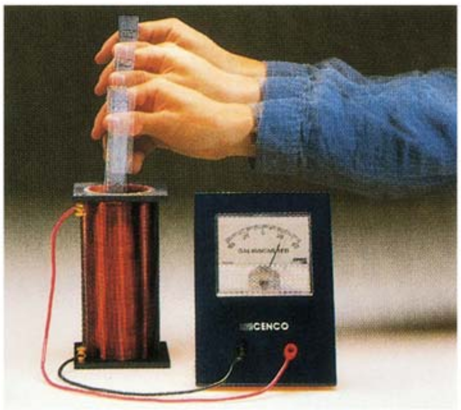
Capítulo 5

Campos electromagnéticos dependientes del tiempo



$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_o}$$

$$\vec{\nabla} \times \vec{E} = 0$$

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{B} = \mu_0 \vec{J}$$

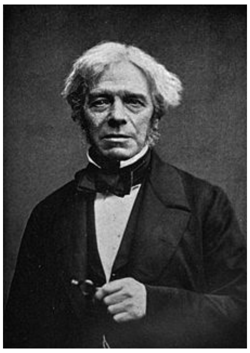
Ecuaciones de Maxwell para campos estáticos

Electricidad

Magnetismo

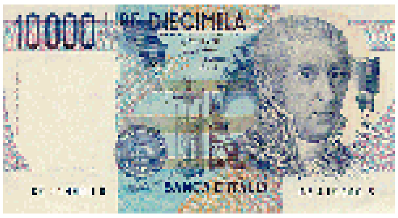
$\Longrightarrow$ **dos fenómenos independientes**

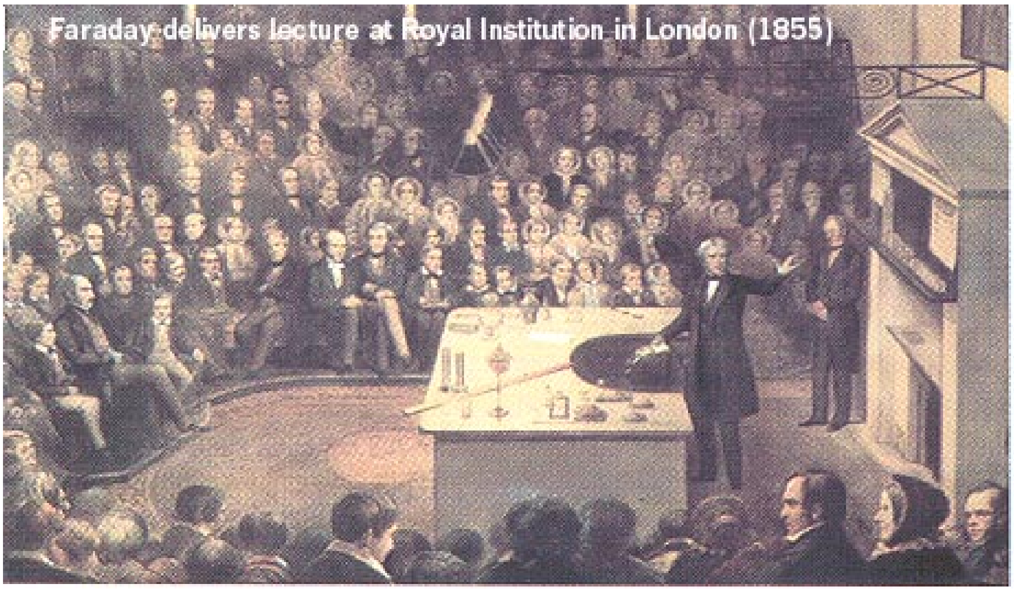
Ley de Faraday-Henry

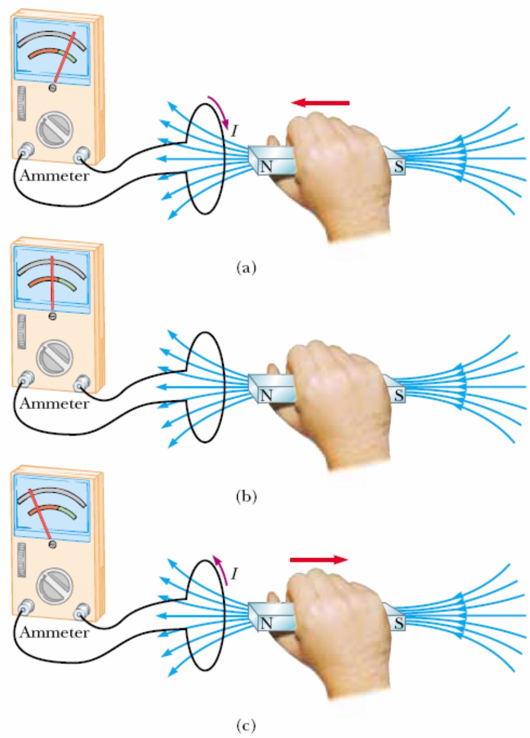


Michael Faraday (1791-1867)

Joseph Henry (1797-1878)



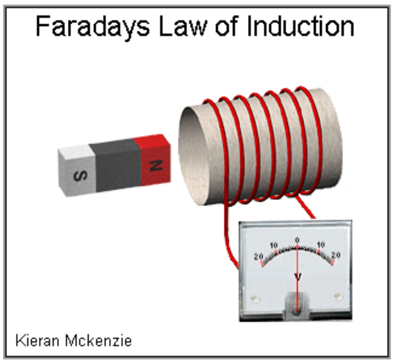




(a)

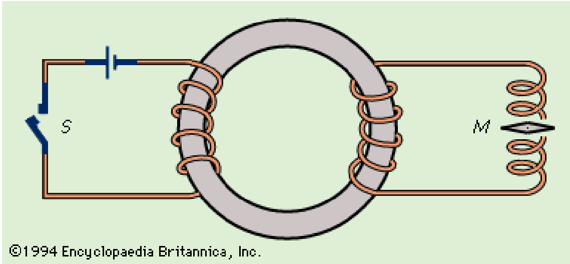
(b)

(c)

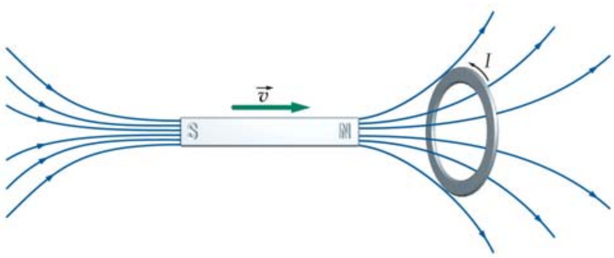


Una corriente eléctrica puede producirse mediante un campo magnético variable

Experimento de Faraday

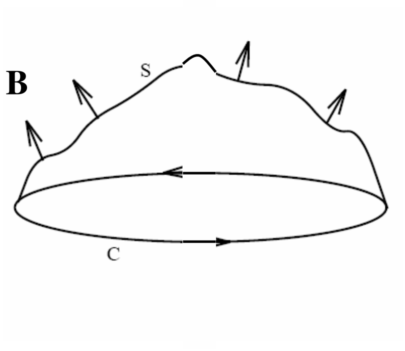


Un campo magnético variable induce una fem en el circuito secundario



$$\varepsilon = -\frac{d\Phi_B}{dt}$$

Enunciado general de la ley de Faraday-Henry



$$\mathcal{E} = -\frac{d\phi_{\mathrm{m}}}{dt}$$

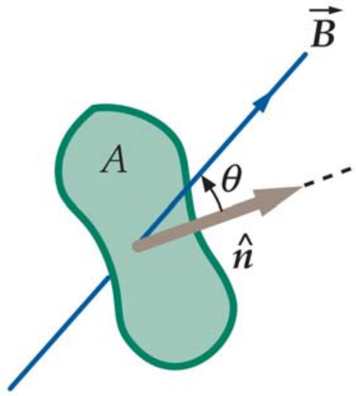
$$\phi_{\mathrm{m}} = \int_S \vec{B} \cdot \hat{n}\, dA = \int_S B_n\, dA$$

$$\boxed{\mathcal{E} = \oint_C \vec{E}_{\mathrm{nc}} \cdot d\vec{\ell} = -\frac{d}{dt}\int_S \vec{B} \cdot \hat{n}\, dA = -\frac{d\phi_{\mathrm{m}}}{dt}}$$

