

Capítulo 3:

Campos Electromagnéticos Estáticos

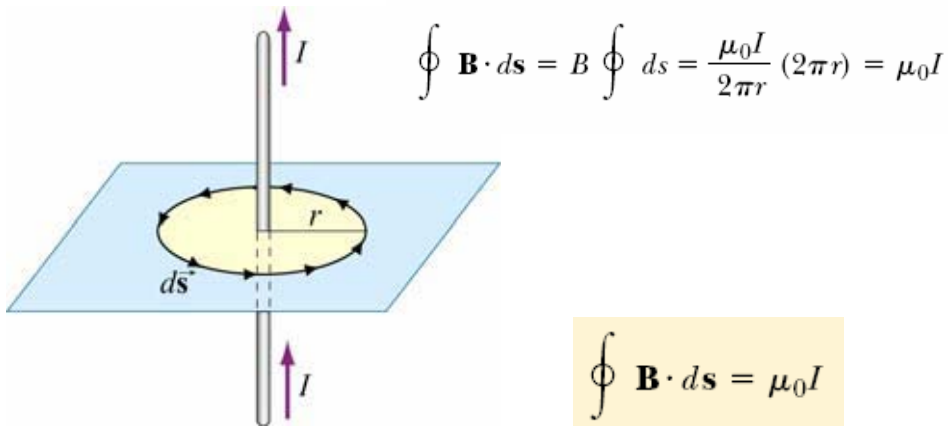
(continuación)

Ley de Ampere para el campo magnético



Andre-Mari Ampere (1775-1836)

Para un conductor rectilíneo infinito, calculamos la circulación de \mathbf{B} a lo largo de una curva circular



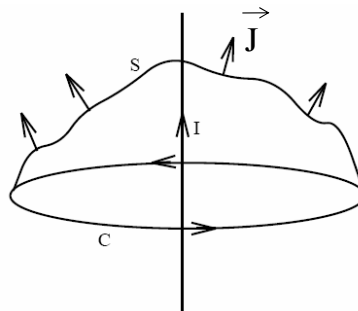
Este resultado se puede generalizar a una curva de cualquier forma

Ley de Ampere: La circulación del campo magnético alrededor de cualquier curva cerrada es igual a $\mu_0 I$, donde I es la corriente neta que atraviesa la superficie encerrada por la curva.

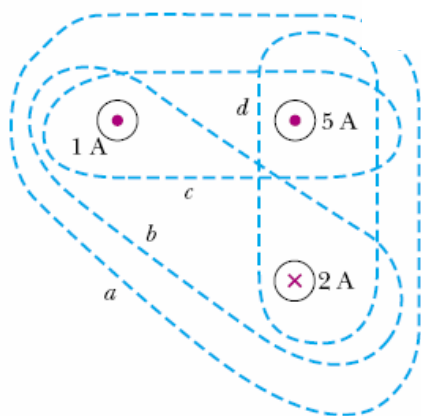
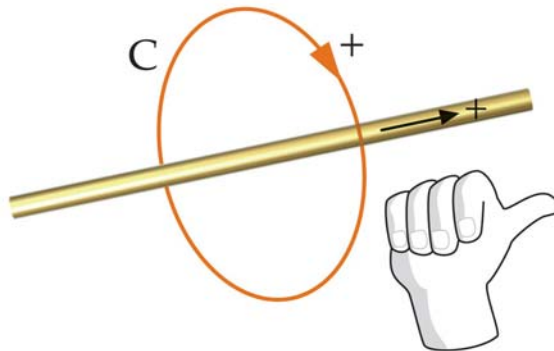
$$\oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 I$$

Otra forma de expresar la ley de Ampere:

$$\oint_C \mathbf{B} \cdot d\mathbf{l} = \mu_0 \int_S \mathbf{J} \cdot d\mathbf{S}$$

