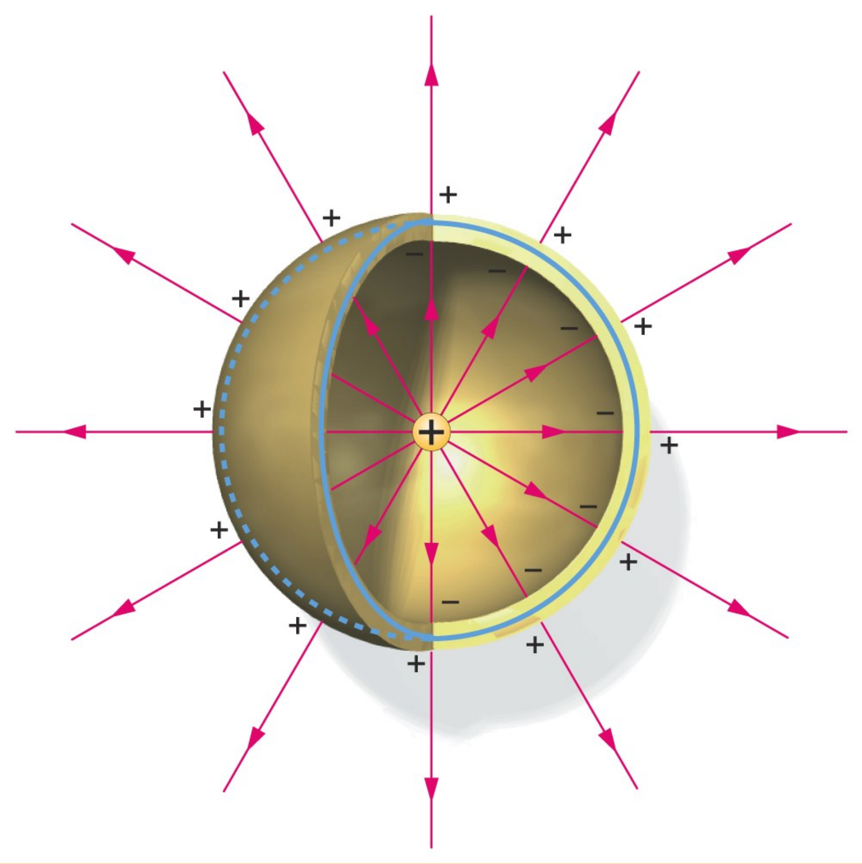
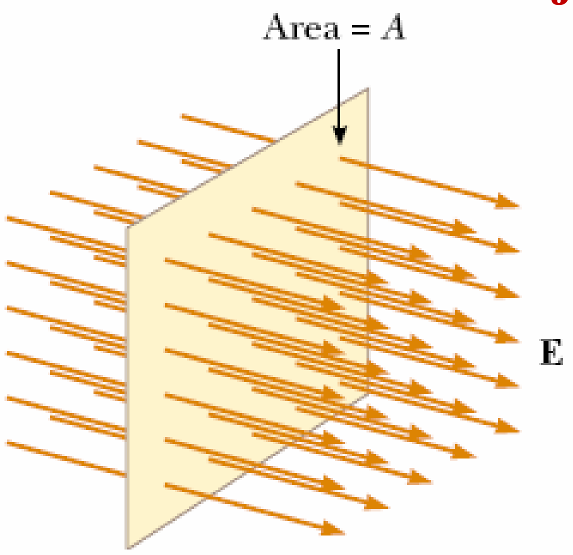
Capítulo 3:

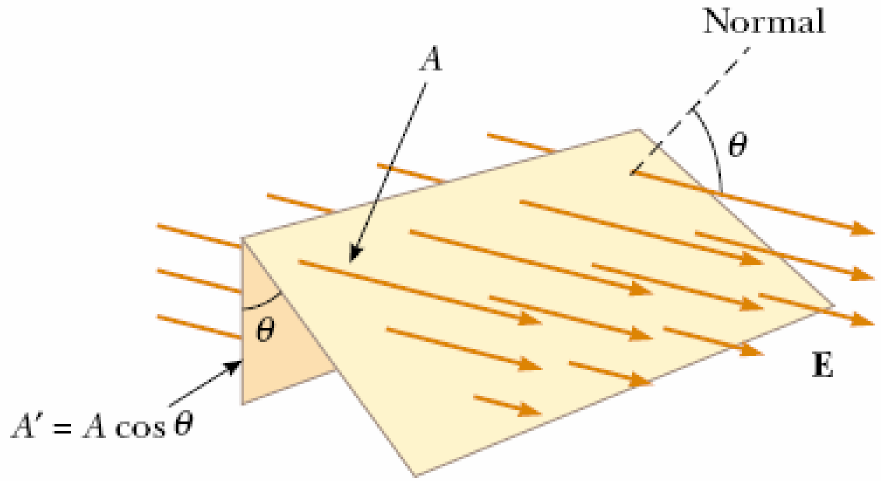
Campos Electromagnéticos Estáticos



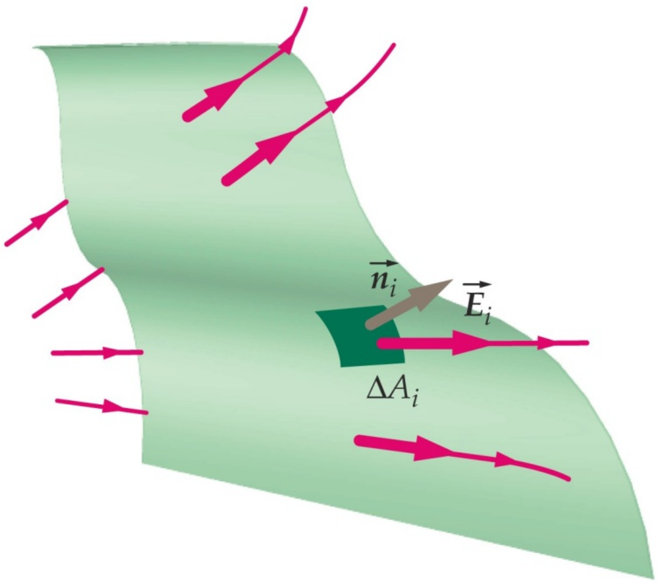
Flujo de un campo vectorial



$$\Phi_E = EA$$



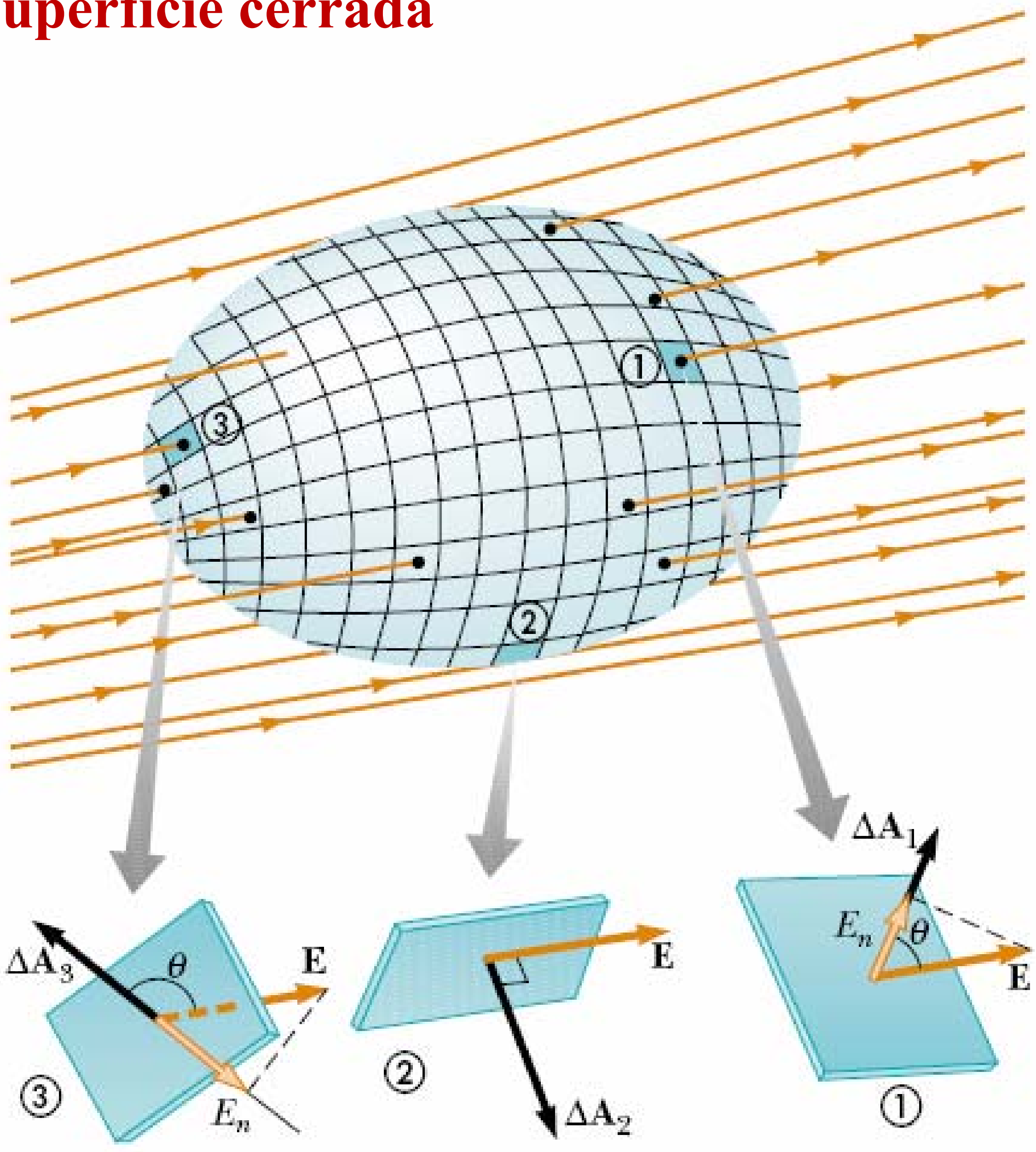
$$\Phi_E = EA\cos\theta.$$



$$\Delta\Phi_E = E_i\,\Delta A_i\cos\theta_i = \mathbf{E}_i\cdot\Delta\mathbf{A}_i$$

$$\Phi_E = \lim_{\Delta A_i\to 0}\sum \mathbf{E}_i\cdot\Delta\mathbf{A}_i = \int_{\text{surface}} \mathbf{E}\cdot d\mathbf{A}$$