En una bobina con N vueltas, todas de la misma área

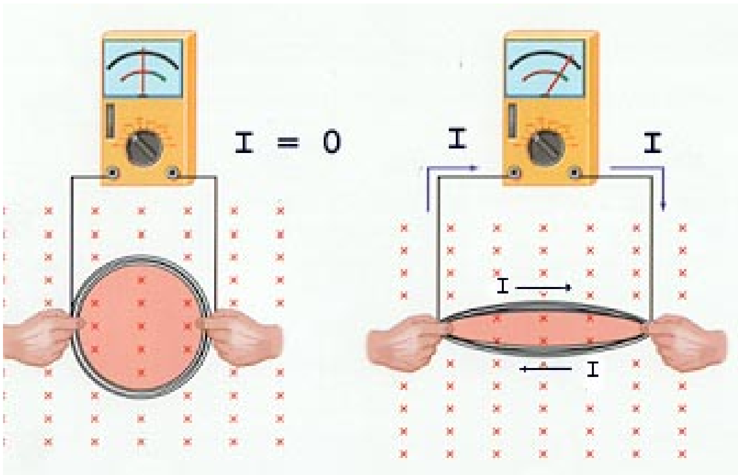
$$\mathcal{E} = -N\frac{d\Phi_B}{dt}$$

$\Phi_B(t)$?

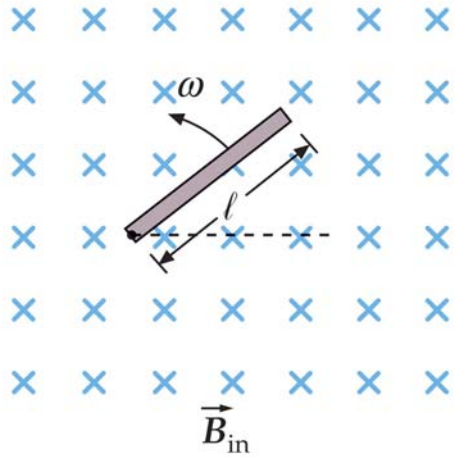


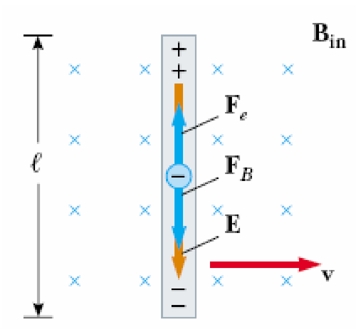
$$\mathcal{E} = -\frac{d}{dt}(BA\cos\theta)$$

- B(t)
- A(t)
- θ(t)



fem de movimiento



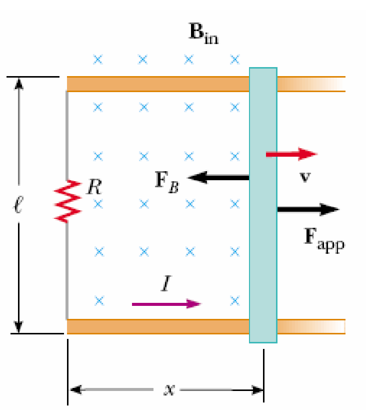


$$qE = qvB$$

$$E = vB$$

$$\Delta V = E\ell = B\ell v$$

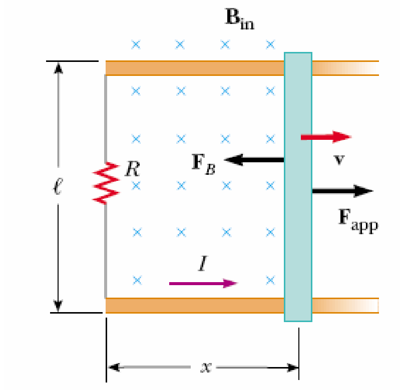
Si se invierte v, se invierte la polaridad de V

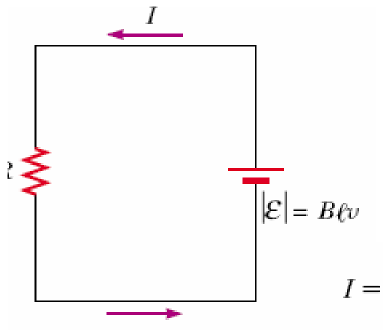


$$\Phi_B = B\ell x$$

$$\mathcal{E} = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}(B\ell x) = -B\ell\frac{dx}{dt}$$

$$\mathcal{E} = -B\ell v$$





$$I = \frac{|\mathcal{E}|}{R} = \frac{B\ell v}{R}$$

Consideración energética: si v = cte $\longrightarrow$ $F_{app} = I\ell B$

La potencia entregada por la fuerza aplicada es:

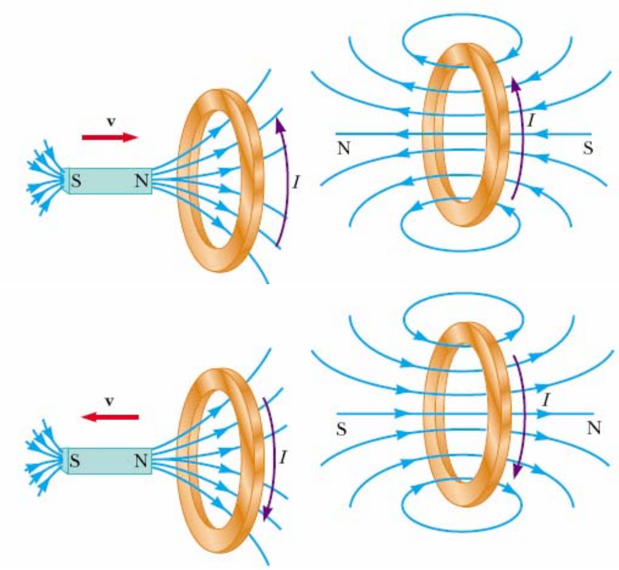
$$\mathcal{P} = F_{app} v = (I\ell B) v = \frac{B^2 \ell^2 v^2}{R} = \frac{\mathcal{E}^2}{R}$$

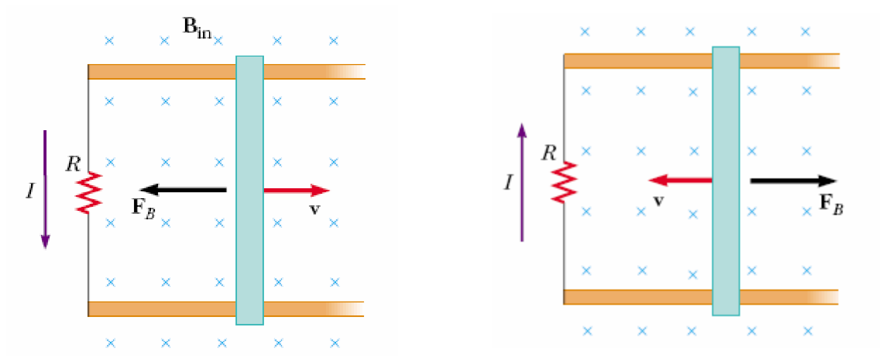
Energía Mecánica → **Energía Eléctrica** → **Energía Térmica**

Ley de Lenz

Heinrich Lenz (1804-1865)

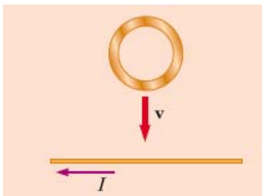
La polaridad de una fem inducida es tal que tiende a producir una corriente eléctrica que creara un flujo magnético que se opone al cambio de Φ_B a través del lazo.