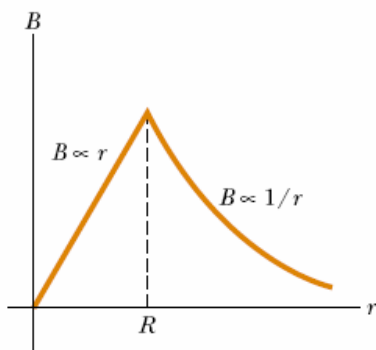
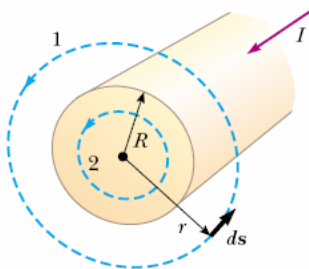


**Convenio de signos:
regla de la mano
derecha**



$$\oint \mathbf{B} \cdot d\mathbf{s} \quad ?$$

Campo creado por un alambre grueso infinito



$$\oint \mathbf{B} \cdot d\mathbf{s} = B \oint ds = B(2\pi r) = \mu_0 I$$

$$B = \frac{\mu_0 I}{2\pi r} \quad (\text{for } r \geq R)$$

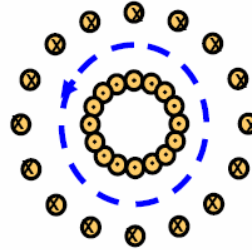
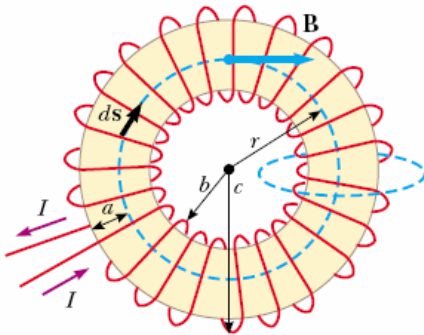
$$\frac{I'}{I} = \frac{\pi r^2}{\pi R^2}$$

$$I' = \frac{r^2}{R^2} I$$

$$\oint \mathbf{B} \cdot d\mathbf{s} = B(2\pi r) = \mu_0 I' = \mu_0 \left(\frac{r^2}{R^2} I \right)$$

$$B = \left(\frac{\mu_0 I}{2\pi R^2} \right) r \quad (\text{for } r < R)$$

Campo creado por un toroide

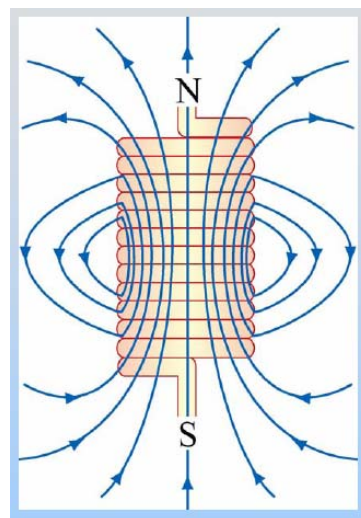
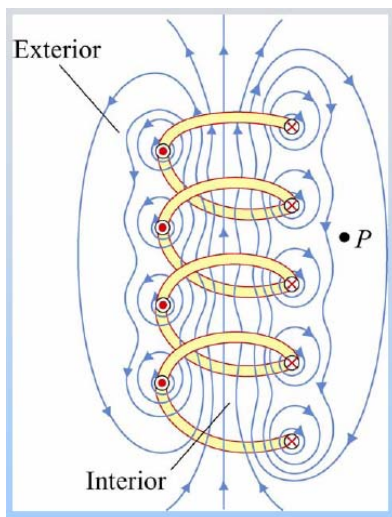


$$\oint \mathbf{B} \cdot d\mathbf{s} = B \oint ds = B(2\pi r) = \mu_0 NI$$

$$\begin{aligned} r < b \\ r > c \end{aligned} \quad \mathbf{B} = 0$$

$$B = \frac{\mu_0 NI}{2\pi r}$$

Campo creado por un solenoide



Para un solenoide ideal, B es uniforme dentro & cero fuera

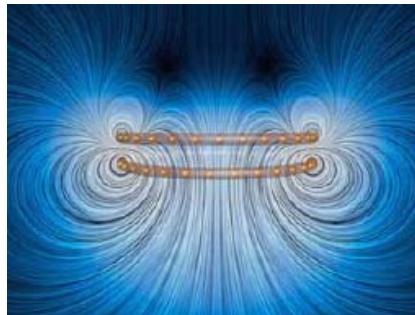
The diagram shows a solenoid with current flowing into the page (marked with 'x'). Blue magnetic field lines (B) are shown as parallel vertical lines pointing upwards. A dashed blue rectangular loop is drawn around the solenoid, with points 1, 2, 3, and 4 labeled. The width of the loop is 'w' and its length is 'ℓ'. The magnetic field is uniform and parallel to the path 1-2-3-4.