Superficie cerrada

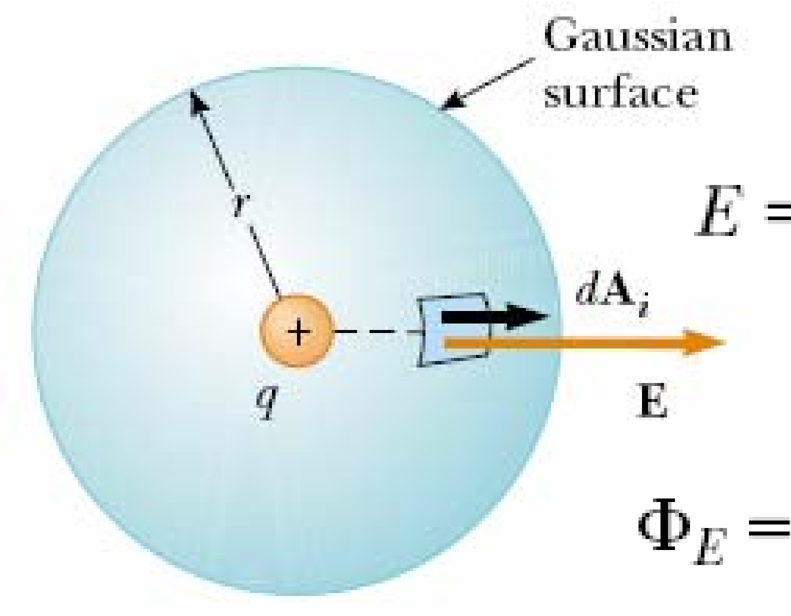


$$\Phi_E = \oint \mathbf{E} \cdot d\mathbf{A} = \oint E_n \, dA$$

Ley de Gauss

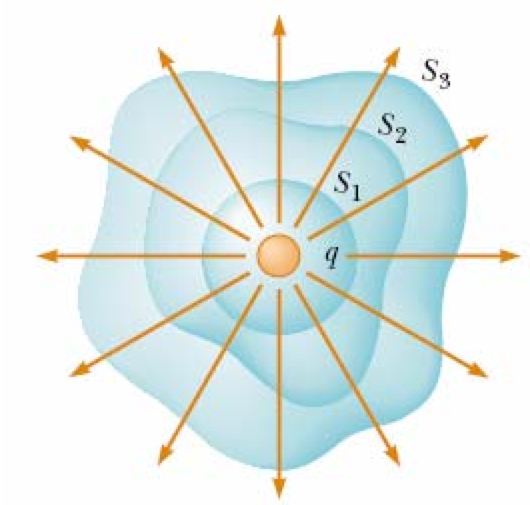
Karl Friedrich Gauss (1777-1855)

Flujo de E generado por una carga puntual



$$E = k_e q / r^2$$

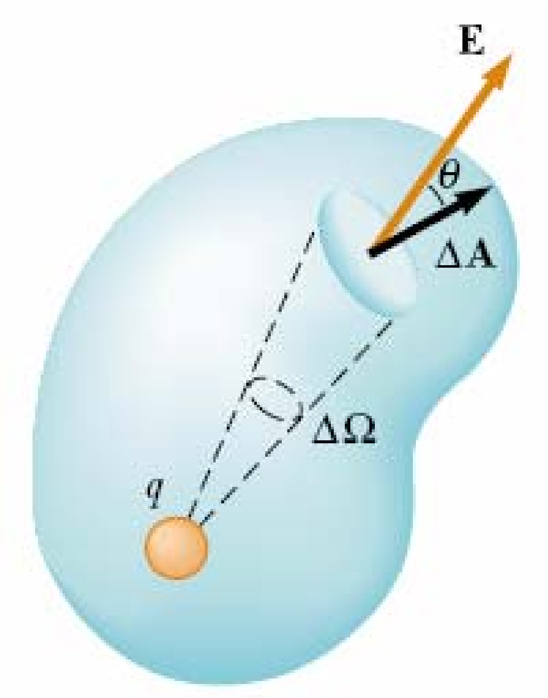
$$\Phi_E = \oint \mathbf{E} \cdot d\mathbf{A} = \oint E \, dA = E \oint dA$$

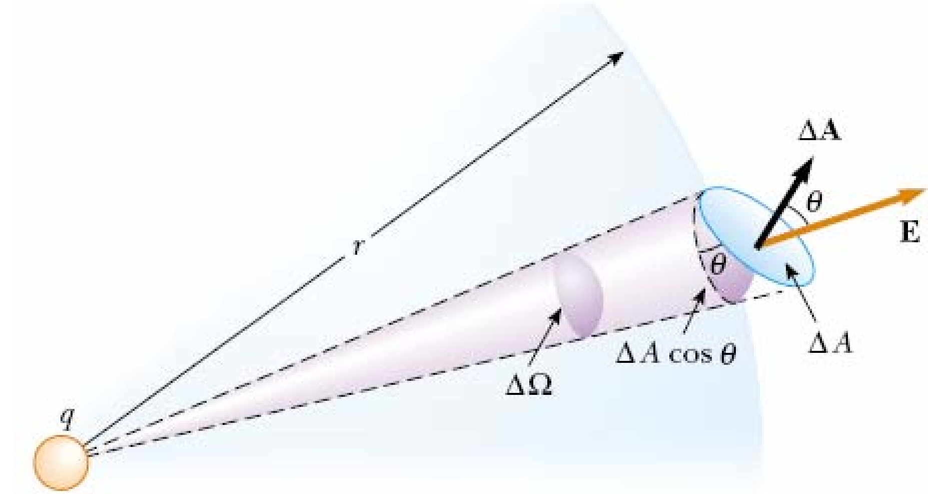


$$\Phi_E = \frac{k_e q}{r^2}(4\pi r^2) = 4\pi k_e q$$

$$\Phi_E = \frac{q}{\epsilon_0}$$

Superficie arbitraria





$$\Delta\Omega = (\Delta A \cos\theta)/r^2$$

Para una esfera ⟹

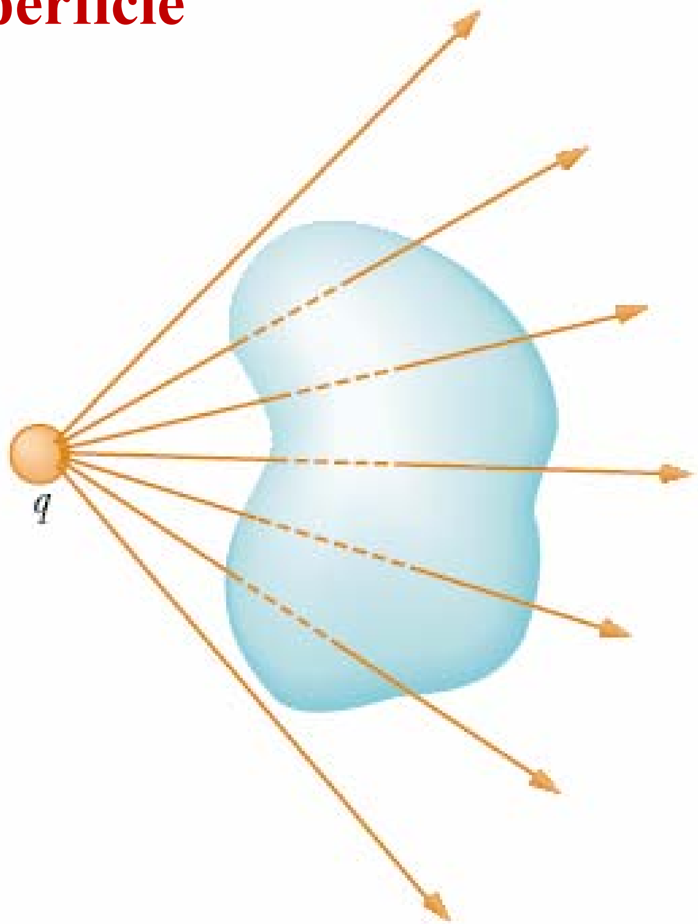
$$\Delta\Omega \equiv \frac{\Delta A}{r^2}$$

$$\Omega = \frac{4\pi r^2}{r^2} = 4\pi \text{ steradians}$$

$$\Delta\Phi_E = \mathbf{E}\cdot\Delta\mathbf{A} = (E\cos\theta)\Delta A = k_e q\,\frac{\Delta A\cos\theta}{r^2}$$

$$\Phi_E = k_e q \oint \frac{dA\cos\theta}{r^2} = k_e q \oint d\Omega = 4\pi k_e q = \frac{q}{\epsilon_0}$$

Carga fuera de la superficie



El flujo neto es nulo

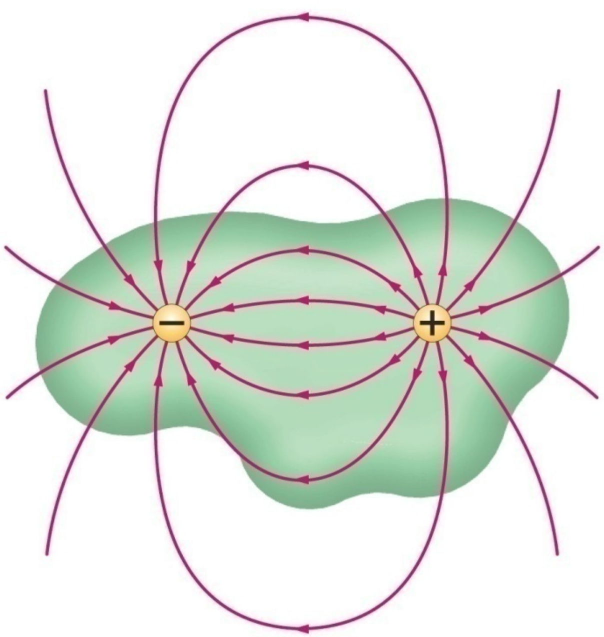
N líneas que entran = N líneas que salen

