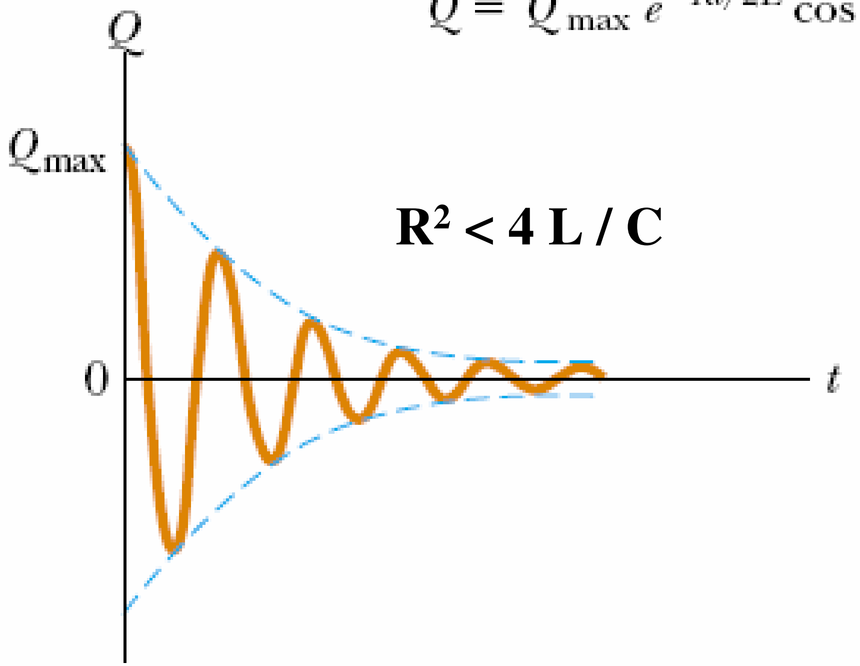
$$\frac{dI}{dt} = \frac{d^2Q}{dt^2} \;\leftrightarrow\; a_x = \frac{dv_x}{dt} = \frac{d^2x}{dt^2}$$

$$U_L = \tfrac{1}{2}\,LI^2 \;\leftrightarrow\; K = \tfrac{1}{2}\,mv^2$$

$$U_C = \tfrac{1}{2}\,\frac{Q^2}{C} \;\leftrightarrow\; U = \tfrac{1}{2}\,kx^2$$

$$I^2R \;\leftrightarrow\; b\,v^2$$

$$Q = Q_{max}\, e^{-Rt/2L} \cos \omega_d t \qquad \omega_d = \left[\frac{1}{LC} - \left(\frac{R}{2L}\right)^2\right]^{1/2}$$

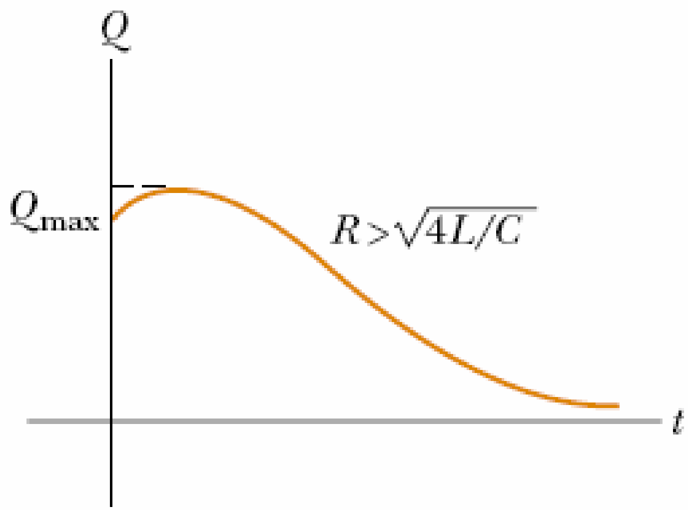


$R^2 < 4\,L\,/\,C$

El sistema esta subamortiguado

$R^2 = 4\,L\,/\,C$

amortiguamiento crítico



$R^2 > 4\,L\,/\,C$

sobreamortiguado