$$\oint \mathbf{B} \cdot d\mathbf{s} = \int_{\text{path 1}} \mathbf{B} \cdot d\mathbf{s} = B \int_{\text{path 1}} ds = B\ell$$

$$\oint \mathbf{B} \cdot d\mathbf{s} = B\ell = \mu_0 NI$$

$$B = \mu_0 \frac{N}{\ell} I = \mu_0 nI$$

Ley de Ampere en forma diferencial



$\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 I$

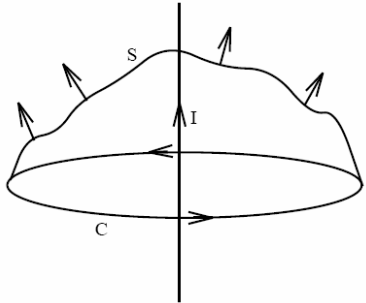
\swarrow $I = I_{\text{enc}}$ \searrow
 $\oint \mathbf{B} \cdot d\mathbf{l} = \int (\nabla \times \mathbf{B}) \cdot d\mathbf{a}$ $I_{\text{enc}} = \int \mathbf{J} \cdot d\mathbf{a}$

Teorema de Stokes

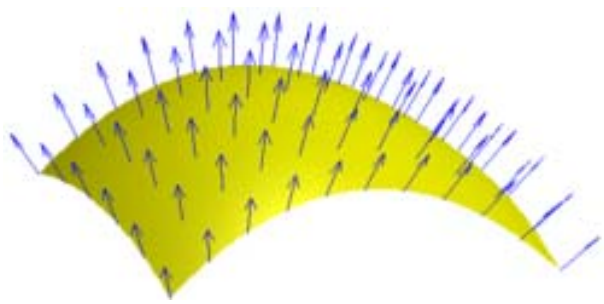
$\int (\nabla \times \mathbf{B}) \cdot d\mathbf{a} = \mu_0 \int \mathbf{J} \cdot d\mathbf{a}$

$\nabla \times \mathbf{B} = \mu_0 \mathbf{J}$

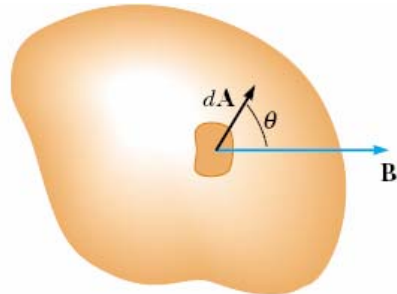
Relación local entre \mathbf{B} en un punto del espacio y la densidad de corriente en el mismo punto



Flujo Magnético



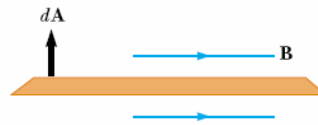
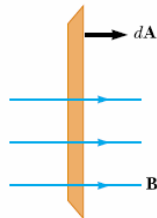
El flujo magnético de B a través de una superficie no cerrada es de gran importancia



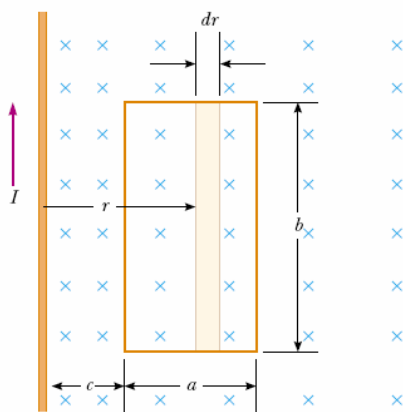
$$\Phi_B = \int \mathbf{B} \cdot d\mathbf{A}$$