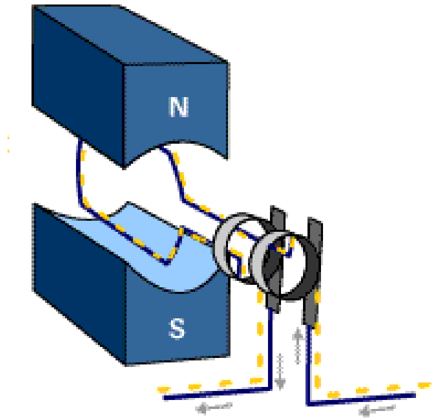


Qué sucede si el sentido de I es contrario al obtenido utilizando la ley de Lenz en los dos casos anteriores ?

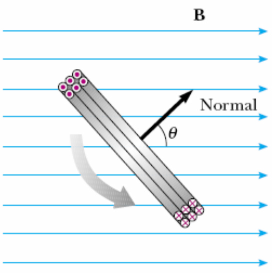
Determinar el sentido de la corriente inducida en la espira circular.

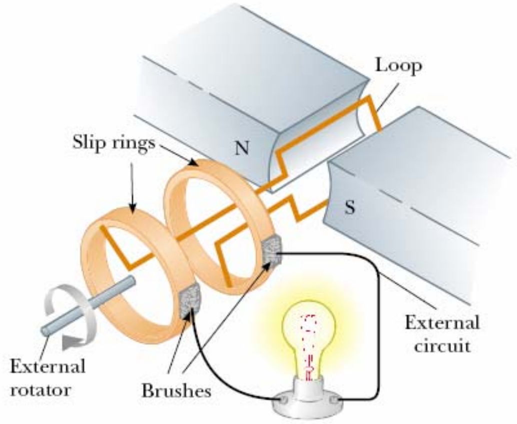


Aplicaciones de la ley de Faraday



Generador eléctrico

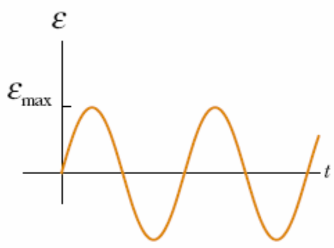


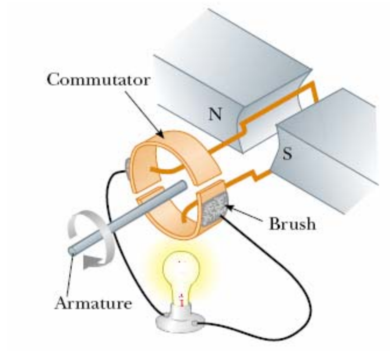


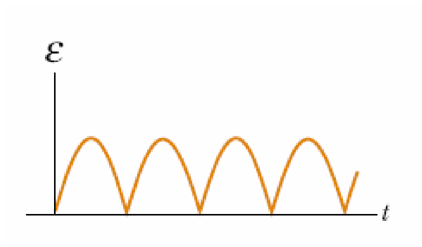
$$\Phi_B = BA\cos\theta = BA\cos\omega t$$

$$\mathcal{E} = -N\frac{d\Phi_B}{dt} = -NAB\frac{d}{dt}(\cos\omega t) = NAB\omega\sin\omega t$$