Varias cargas puntuales

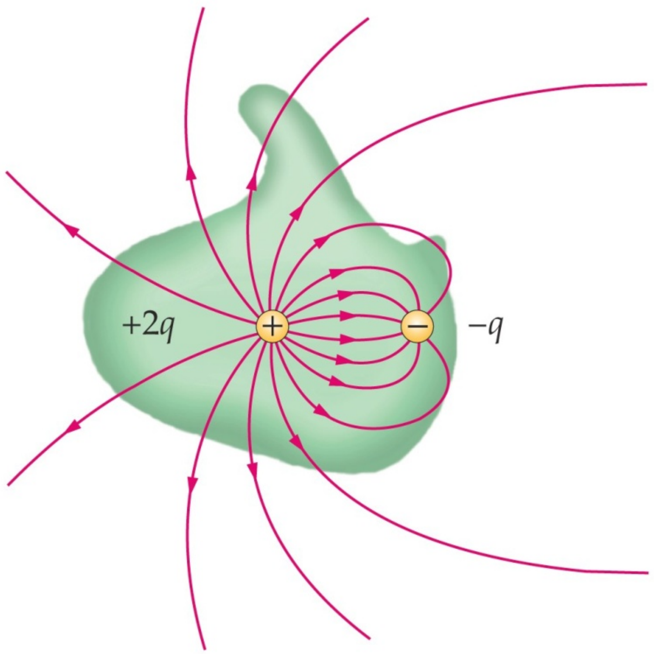
Principio de superposición

$$\oint \mathbf{E} \cdot d\mathbf{A} = \oint (\mathbf{E}_1 + \mathbf{E}_2 + \cdots) \cdot d\mathbf{A}$$

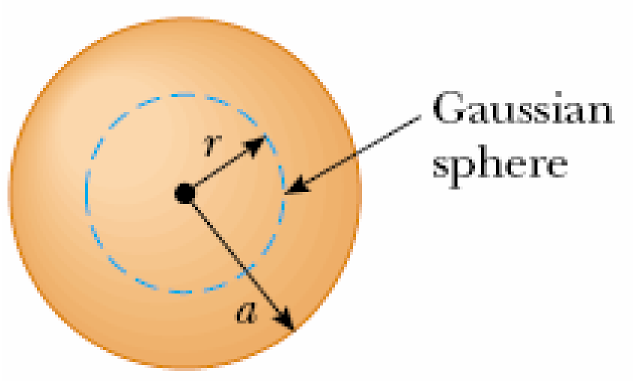
$$\Phi_E = \oint \mathbf{E} \cdot d\mathbf{A} = \frac{q_{\text{in}}}{\epsilon_0}$$



Φ_E ?



Análogamente para una distribución de carga



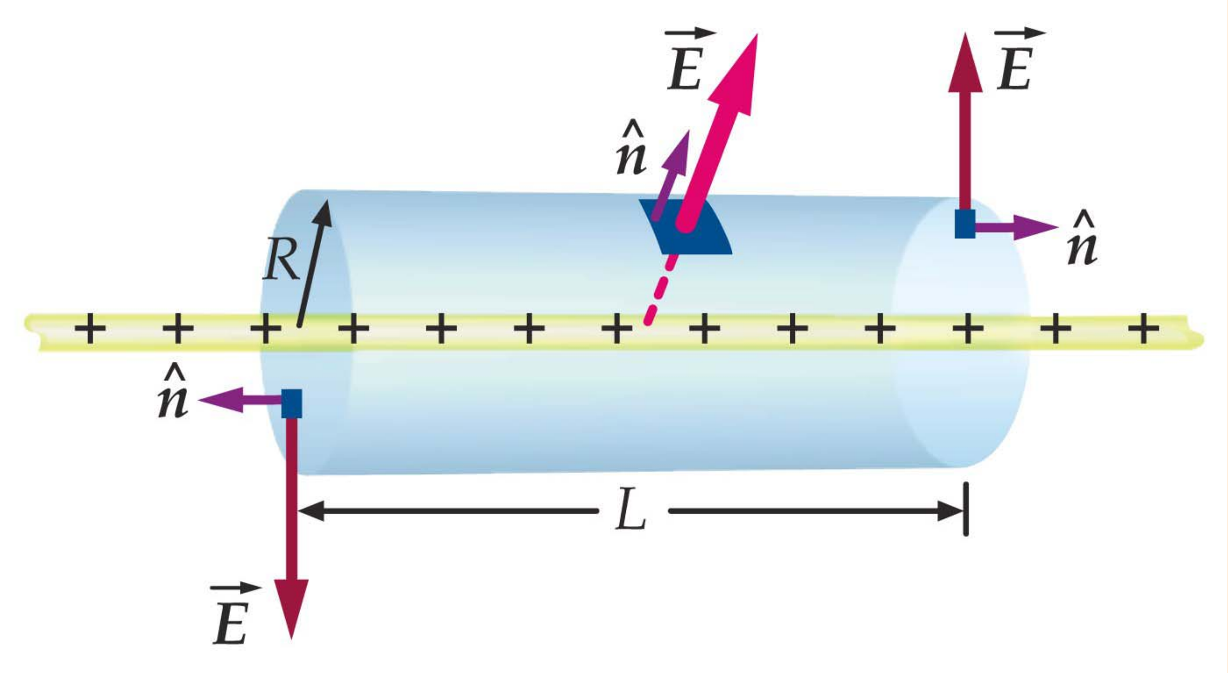
$$\oint E\,dA = E\oint dA = E\,(4\pi r^2) = \frac{q_{in}}{\epsilon_0}$$

q_{in} : carga neta encerrada por la superficie gaussiana

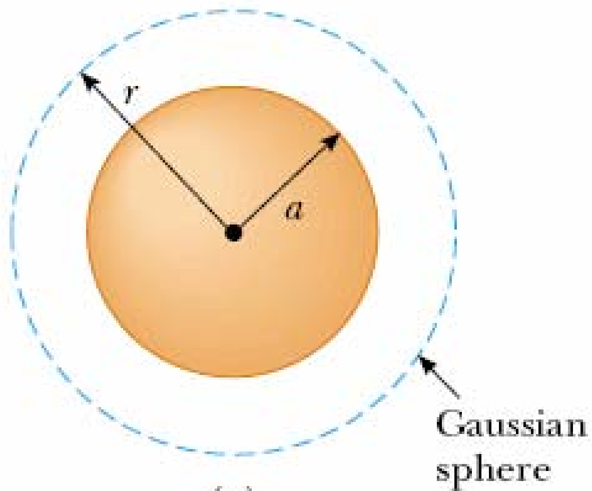
Ley de Gauss: el flujo eléctrico neto a través de cualquier superficie cerrada es igual a la carga neta dentro de la superficie dividido ε_0

$$\Phi_E = \oint \mathbf{E}\cdot d\mathbf{A} = \frac{q_{in}}{\epsilon_0}$$

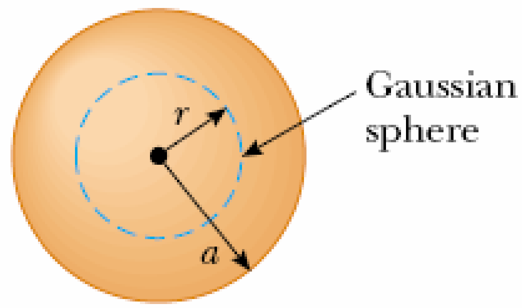
Cálculo de E utilizando la Ley de Gauss



Esfera uniformemente cargada



(a)



(b)

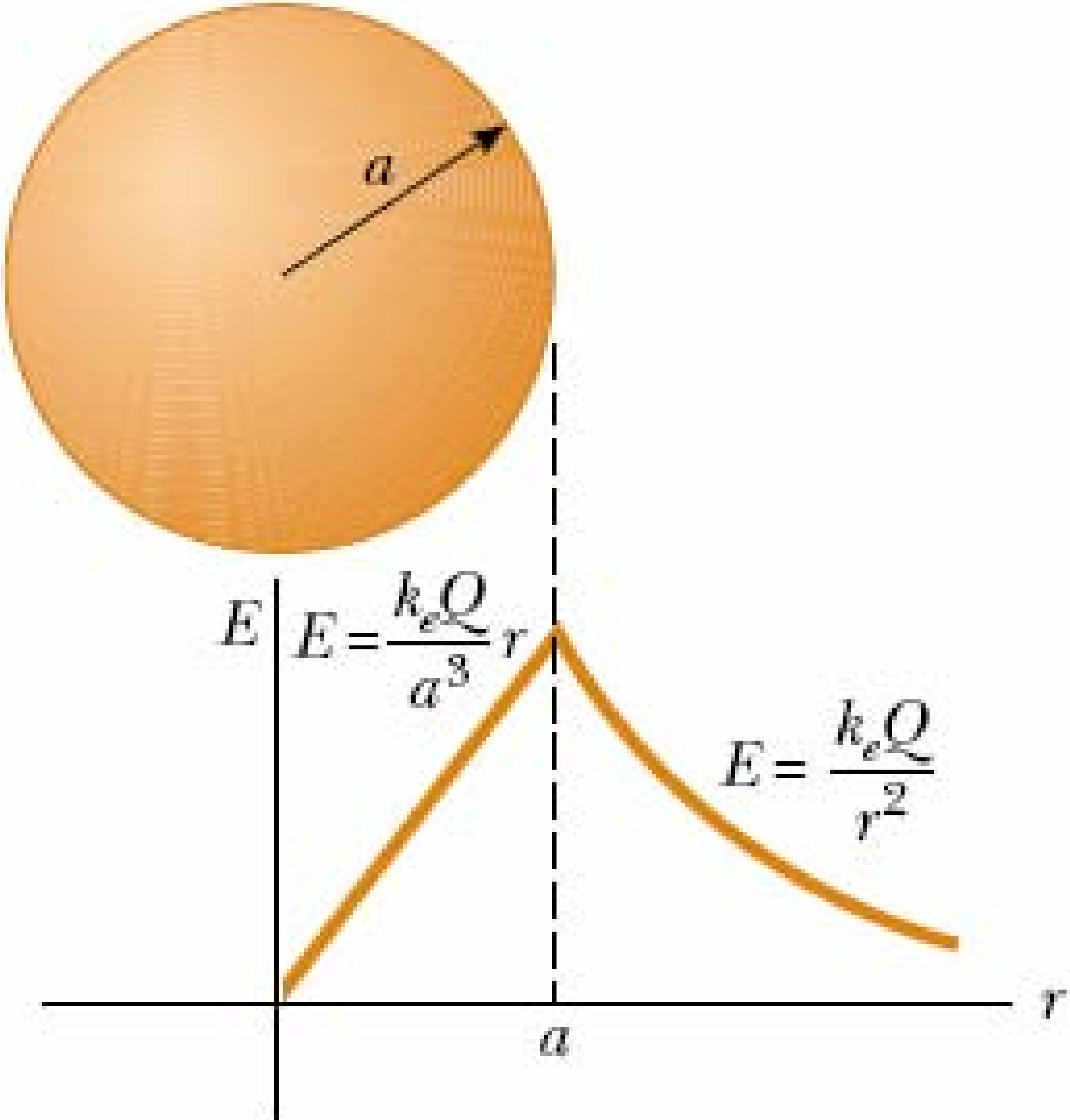
$$\oint E\,dA = E \oint dA = E\,(4\pi r^2) = \frac{q_{\text{in}}}{\epsilon_0}$$