Para una superficie plana

$$\Phi_B = BA \cos \theta$$



Ejemplo:



$$B = \frac{\mu_0 I}{2\pi r}$$

$$\Phi_B = \int B dA = \int \frac{\mu_0 I}{2\pi r} dA$$

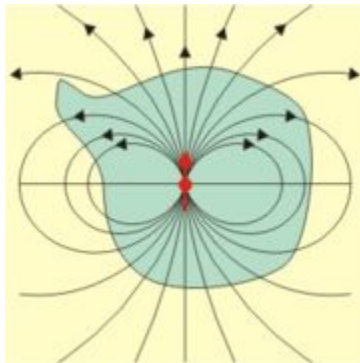
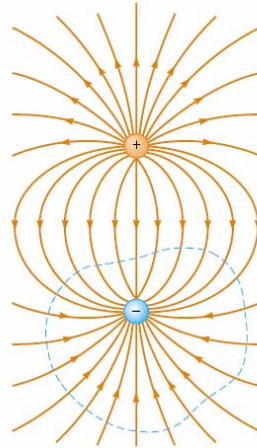
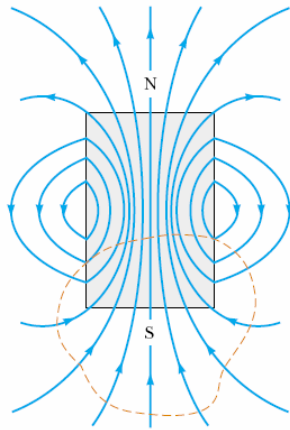
$$\Phi_B = \frac{\mu_0 I b}{2\pi} \int_c^{a+c} \frac{dr}{r} = \frac{\mu_0 I b}{2\pi} \ln r \Big|_c^{a+c}$$

$$= \frac{\mu_0 I b}{2\pi} \ln \left(\frac{a+c}{c} \right) = \frac{\mu_0 I b}{2\pi} \ln \left(1 + \frac{a}{c} \right)$$

$$[\Phi] = [B] [A] = \text{T m}^2 = \text{Wb} \quad (\text{Weber})$$

$$[B] = \text{T} = \text{Wb/m}^2$$

Ley de Gauss para el Magnetismo



Las líneas de B no empiezan ni terminan en ningún lado. Son cerradas. No existen monopolos magnéticos.

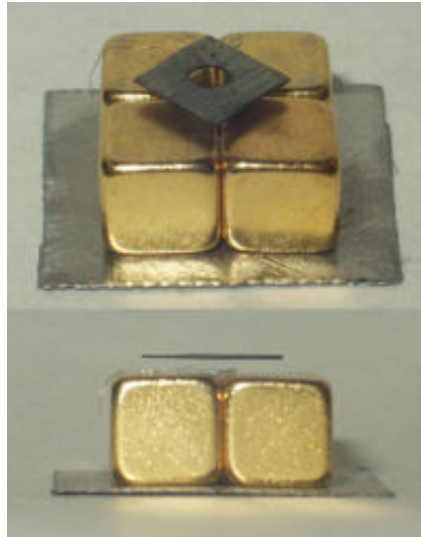
$$\oint \vec{B}(\vec{r}) \cdot d\vec{S} = 0$$

Usando el Teorema de la divergencia

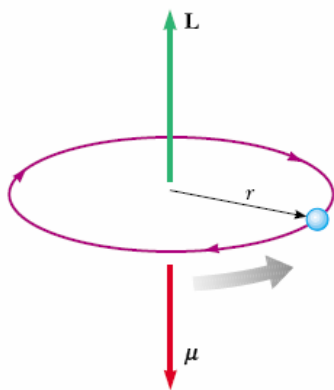
$$\iiint_V \vec{\nabla} \cdot \vec{F} dV = \iint_S \vec{F} \cdot \vec{n} dS$$

$$\vec{\nabla} \cdot \vec{B} = 0$$

Magnetización de la Materia



Momento magnético de los átomos



$$I = \frac{e}{T} = \frac{e\omega}{2\pi} = \frac{ev}{2\pi r}$$

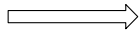
$$\mu = IA = \left(\frac{ev}{2\pi r}\right) \pi r^2 = \frac{1}{2}evr$$