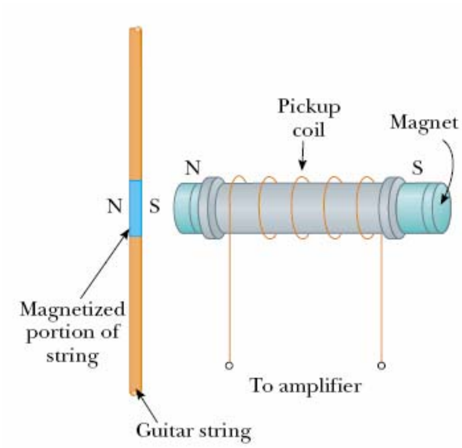
$$\mathcal{E}_{max} = NAB\omega$$



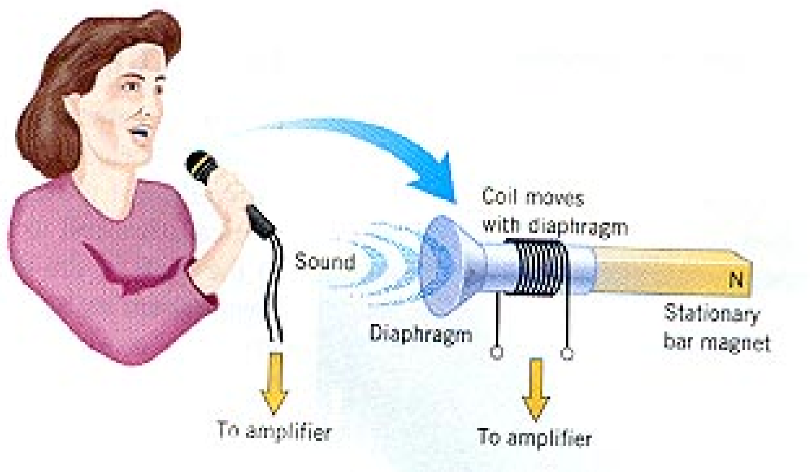




Guitarra eléctrica

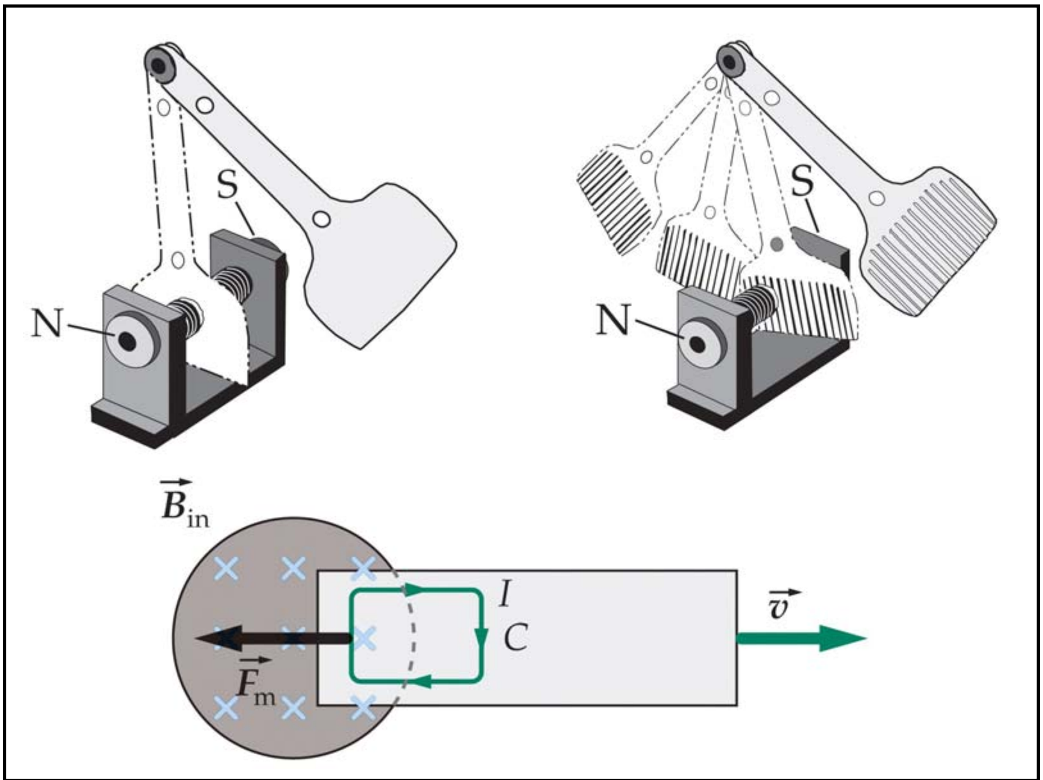


Micrófono de bobina móvil

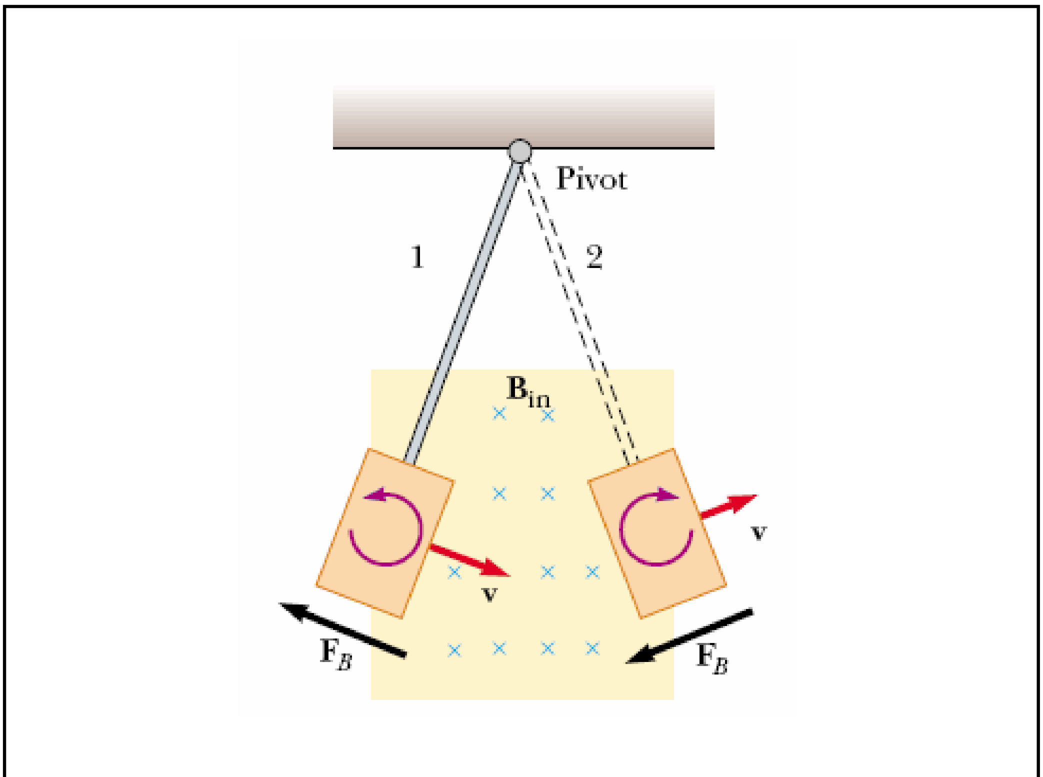


Corrientes de Foucault

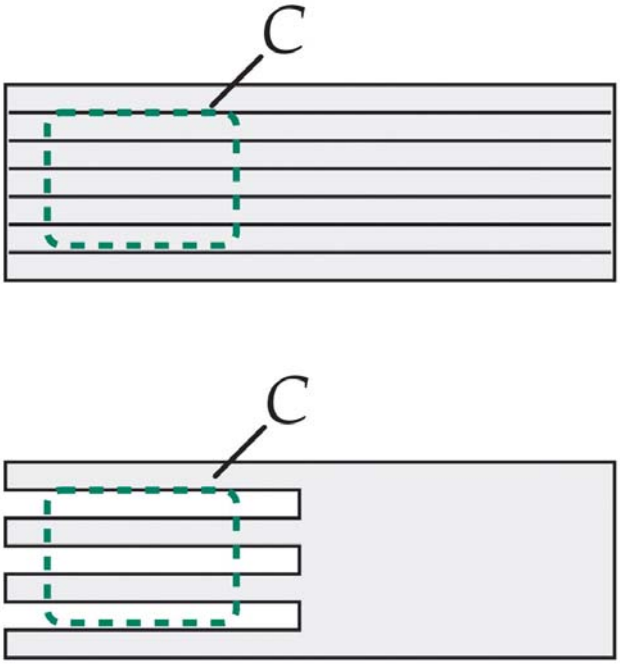
Jean Leon Foucault (1819-1868)



S
N
S
N
$\vec{B}_{in}$
I
C
$\vec{v}$
$\vec{F}_m$

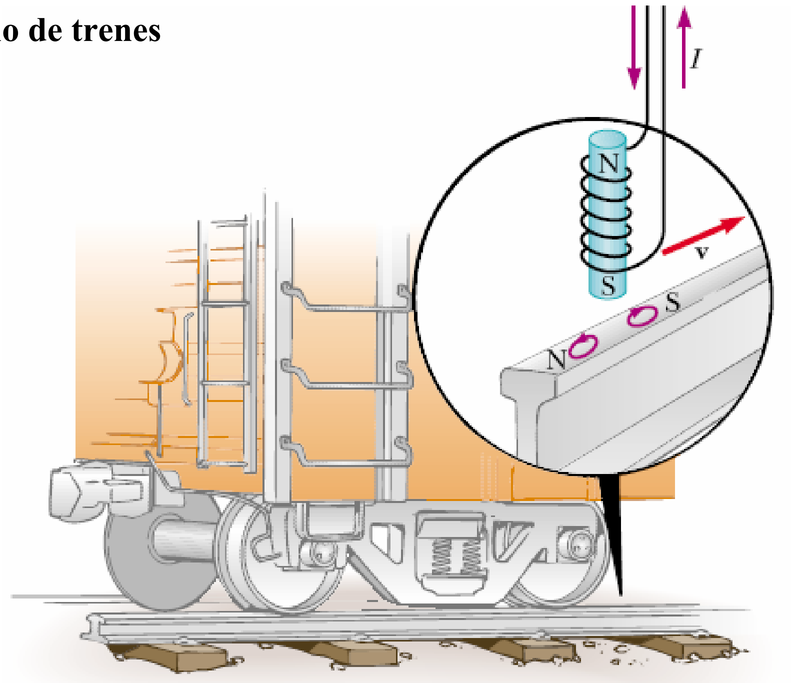


Pivot
1
2
$\mathbf{B}_{in}$
v
v
$\mathbf{F}_B$
$\mathbf{F}_B$

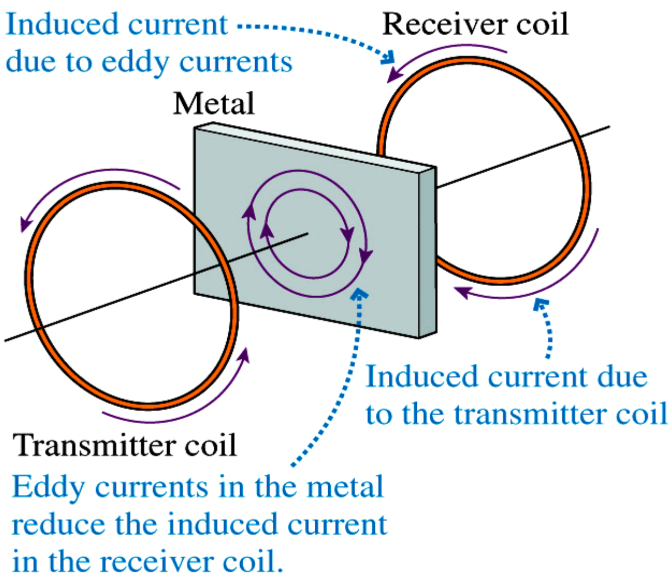


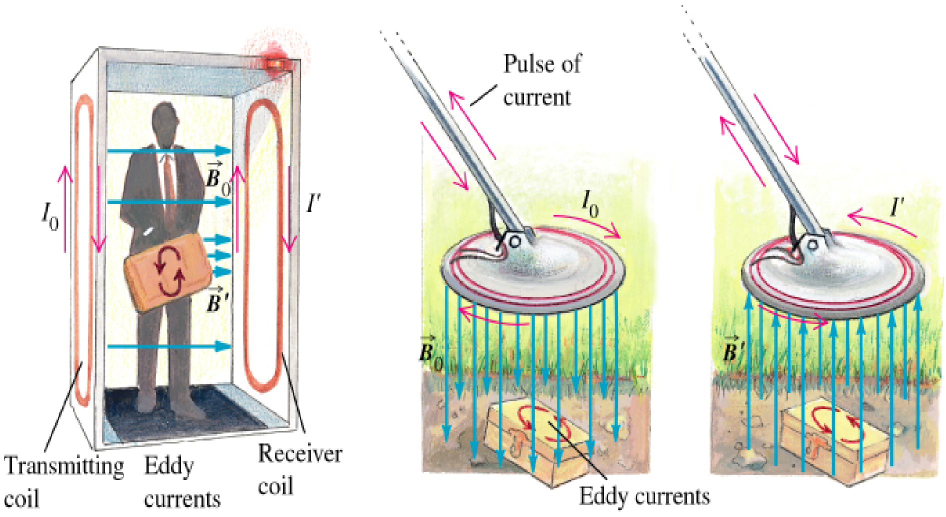
Son útiles para amortiguar oscilaciones mecánicas