$$E = \boxed{k_e \frac{Q}{r^2}} \qquad (\text{for } r > a)$$

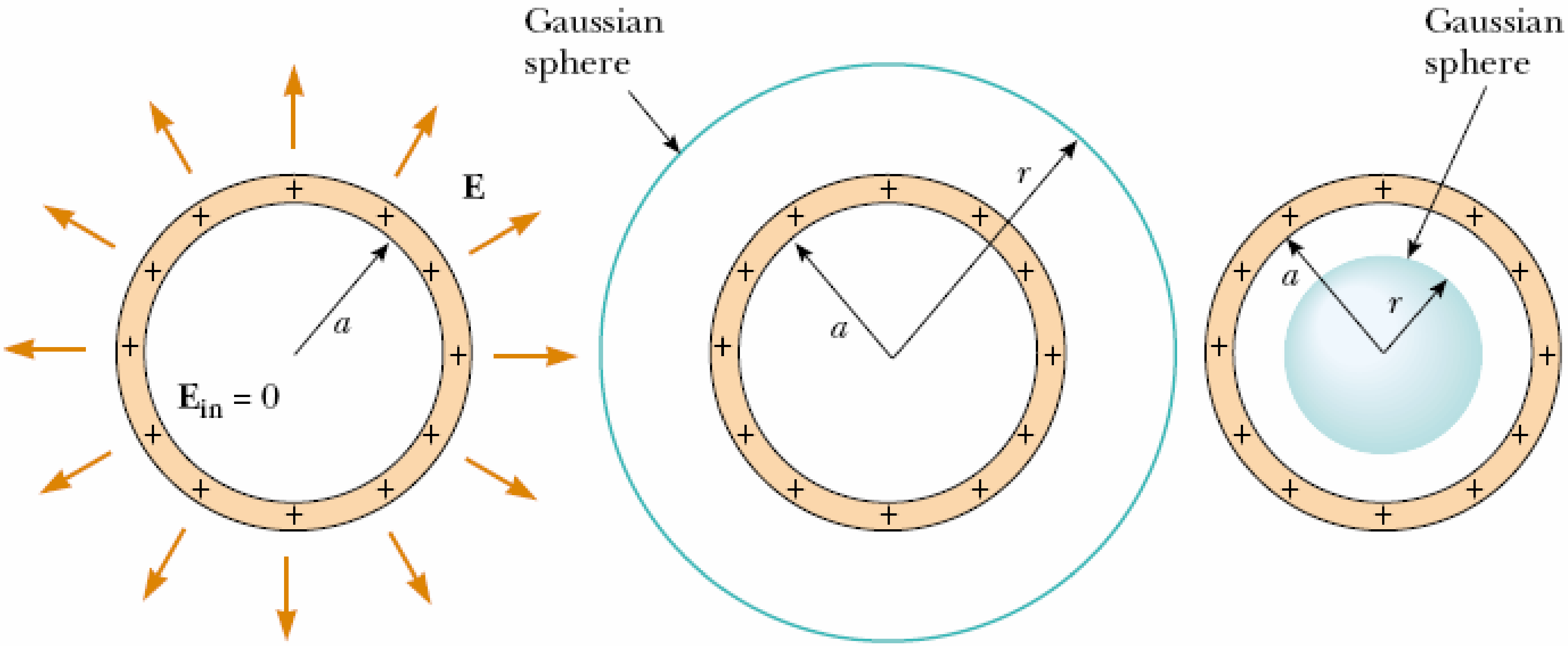
$$E = \frac{q_{\text{in}}}{4\pi\epsilon_0 r^2} = \frac{\rho\left(\frac{4}{3}\pi r^3\right)}{4\pi\epsilon_0 r^2} = \frac{\rho}{3\epsilon_0}\,r$$

$$\rho = Q/\tfrac{4}{3}\pi a^3$$

$$E = \frac{Qr}{4\pi\epsilon_0 a^3} = \boxed{k_e \frac{Q}{a^3}\,r} \qquad (\text{for } r < a)$$

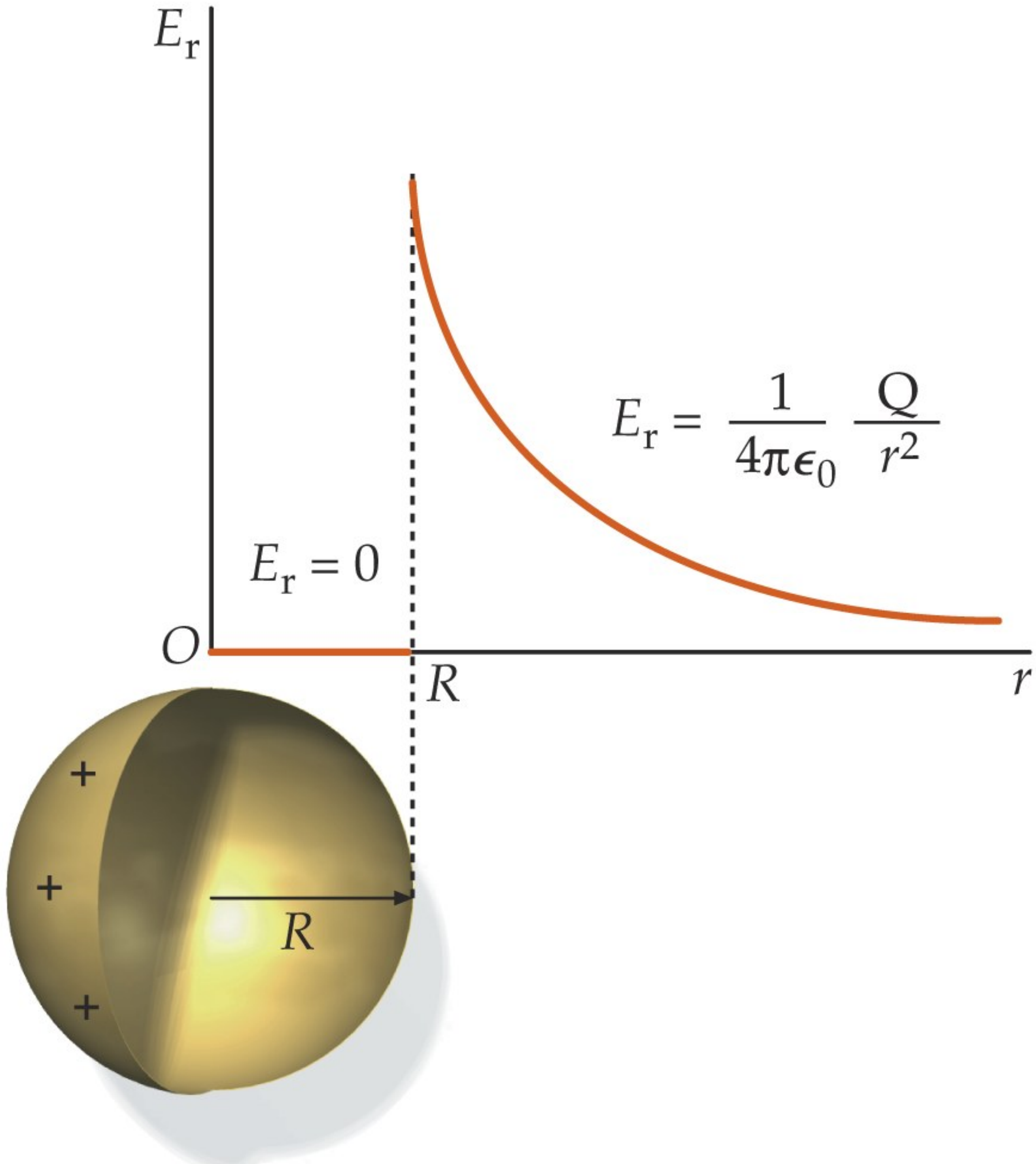

a
E
$E = \frac{k_e Q}{a^3} r$
$E = \frac{k_e Q}{r^2}$
r
a

Cascaron esférico delgado



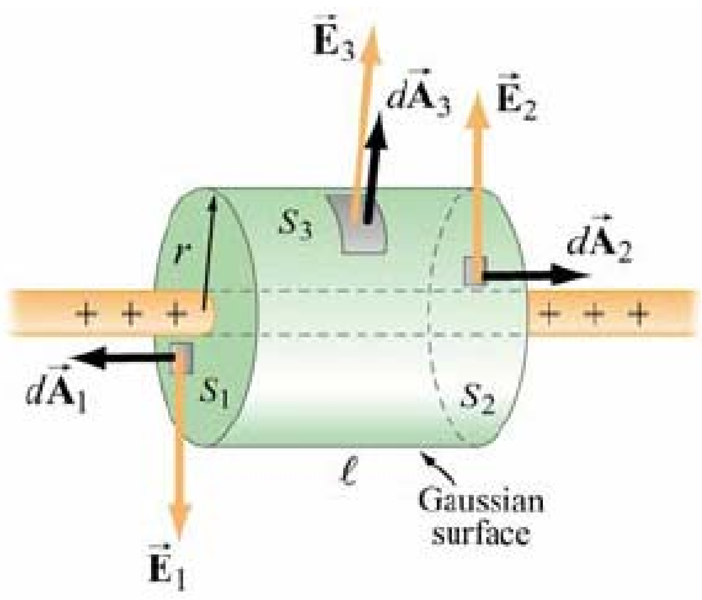
$$E = \frac{Q}{4\pi\varepsilon_0 r^2} = k_e \frac{Q}{r^2}, \quad r \geq a$$

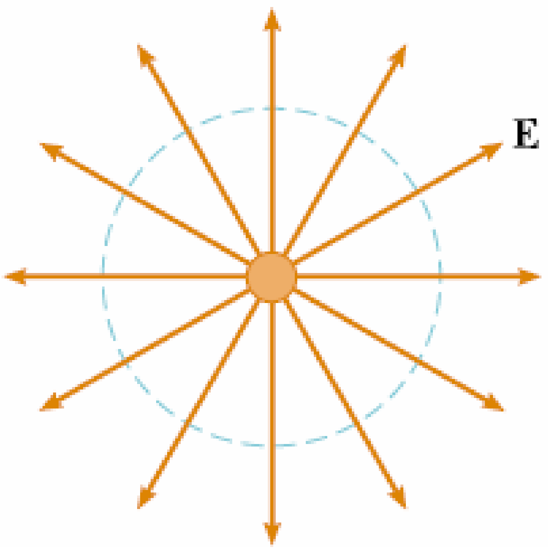
$$E = 0, \quad r < a$$



E_r
$E_r = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}$
$E_r = 0$
O
R
r
+
+
+
R