$$\mu = \left(\frac{e}{2m_e}\right) L$$

$$\vec{\mu} = \left(\frac{-e}{2m_e}\right) \vec{L}$$

Momento magnético orbital

Propiedad intrínseca
de los electrones

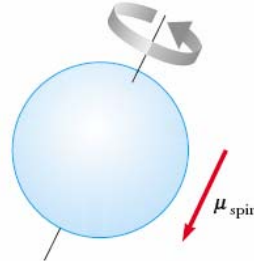


Momento magnético de
spin

$$\mu_{\text{spin}} = \frac{e\hbar}{2m_e}$$

$$\mu_B = \frac{e\hbar}{2m_e} = 9.27 \times 10^{-24} \text{ J/T}$$

se lo denomina magnetón de Bohr



Átomos con muchos
electrones



$$\mu_{\text{TOT}} = \sum \mu$$

Momento magnético total

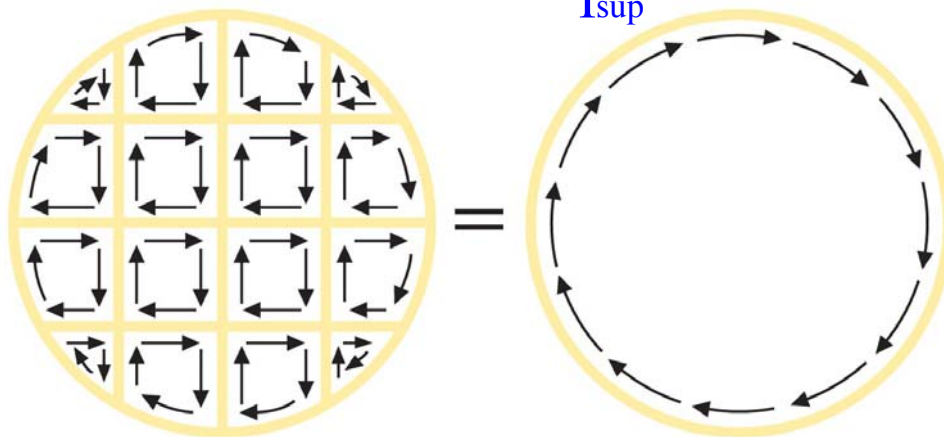
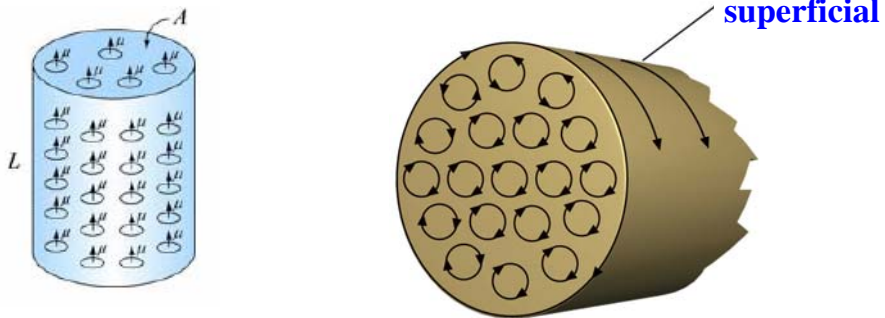
Magnetic Moments of Some Atoms and Ions

Atom or Ion	Magnetic Moment (10^{-24} J/T)
H	9.27
He	0
Ne	0
Ce ³⁺	19.8
Yb ³⁺	37.1

Definimos vector de magnetización de un material como el momento magnético por unidad de volumen

$$\vec{M} = \frac{1}{V} \sum_i \vec{\mu}_i$$

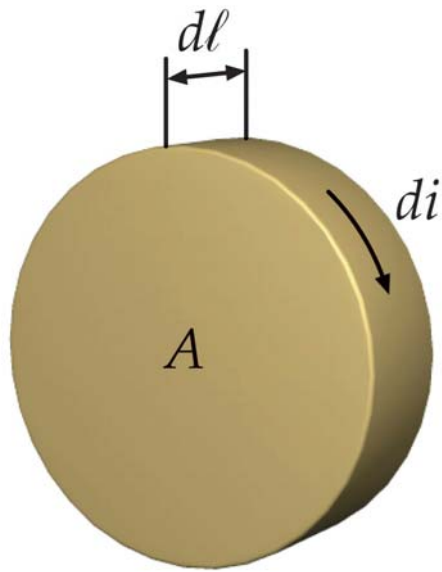
Consideremos una substancia magnetizada con forma de cilindro con M paralelo al eje del cilindro