Freno de trenes



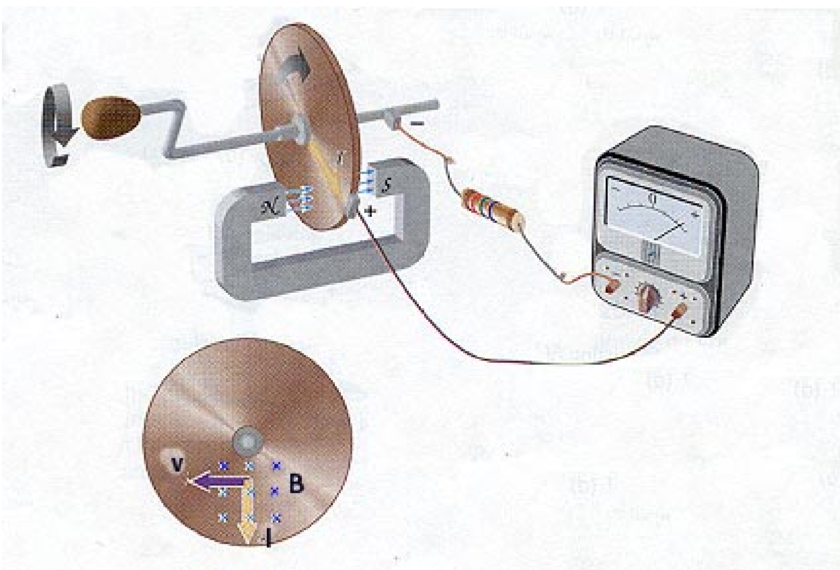
Detector de metales

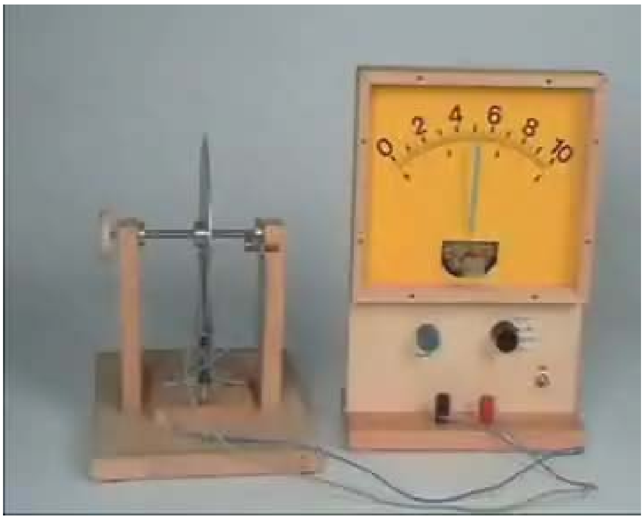




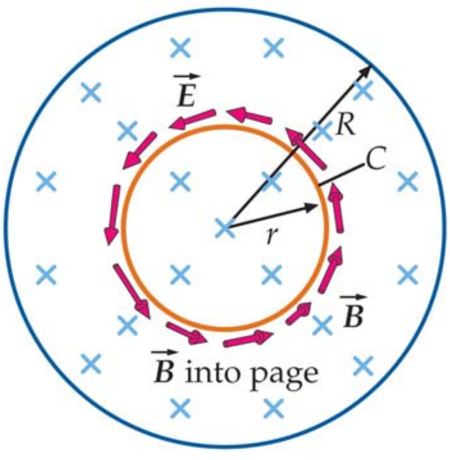
(a) (b)

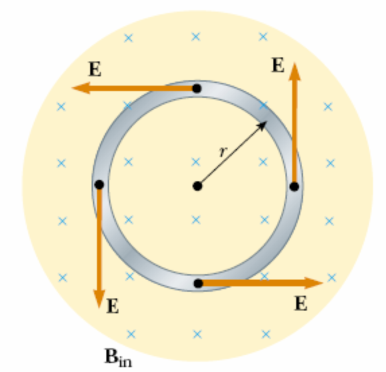
El primer generador





fem inducidas y campos eléctricos





$$\mathcal{E} = -\frac{d\phi_m}{dt}$$

$$\phi_m = \int_S \vec{B} \cdot \hat{n}\, dA = \int_S B_n\, dA$$

$$\mathcal{E} = \oint_C \vec{E}_{nc} \cdot d\vec{\ell} = -\frac{d}{dt}\int_S \vec{B} \cdot \hat{n}\, dA = -\frac{d\phi_m}{dt}$$

El campo eléctrico inducido no es conservativo

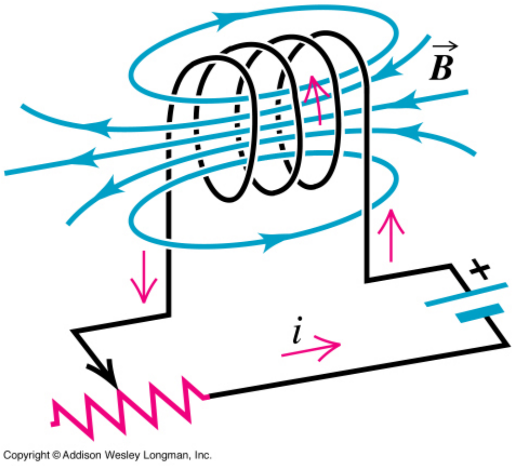
$$\oint_C \vec{E} \cdot d\vec{l} = -\frac{d}{dt}\int_S \vec{B} \cdot d\vec{A}$$

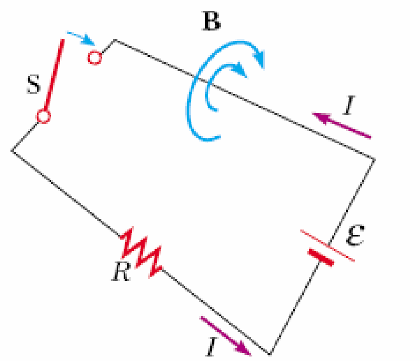
$$\left.\begin{aligned} \oint_L \vec{E} \cdot d\vec{l} &= \int_S \nabla \wedge \vec{E} \cdot d\vec{s} \\ \frac{d}{dt}\int_S \vec{B} \cdot d\vec{s} &= \int_S \frac{\partial \vec{B}}{\partial t} \cdot d\vec{s} \end{aligned}\right\} \rightarrow \nabla \wedge \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\boxed{\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}}$$

La ley de Faraday-Henry en forma diferencial

Autoinducción





S se cierra $\Longrightarrow$ I (t)

$\Downarrow$

B(t)

Variación temporal de Φ_B a través del área del circuito

fem autoinducida

$$\varepsilon = -\frac{d\Phi_B}{dt}$$

B α I

Φ_B α I

Definimos: $\Phi_B = L\ I$

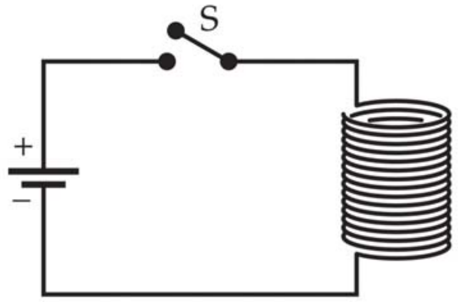
L: coeficiente de autoinductancia o inductancia $\Longrightarrow$