Alambre infinito





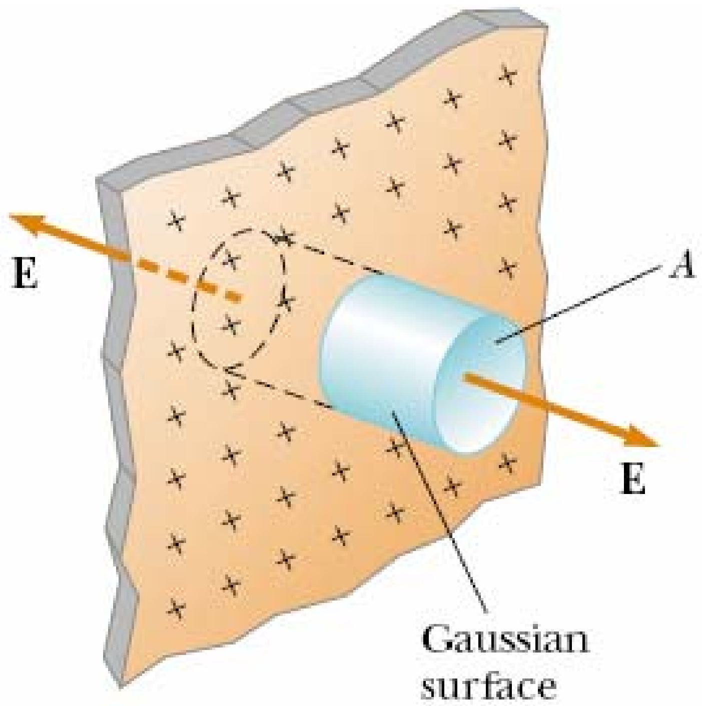
$$\Phi_E = \oiint_S \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \iint_{S_1} \vec{\mathbf{E}}_1 \cdot d\vec{\mathbf{A}}_1 + \iint_{S_2} \vec{\mathbf{E}}_2 \cdot d\vec{\mathbf{A}}_2 + \iint_{S_3} \vec{\mathbf{E}}_3 \cdot d\vec{\mathbf{A}}_3$$

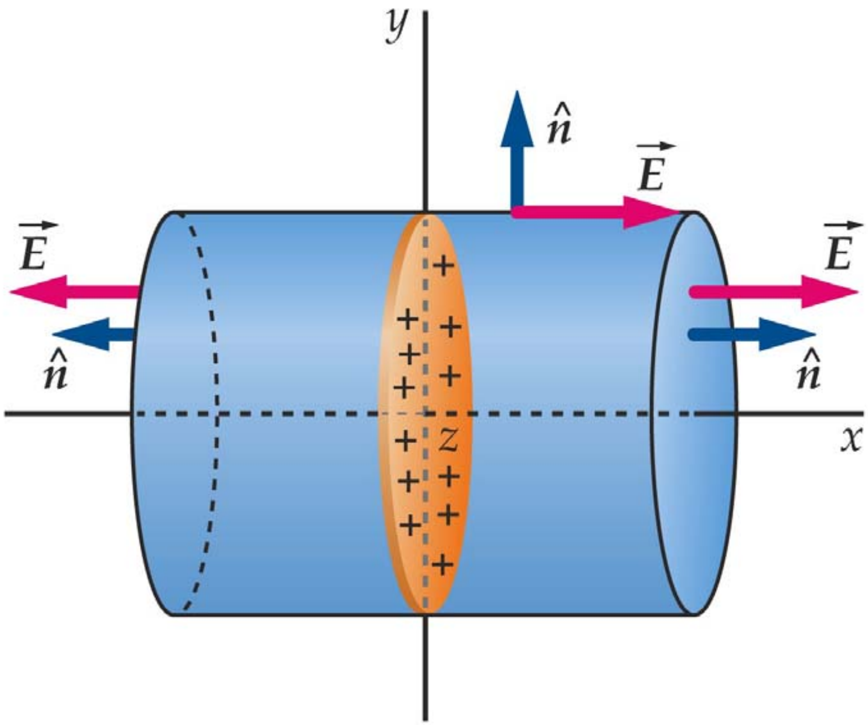
$$= 0 + 0 + E_3 A_3 = E\left(2\pi r \ell\right)$$