Llamamos corriente de magnetización

$$I_M = I_{\text{sup}} / L$$

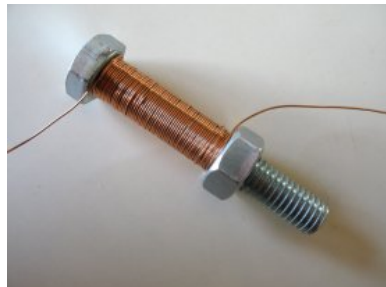
Corriente superficial por unidad de longitud

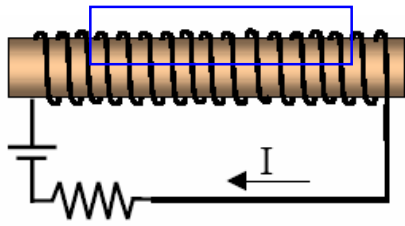


$$M = \frac{dm}{dV} = \frac{A di}{A dl} = \frac{di}{dl}$$

$$M = I_M = I_{\text{sup}} / L$$

Generalización de la Ley de Ampere





I debida a cargas libres

I_{Mag} debida a cargas ligadas

$$M = I_{Mag} = I_{sup} / L$$

Aplicando Ampere: $B = \mu_0(nI + M) \Rightarrow \frac{B}{\mu_0} - M = nI$

↙
↘

solenoid **material**

Definimos:

$$\boxed{\vec{H} = \frac{\vec{B}}{\mu_0} - \vec{M}} \iff \boxed{\vec{B} = \mu_0(\vec{H} + \vec{M})}$$

campo magnetizante

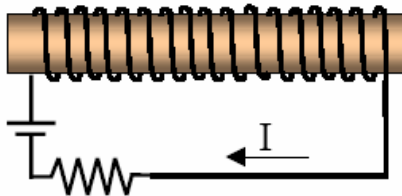
$$\oint_c \vec{B} \cdot d\vec{\ell} = \mu_0 (I_{libre} + I_{magnetiz.})$$

$$\boxed{\oint_c \vec{H} \cdot d\vec{\ell} = I_{libre}}$$

**Ley de
Ampere
generalizada**

$$\boxed{\oint_c \vec{M} \cdot d\vec{\ell} = I_{Magnetiz.}}$$

Materiales Magnéticos



$$\vec{M} = \chi_m \vec{H}$$

χ_m susceptibilidad magnética

$$\vec{B} = \mu_0 (\vec{H} + \vec{M}) = \mu_0 (\vec{H} + \chi_m \vec{H}) = \mu_0 (1 + \chi_m) \vec{H} \Rightarrow \vec{B} = \mu \vec{H}$$

$$\mu = \mu_0 (1 + \chi_m) \quad \mu \text{ permeabilidad magnética del material}$$

$$\mu_r = \frac{\mu}{\mu_0} = (1 + \chi_m) \quad \mu_r \text{ permeabilidad magnética relativa}$$

$$\oint_C \vec{B} \cdot d\vec{\ell} = \mu_0 (\vec{I}_{\text{libre}} + \vec{I}_{\text{magnetiz.}})$$

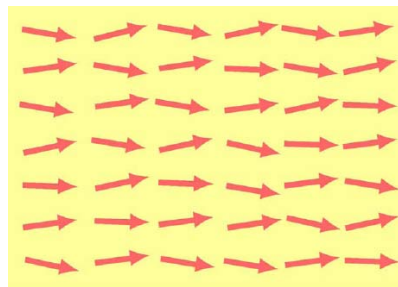
$$\oint_C \vec{H} \cdot d\vec{\ell} = I_{\text{libre}}$$

$$\oint_C \vec{H} \cdot d\vec{\ell} = \oint_C \frac{\vec{B}}{\mu} \cdot d\vec{\ell} = I_{\text{libre}} \Rightarrow \oint_C \vec{B} \cdot d\vec{\ell} = \mu I_{\text{libre}}$$