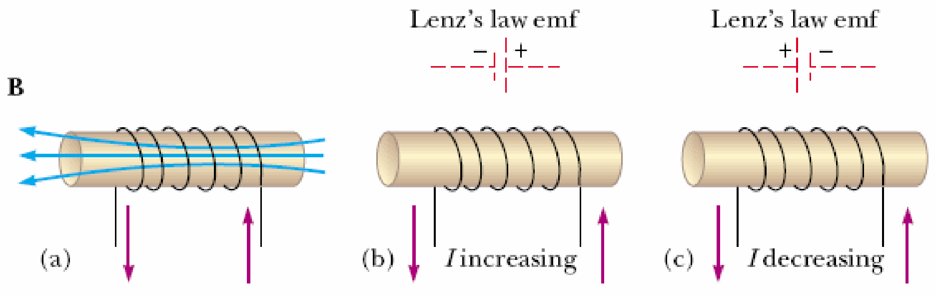
$$\varepsilon_L = -L\frac{dI}{dt}$$

$L = \Phi_B \; / \; I = -\mathcal{E}_L \, / \, (dI/dt)$

[L] = T m² / A = Wb / A = V s / A = H (Henry)

La inductancia de un circuito depende de su geometría





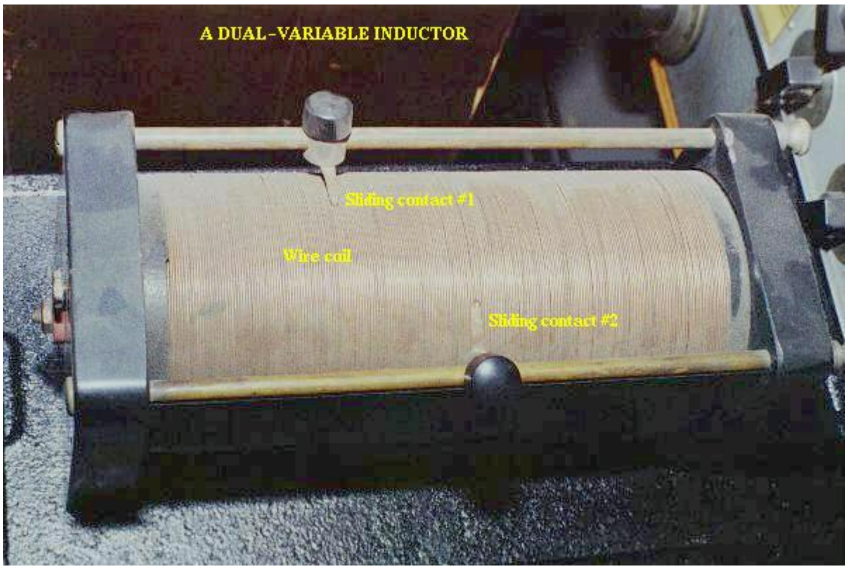
$$L = \frac{N\Phi_B}{I}$$

$$B = \mu_0 n I = \mu_0 \frac{N}{\ell} I \qquad n = N/\ell$$

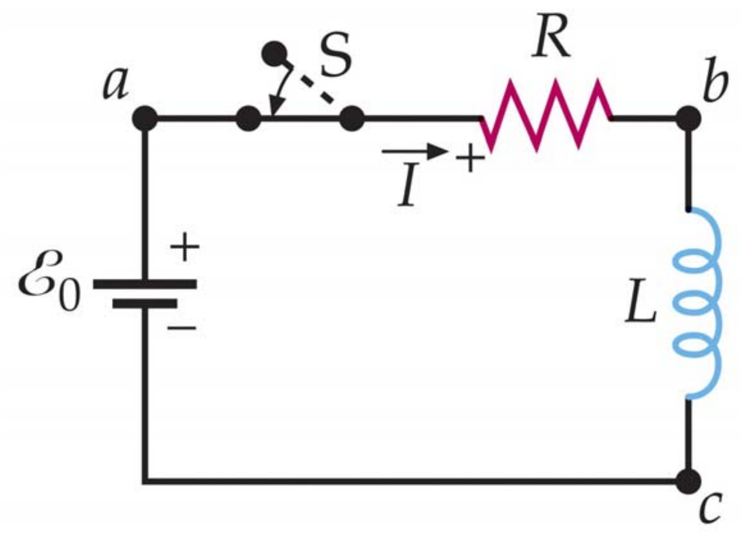
$$\Phi_B = BA = \mu_0 \frac{NA}{\ell} I$$

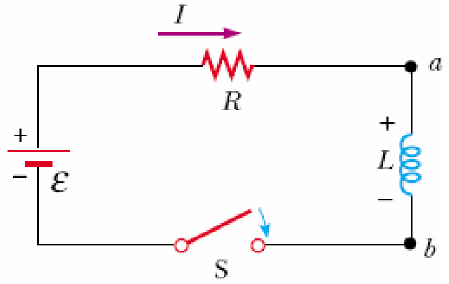
$$L = \frac{N\Phi_B}{I} = \frac{\mu_0 N^2 A}{\ell} \qquad L = \mu_0 \frac{(n\ell)^2}{\ell} A = \mu_0 n^2 A\ell = \mu_0 n^2 V$$

$$V = A\ell$$


A DUAL-VARIABLE INDUCTOR
Sliding contact #1
Wire coil
Sliding contact #2

Circuito *RL*


a
S
R
b
I
+
$\mathcal{E}_0$
+
−
L
c



$$\mathcal{E} - IR - L\frac{dI}{dt} = 0$$

$$x = (\mathcal{E}/R) - I, \quad dx = -\,dI.$$

$$x + \frac{L}{R}\frac{dx}{dt} = 0 \qquad \frac{dx}{x} = -\frac{R}{L}\,dt \qquad \int_{x_0}^{x}\frac{dx}{x} = -\frac{R}{L}\int_0^t dt$$

$$\ln\frac{x}{x_0} = -\frac{R}{L}\,t \quad \Longrightarrow \quad x = x_0 e^{-Rt/L}$$

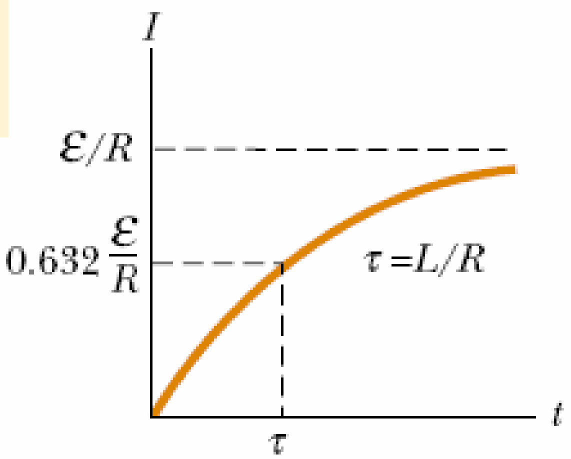
$x_0 = x\,(t=0) = \mathcal{E} / R$

$$\frac{\mathcal{E}}{R} - I = \frac{\mathcal{E}}{R}\,e^{-Rt/L} \qquad I = \frac{\mathcal{E}}{R}\left(1 - e^{-Rt/L}\right)$$

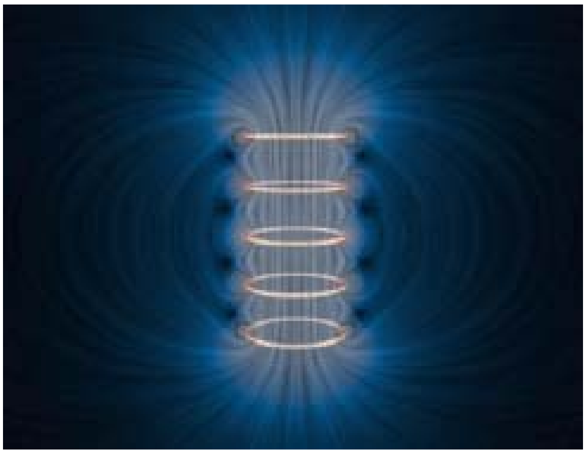
$$I = \frac{\mathcal{E}}{R}\left(1 - e^{-t/\tau}\right)$$

$$\tau = \frac{L}{R}$$

Constante de tiempo del circuito RL



Energía del Campo Magnético



La fem inducida evita que la batería establezca una corriente: la batería efectúa trabajo contra el inductor