$$E(2\pi r \ell) = \frac{\lambda \ell}{\epsilon_0}$$

$$E = \frac{\lambda}{2\pi\epsilon_0 r} = 2k_e \frac{\lambda}{r}$$

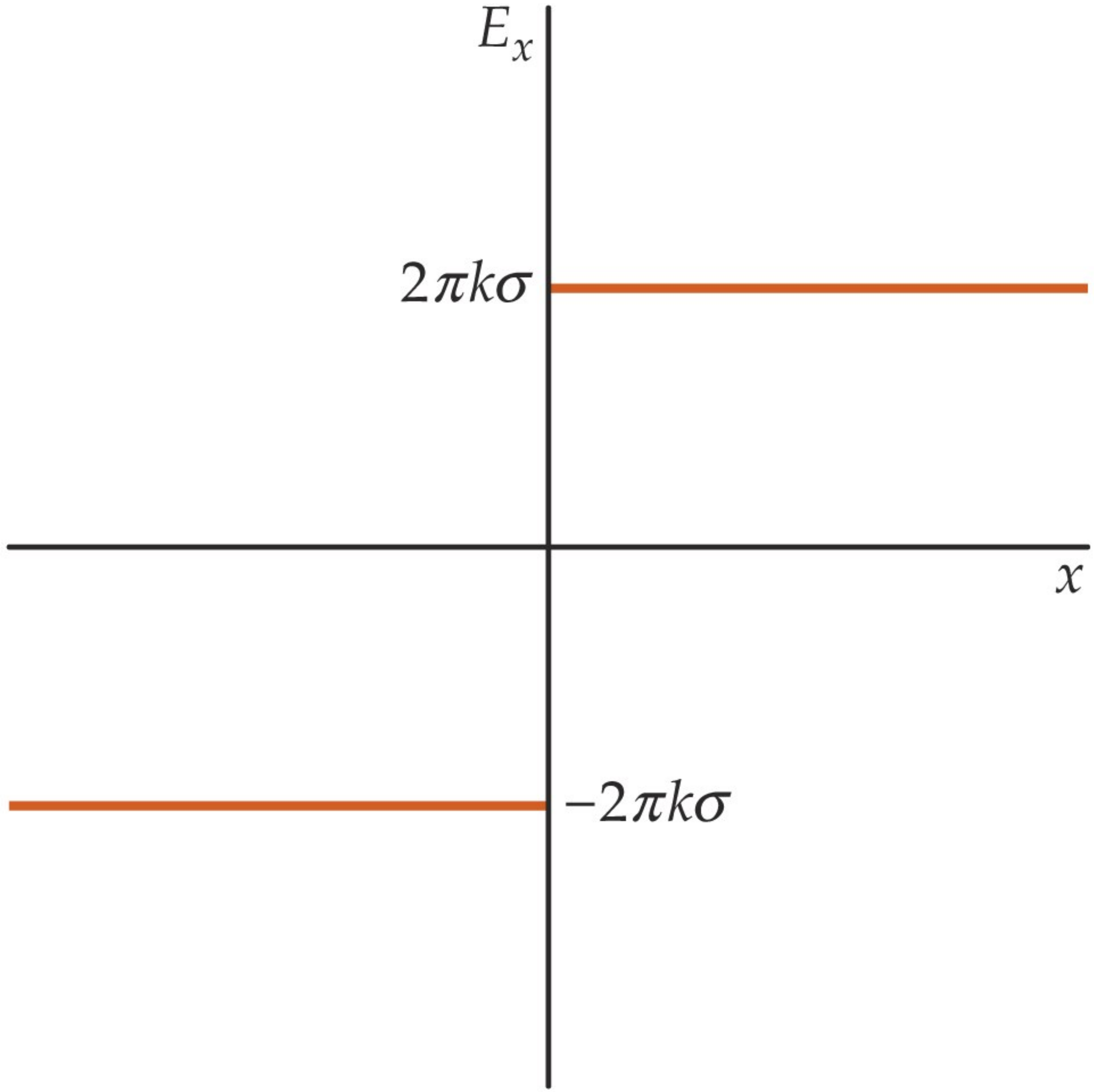
Plano infinito



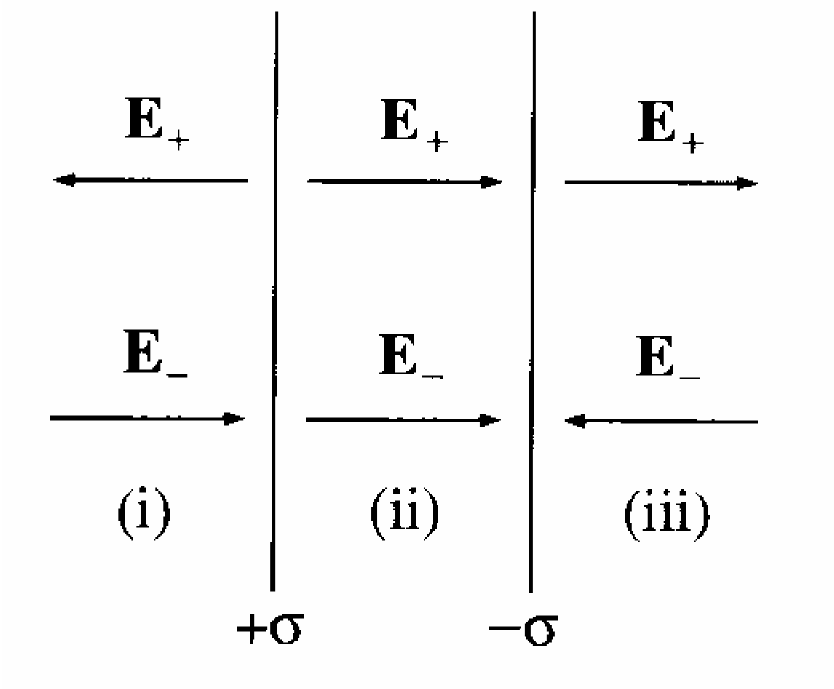


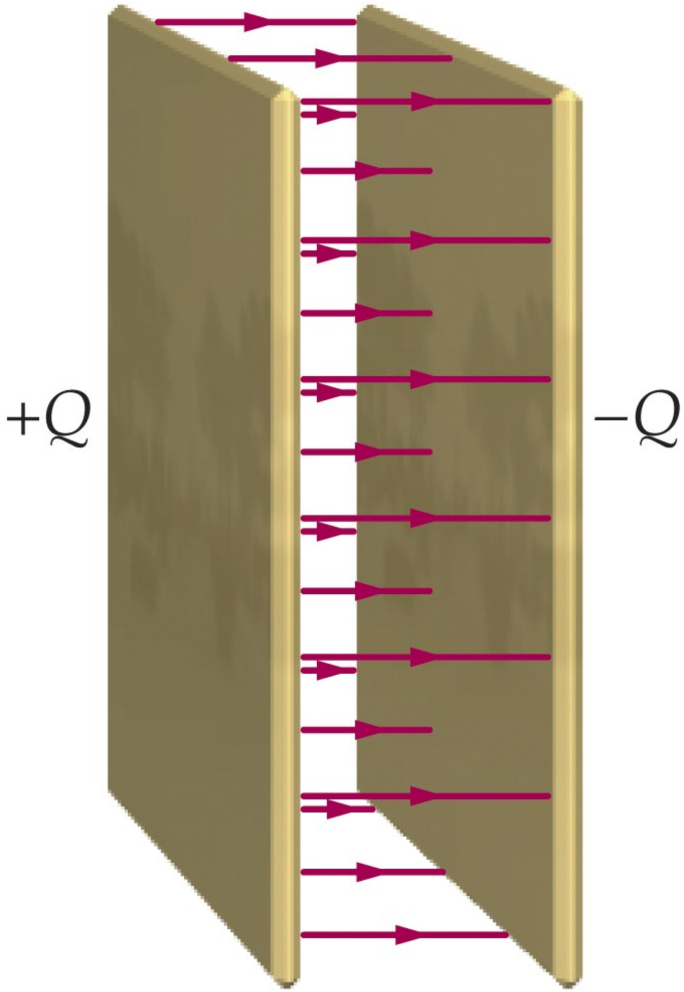
$$\Phi_E = 2EA = \frac{q_{in}}{\epsilon_0} = \frac{\sigma A}{\epsilon_0}$$

$$E = \frac{\sigma}{2\epsilon_0}$$


E_x
$2\pi k\sigma$
x
$-2\pi k\sigma$

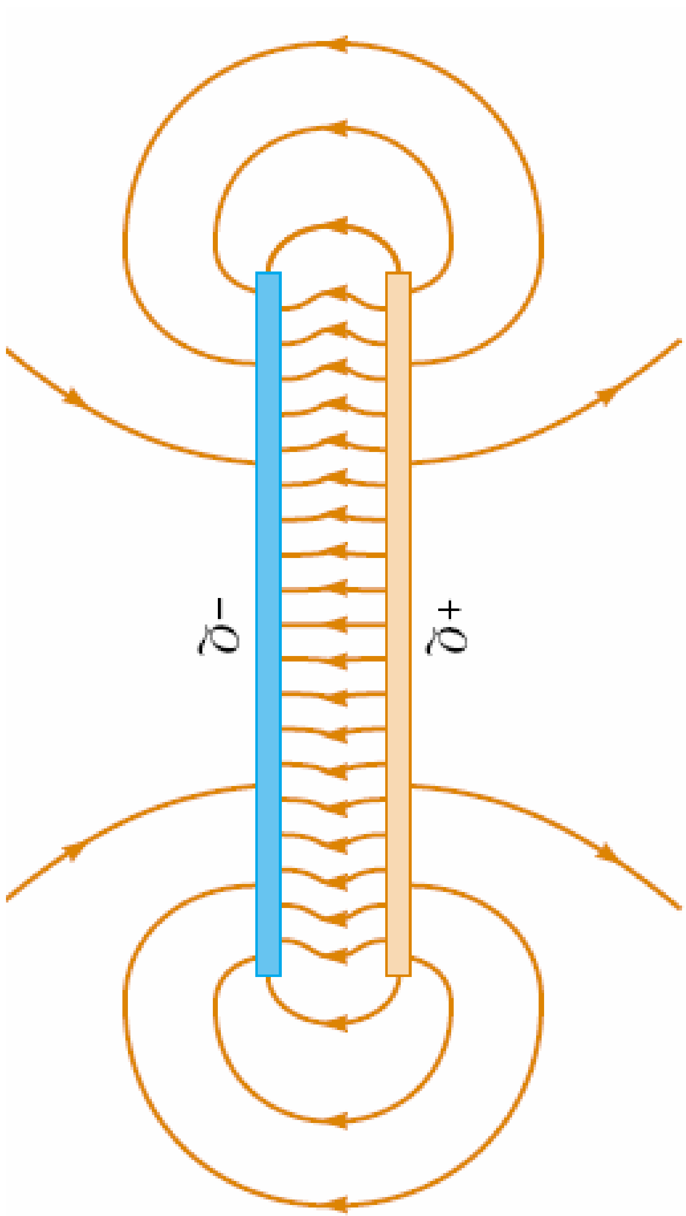
Dos planos infinitos paralelos +σ y -σ

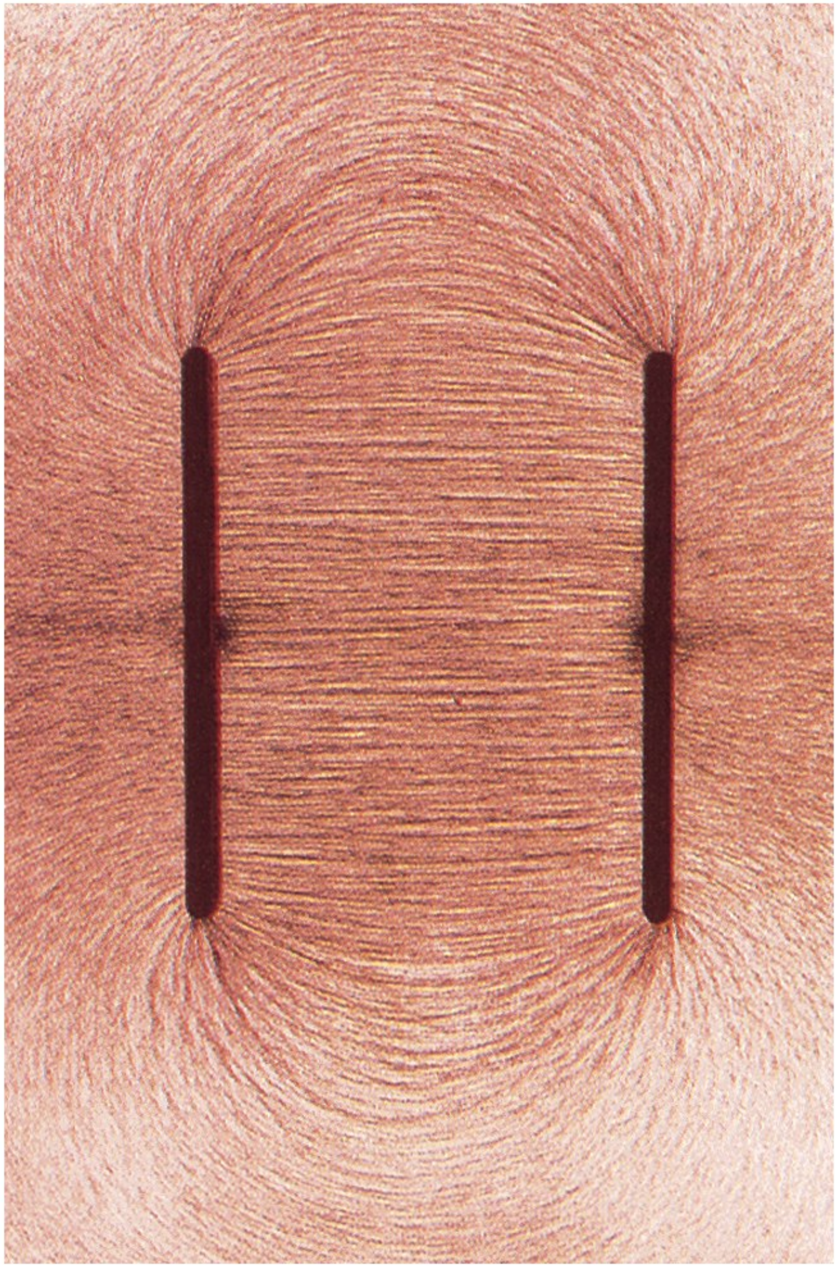




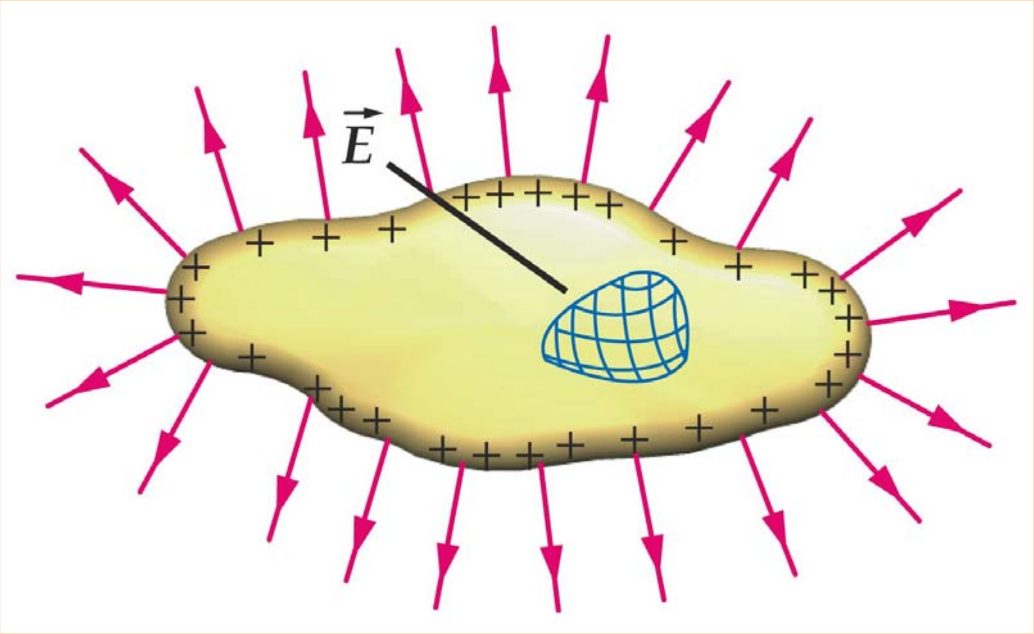
$E = 0$ en (i) y (iii)

$E = 2\,\sigma / \varepsilon_0$ en (ii)