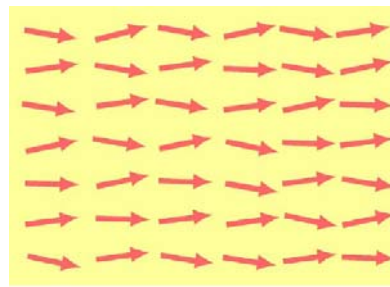
$$\oint_C \vec{B} \cdot d\vec{\ell} = \mu I_{\text{libre}}$$

Ley de Ampere generalizada

Materiales $\left\{ \begin{array}{l} \text{diamagnéticos } (\mu < \mu_0, \mu_r < 1, \chi_m < 0) \\ \text{paramagnéticos } (\mu > \mu_0, \mu_r > 1, \chi_m > 0) \end{array} \right.$



paramagnético

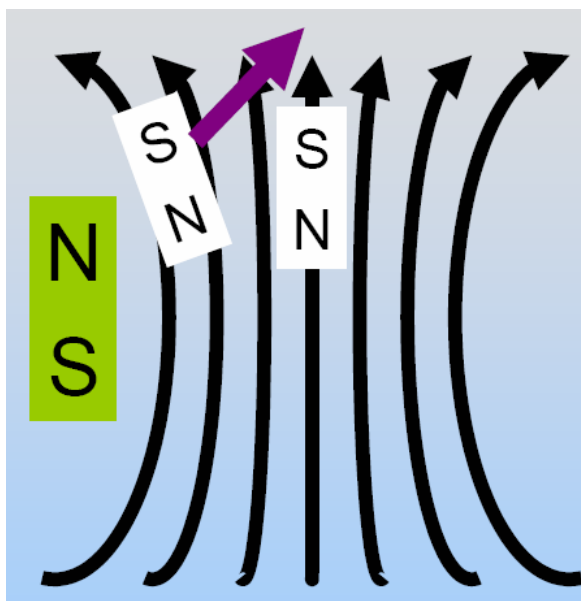


diamagnético

Susceptibilidades magnéticas de diversas sustancias a temperatura ambiente

Sustancias Diamagnéticas	χ_m	Sustancias Paramagnéticas	χ_m
Hidrógeno (1 atm)	-2.1×10^{-9}	Oxígeno (1 atm)	2.1×10^{-6}
Nitrógeno (1 atm)	-5.1×10^{-9}	Magnesio	1.2×10^{-5}
Sodio	-2.4×10^{-6}	Aluminio	2.3×10^{-5}
Cobre	-1.0×10^{-5}	Tungsteno	6.8×10^{-5}
Bismuto	-1.7×10^{-5}	Titanio	7.1×10^{-5}
Diamante	-2.2×10^{-5}	Platino	3.0×10^{-4}
Mercurio	-3.2×10^{-5}	Cloruro de Gadolinio	2.8×10^{-3}

Levitación magnética





Rana y frutilla en campo magnético de 16 T

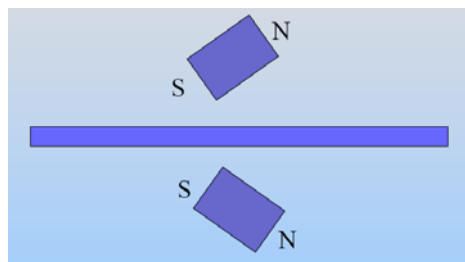
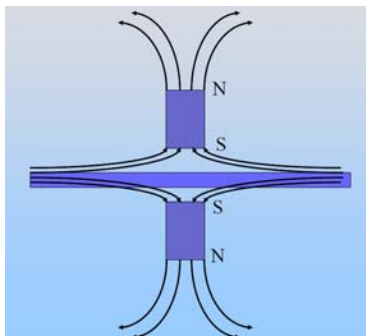
La fuerza de levitación es ejercida sobre las moléculas *diamagnéticas* del *agua* existentes en el cuerpo de la rana y en la frutilla.