Energía suministrada por la batería $\Longrightarrow$ **Calor Joule** + **Energía almacenada en el inductor**

$$\mathcal{E} - IR - L\frac{dI}{dt} = 0$$

$$I\mathcal{E} = I^2R + LI\frac{dI}{dt}$$

U: energía almacenada en el inductor

$$\frac{dU}{dt} = LI\frac{dI}{dt}$$

$$U = \int dU = \int_0^I LI\,dI = L\int_0^I I\,dI$$

$$U = \tfrac{1}{2}LI^2$$

$$U = \tfrac{1}{2}C(\Delta V)^2.$$

Análogo eléctrico

Para calcular la densidad de energía almacenada en el campo magnético, consideramos un solenoide:

$$L = \mu_0 n^2 A\ell$$

El campo magnético dentro del solenoide es $B = \mu_0 n I$

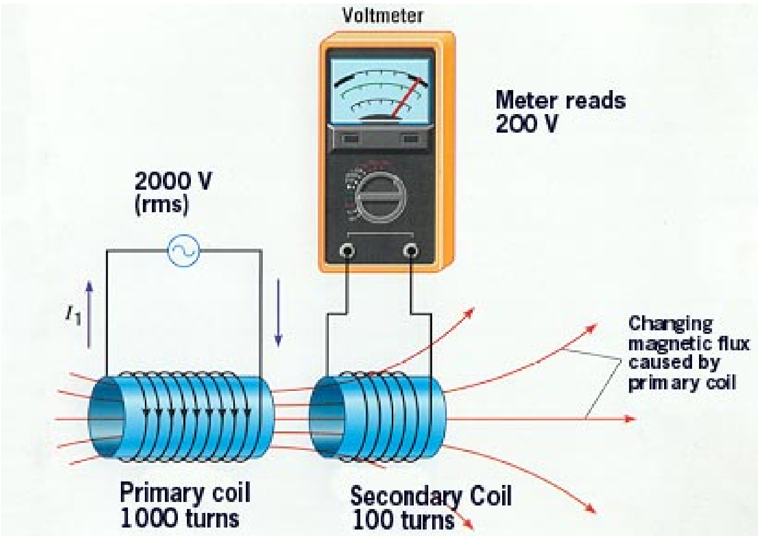
$$U = \tfrac{1}{2} L I^2 = \tfrac{1}{2}\mu_0 n^2 A\ell \left(\frac{B}{\mu_0 n}\right)^2 = \frac{B^2}{2\mu_0} A\ell$$

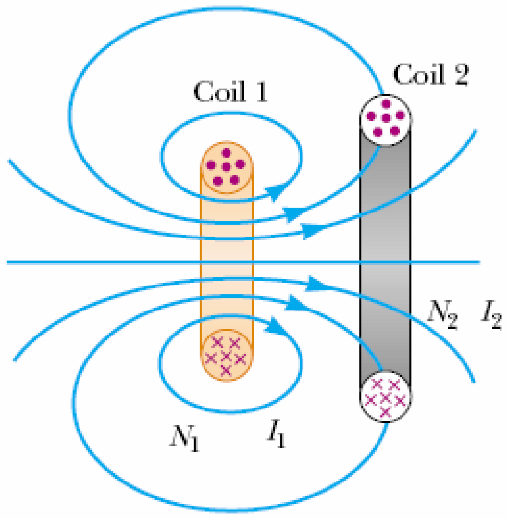
$A\ell$ **volumen del solenoide donde se encuentra confinado B**

$$u_B = \frac{U}{A\ell} = \frac{B^2}{2\mu_0}$$

Es válida para cualquier región del espacio donde haya B

Inductancia mutua





Φ_{12}.

Flujo magnético a través de la bobina 2 producido por la bobina 1.

Definimos inductancia mutua de la bobina 2 respecto a la 1

$$M_{12} \equiv \frac{N_2\Phi_{12}}{I_1}$$

M12 depende de la geometría de ambos circuitos y de la orientación de uno respecto del otro.

La fem inducida en la bobina 2 es:

$$\mathcal{E}_2 = -N_2\frac{d\Phi_{12}}{dt} = -N_2\frac{d}{dt}\left(\frac{M_{12}I_1}{N_2}\right) = -M_{12}\frac{dI_1}{dt}$$

De la misma manera:

$$\mathcal{E}_1 = -M_{21}\frac{dI_2}{dt}$$

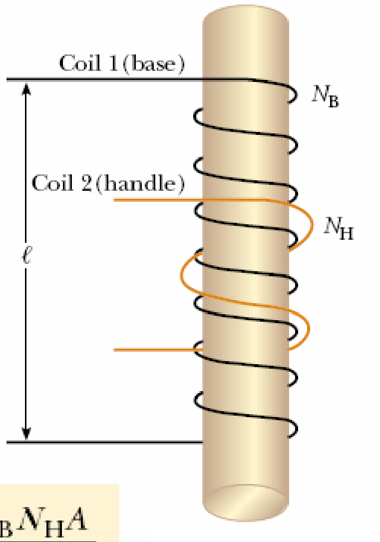
Se puede demostrar que $M_{12} = M_{21} = M$

$$\mathcal{E}_2 = -M\frac{dI_1}{dt} \qquad \mathcal{E}_1 = -M\frac{dI_2}{dt}$$

[M] = Hy

Ejemplo:

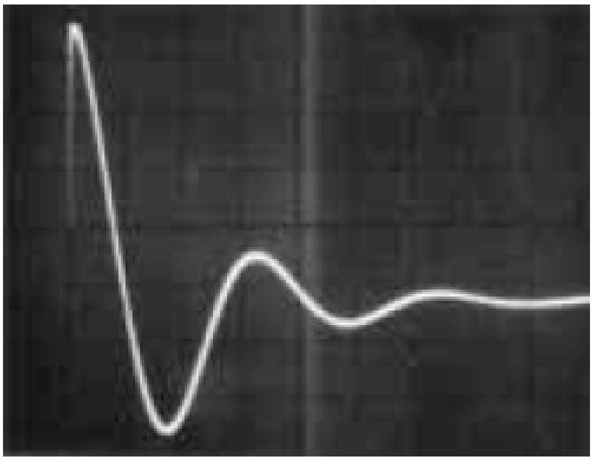
$$B = \frac{\mu_0 N_B I}{\ell}$$

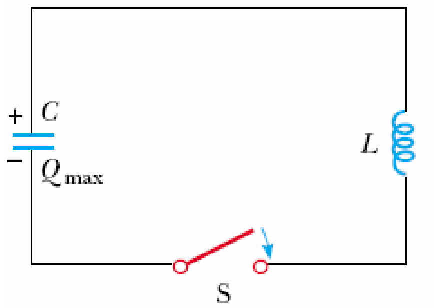


$$M = \frac{N_H \Phi_{BH}}{I} = \frac{N_H B A}{I} = \mu_0 \frac{N_B N_H A}{\ell}$$

Usualmente M se determina experimentalmente.