−q
+q



Conductores en equilibrio electrostático



Propiedades básicas de los conductores

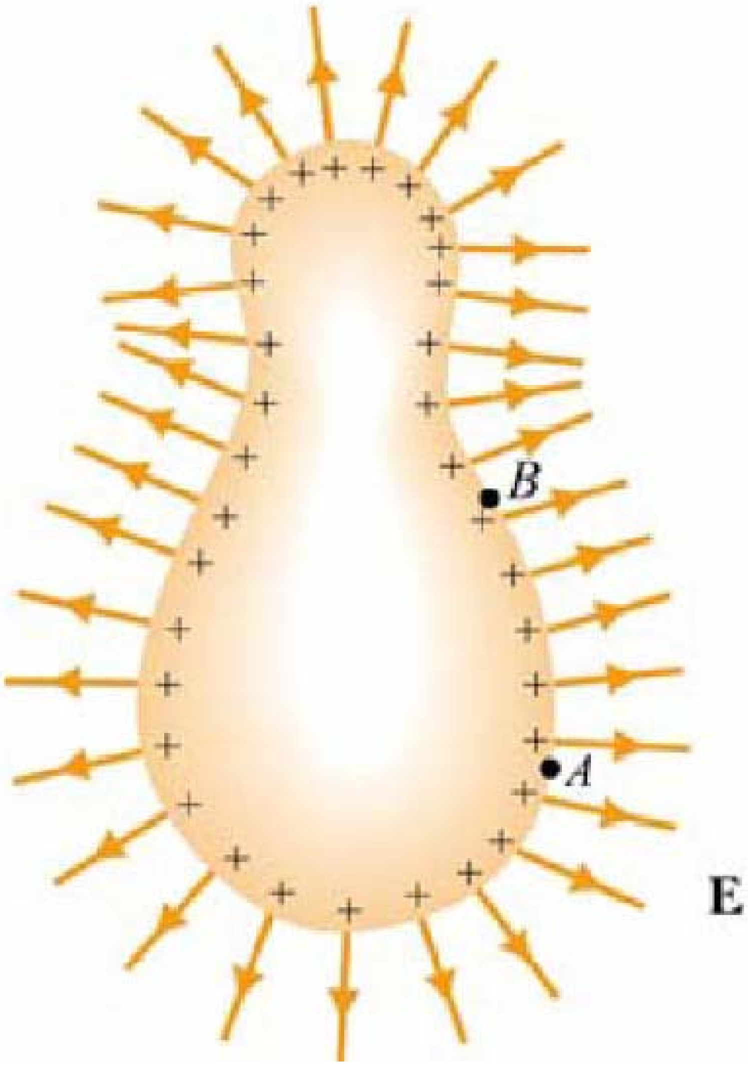
1- E = 0 dentro del conductor
(V cte)

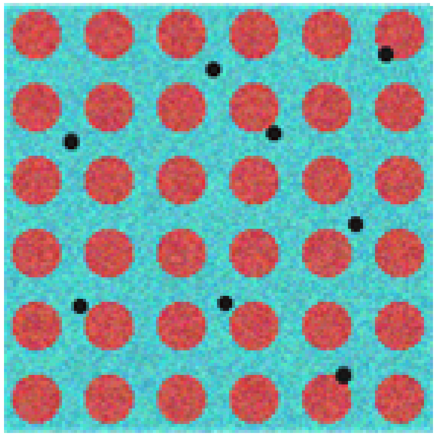
2- La carga neta dentro del conductor es cero.

3- El exceso de carga se ubica en la superficie

4- E es perpendicular a la superficie

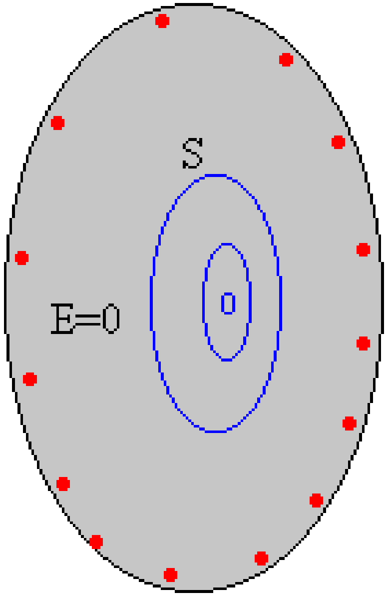
$$E = \frac{\sigma}{\epsilon_0}$$



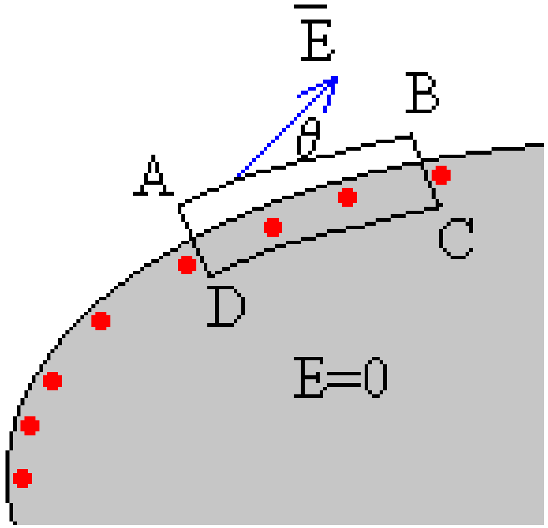


Modelo: gas de electrones libres

Cu: [Ar] $3d^{10}$ $4s^{1}$ → electrones de conducción



Un conductor se caracteriza por que los portadores de carga se pueden mover libremente por el interior del mismo. Si las cargas en un conductor en equilibrio están en reposo, la intensidad del campo eléctrico en todos los puntos interiores del mismo deberá ser cero, de otro modo, las cargas se moverían originado una corriente eléctrica.



$$\oint \mathbf{E}\cdot d\mathbf{l} = 0$$

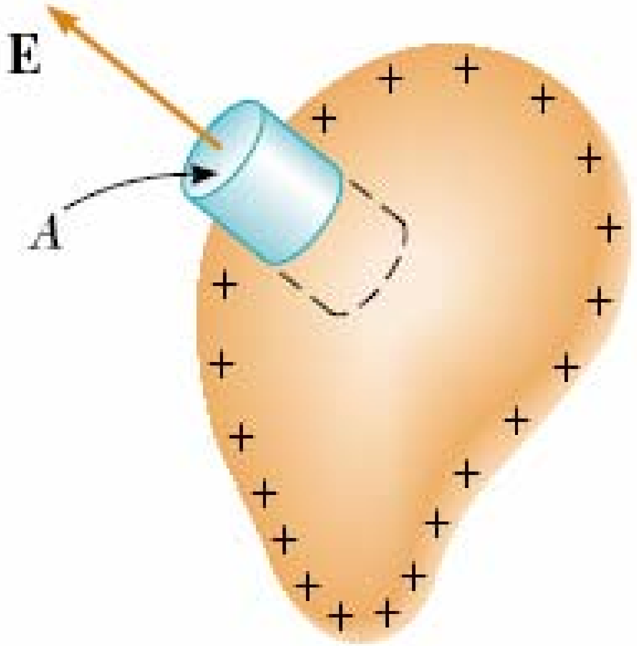
$$\oint \mathbf{E}\cdot d\mathbf{l} = \int_A^B \mathbf{E}\cdot d\mathbf{l} + \int_B^C \mathbf{E}\cdot d\mathbf{l} + \int_C^D \mathbf{E}\cdot d\mathbf{l} + \int_D^A \mathbf{E}\cdot d\mathbf{l} = \int_A^B E\cdot dl\cdot\cos\theta + 0 + 0 + 0$$

La circulación de E es la suma de cuatro contribuciones:

-Tramo CD es nula, por ser el campo en el interior de un conductor cero.

-Tramos AD y BC son aproximadamente cero por ser sus longitudes muy pequeñas |AD|=|BC|≈0.

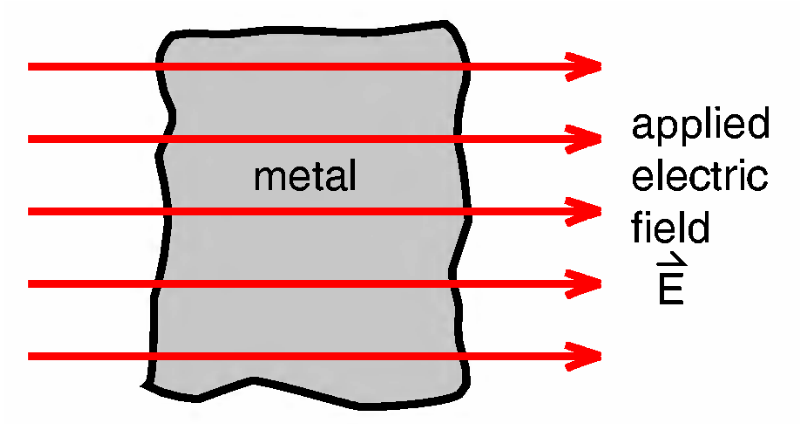
-La contribución en el lado AB deberá ser por tanto cero para que la suma total sea cero. Esto solamente es posible, si el campo E es perpendicular a la superficie del conductor.

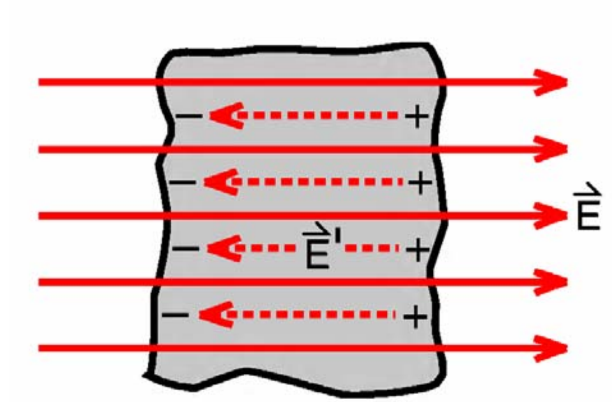


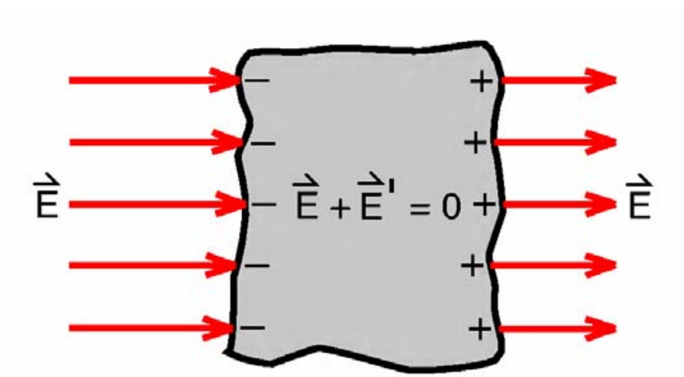
$$\Phi_E = \oint E\,dA = EA = \frac{q_{in}}{\epsilon_0} = \frac{\sigma A}{\epsilon_0}$$