<http://www.hfml.ru.nl/levitation-movies.html>

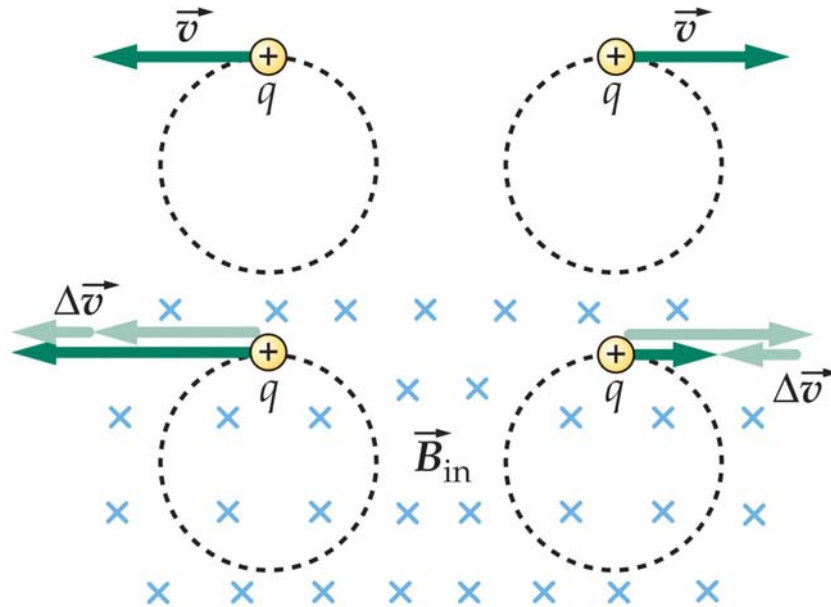


Superconductores Diamagnéticos perfectos

Espejo magnético

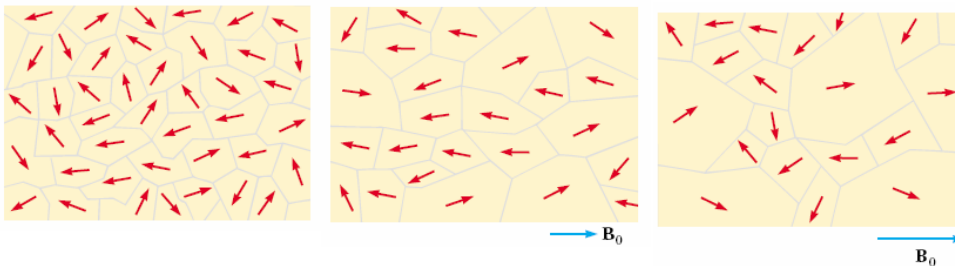


Modelo clásico para el diamagnetismo

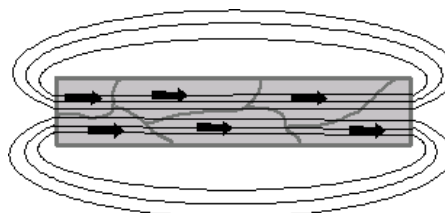


Materiales ferromagnéticos ($\mu \gg \mu_0$, $\mu_r \gg 1$, $\chi_m \gg 0$)

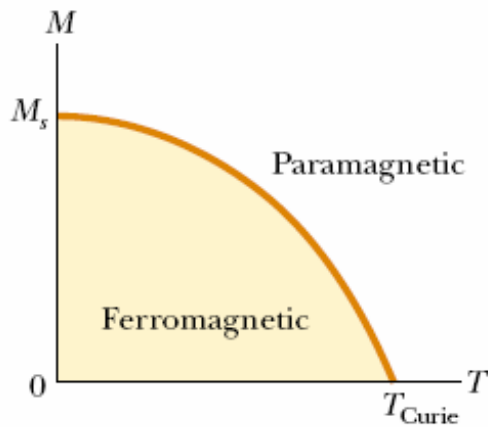
Poseen momento magnético permanente que se puede reorientar mediante la aplicación de un campo magnético externo



Imán de barra



El ferromagnetismo se destruye con la temperatura



Curie Temperatures for Several Ferromagnetic Substance

Substance	$T_{Curie}(K)$
Iron	1 043
Cobalt	1 394
Nickel	631
Gadolinium	317
Fe_2O_3	893