Oscilaciones eléctricas
Circuitos LC y RLC





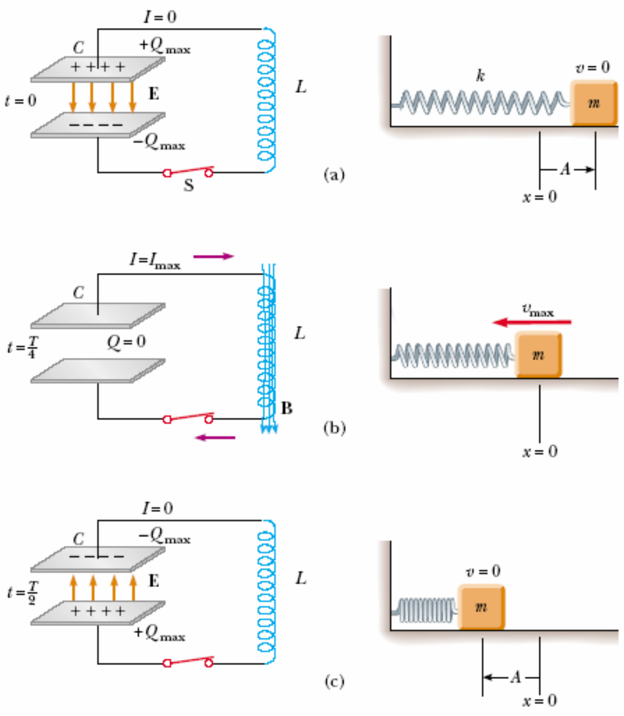
$$U = U_C + U_L = \frac{Q^2}{2C} + \frac{1}{2}LI^2$$

$$\frac{dU}{dt} = \frac{d}{dt}\left(\frac{Q^2}{2C} + \frac{1}{2}LI^2\right) = \frac{Q}{C}\frac{dQ}{dt} + LI\,\frac{dI}{dt} = 0$$

$I = dQ/dt.$ $\longrightarrow$ $\frac{Q}{C} + L\,\frac{d^2Q}{dt^2} = 0$

$$\frac{d^2Q}{dt^2} = -\frac{1}{LC}Q$$

Ecuación diferencial del oscilador armónico



$$\frac{d^2x}{dt^2} = -\frac{k}{m}x = -\omega^2 x$$

$$\omega = \sqrt{k/m}.$$

$$x = A\cos(\omega t + \phi)$$

$$Q = Q_{\max}\cos(\omega t + \phi)$$

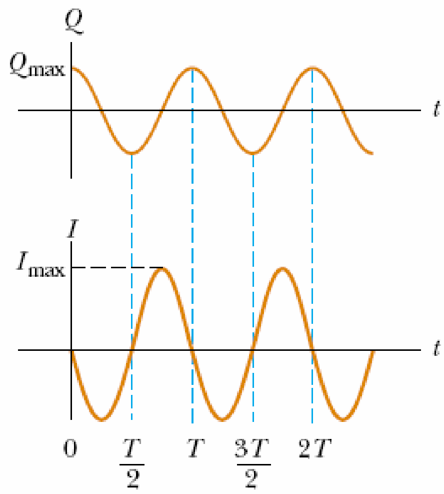
$$\omega = \frac{1}{\sqrt{LC}}$$

$$Q = Q_{\max}\cos(\omega t + \phi) \qquad I = \frac{dQ}{dt} = -\omega Q_{\max}\sin(\omega t + \phi)$$

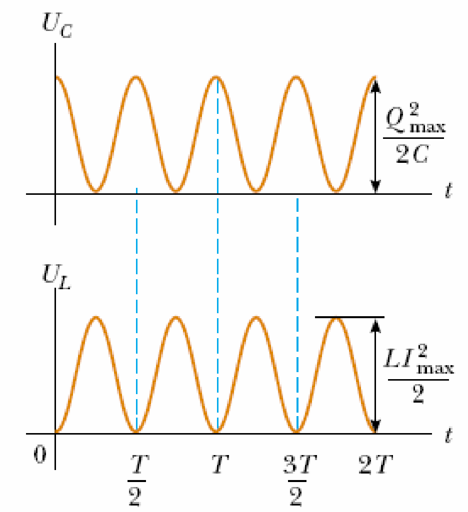
$$I = 0 \text{ at } t = 0 \qquad 0 = -\omega Q_{\max}\sin\phi$$

$$Q = Q_{\max}\cos\omega t$$

$$I = -\omega Q_{\max}\sin\omega t = -I_{\max}\sin\omega t$$



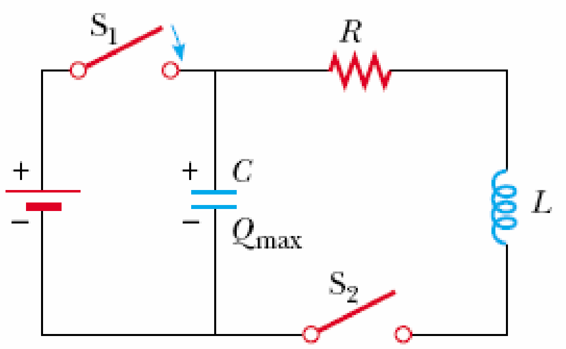
$$U = U_C + U_L = \frac{Q_{\max}^2}{2C}\cos^2\omega t + \tfrac{1}{2}LI_{\max}^2\sin^2\omega t$$



$$\frac{Q_{\max}^2}{2C} = \frac{LI_{\max}^2}{2}$$

$$U = \frac{Q_{\max}^2}{2C}(\cos^2\omega t + \sin^2\omega t) = \frac{Q_{\max}^2}{2C}$$

Que ocurre si agrego R??



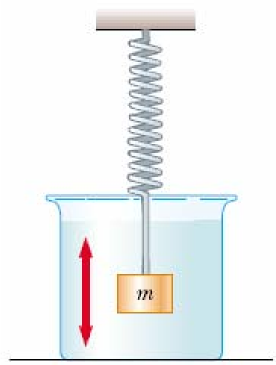
$$\frac{dU}{dt} = -I^2R$$

$$LI\frac{dI}{dt} + \frac{Q}{C}\frac{dQ}{dt} = -I^2R$$

$$LI\frac{d^2Q}{dt^2} + I^2R + \frac{Q}{C}I = 0$$

$$L\frac{d^2Q}{dt^2} + IR + \frac{Q}{C} = 0$$

$$L\frac{d^2Q}{dt^2} + R\frac{dQ}{dt} + \frac{Q}{C} = 0$$



$$L\frac{d^2Q}{dt^2} + R\frac{dQ}{dt} + \frac{Q}{C} = 0 \leftrightarrow m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = 0$$

$$\begin{aligned} Q &\leftrightarrow x \\ I &\leftrightarrow v_x \\ \Delta V &\leftrightarrow F_x \\ R &\leftrightarrow b \\ C &\leftrightarrow 1/k \\ L &\leftrightarrow m \end{aligned}$$

$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = 0$$

$$\boxed{Q = Q_{\max} e^{-Rt/2L} \cos \omega_d t}$$

donde

$$\omega_d = \left[\frac{1}{LC} - \left(\frac{R}{2L}\right)^2\right]^{1/2}$$

$$I = \frac{dQ}{dt} \leftrightarrow v_x = \frac{dx}{dt}$$

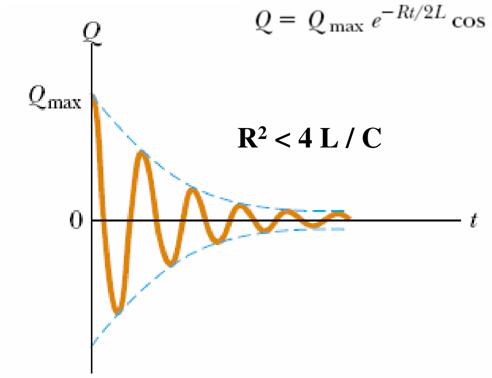
$$\frac{dI}{dt} = \frac{d^2Q}{dt^2} \leftrightarrow a_x = \frac{dv_x}{dt} = \frac{d^2x}{dt^2}$$

$$U_L = \tfrac{1}{2}LI^2 \leftrightarrow K = \tfrac{1}{2}mv^2$$

$$U_C = \tfrac{1}{2}\frac{Q^2}{C} \leftrightarrow U = \tfrac{1}{2}kx^2$$

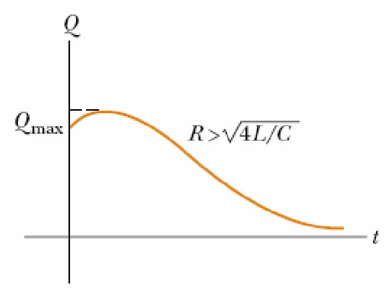
$$I^2R \leftrightarrow bv^2$$

$$Q = Q_{\max} e^{-Rt/2L} \cos \omega_d t \qquad \omega_d = \left[\frac{1}{LC} - \left(\frac{R}{2L}\right)^2\right]^{1/2}$$



$R^2 < 4\ L / C$

El sistema esta subamortiguado



$R^2 = 4\ L / C$ amortiguamiento crítico

$R^2 > 4\ L / C$ sobreamortiguado