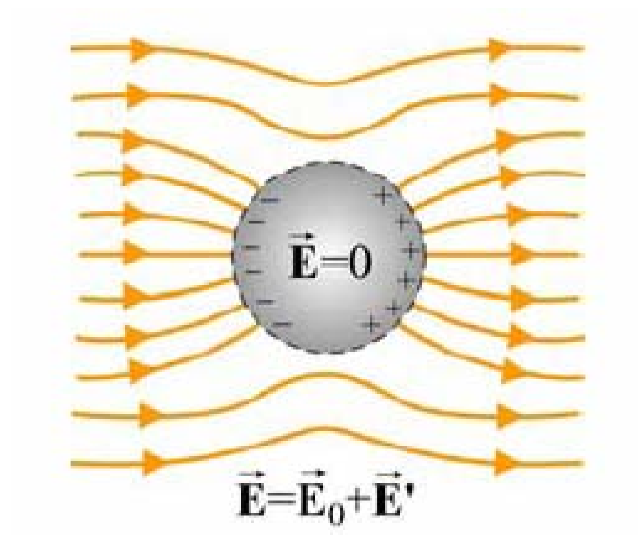
$$E = \frac{\sigma}{\epsilon_0}$$

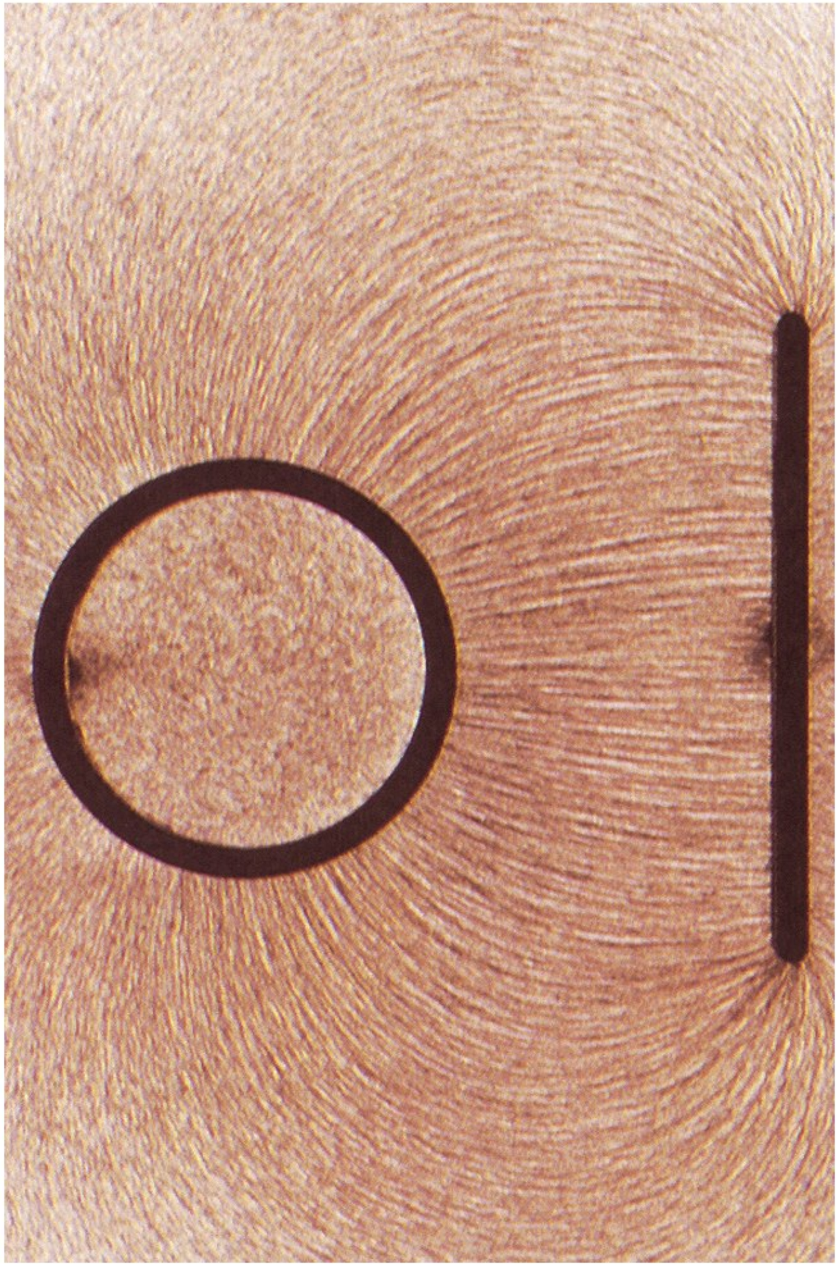
Módulo del campo eléctrico en las proximidades de la superficie de un conductor


metal
applied
electric
field
$\vec{E}$

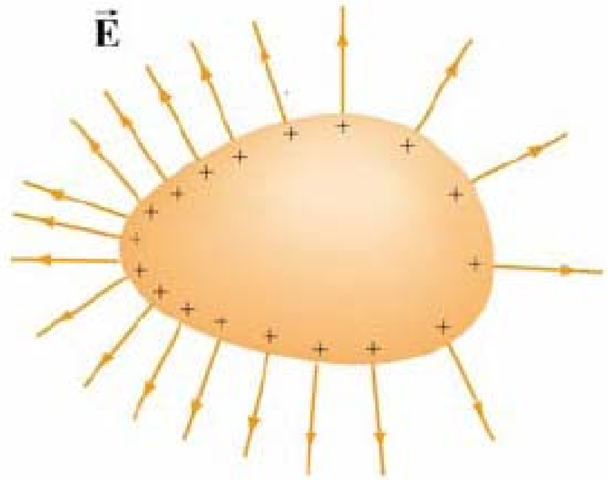

$\vec{E}'$
$\vec{E}$

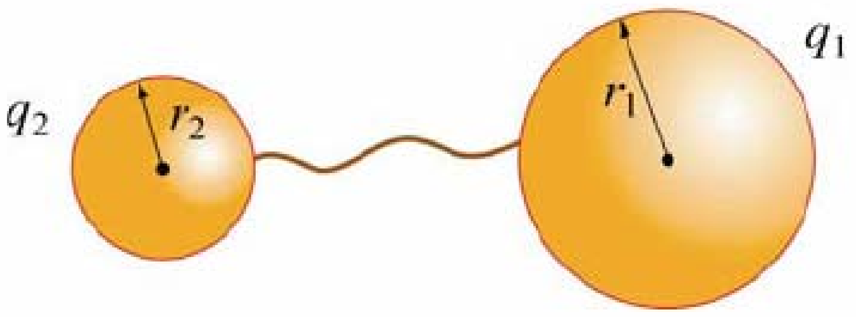

$\vec{E}$
$\vec{E} + \vec{E}' = 0$
$\vec{E}$


$\vec{\mathbf{E}}=0$
$\vec{\mathbf{E}}=\vec{\mathbf{E}}_0+\vec{\mathbf{E}}'$



Efecto punta





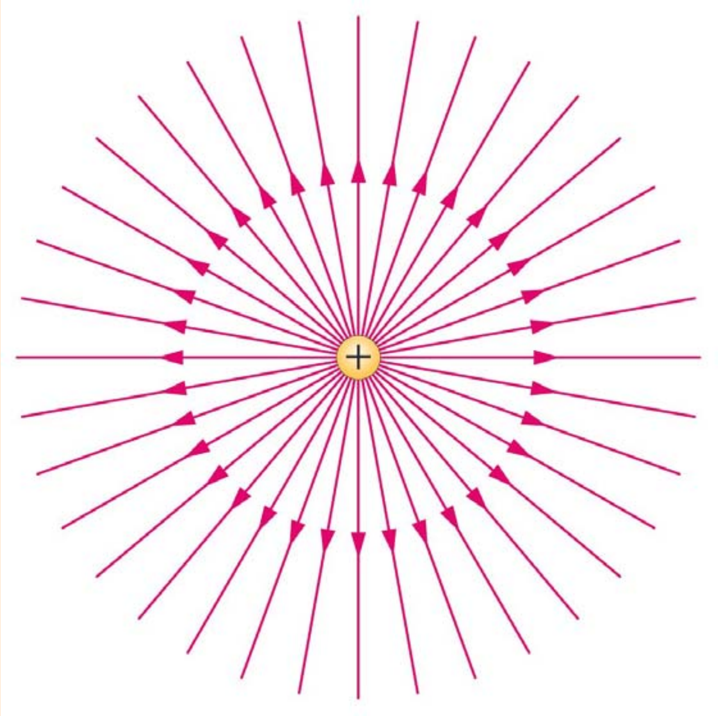
$$V = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_1} = \frac{1}{4\pi\varepsilon_0}\frac{q_2}{r_2}$$

$$\frac{q_1}{r_1} = \frac{q_2}{r_2}$$

$$E_1 = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_1^2} = \frac{\sigma_1}{\varepsilon_0}, \qquad E_2 = \frac{1}{4\pi\varepsilon_0}\frac{q_2}{r_2^2} = \frac{\sigma_2}{\varepsilon_0}$$

$$\boxed{\frac{E_1}{E_2} = \frac{\sigma_1}{\sigma_2} = \frac{r_2}{r_1}}$$

Ley de Gauss en forma diferencial



Su forma integral utilizada en el caso de una distribución extensa de carga puede escribirse de la manera siguiente:

$$\Phi = \oint_S \vec{E} \cdot d\vec{A} = \frac{1}{\epsilon_o} \int_V \rho \, dV = \frac{Q_A}{\epsilon_o}$$

Forma integral de la ley de Gauss

Aplicando al primer término el teorema divergencia queda

$$\oint_V (\vec{\nabla} \cdot \vec{E}) dV = \frac{1}{\epsilon_o} \int_V \rho \, dV$$

Como ambos lados de la igualdad poseen diferenciales volumétricas, y esta expresión debe ser cierta para cualquier volumen, solo puede ser que:

$$\boxed{\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_o}}$$

Forma diferencial de la ley de Gauss

Las cargas eléctricas son las fuentes de los campos eléctricos

Esta ecuacion se puede escribir en términos del potencial eléctrico:

$$\vec{E} = -\nabla V$$

Reemplazando, se obtiene:

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = -\frac{\rho}{\varepsilon_0}$$

Ecuación de Poisson

Si se toma en una región del campo donde la densidad de carga es cero:

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0$$

Ecuación de Laplace

$\nabla \times \mathbf{E} = ?$

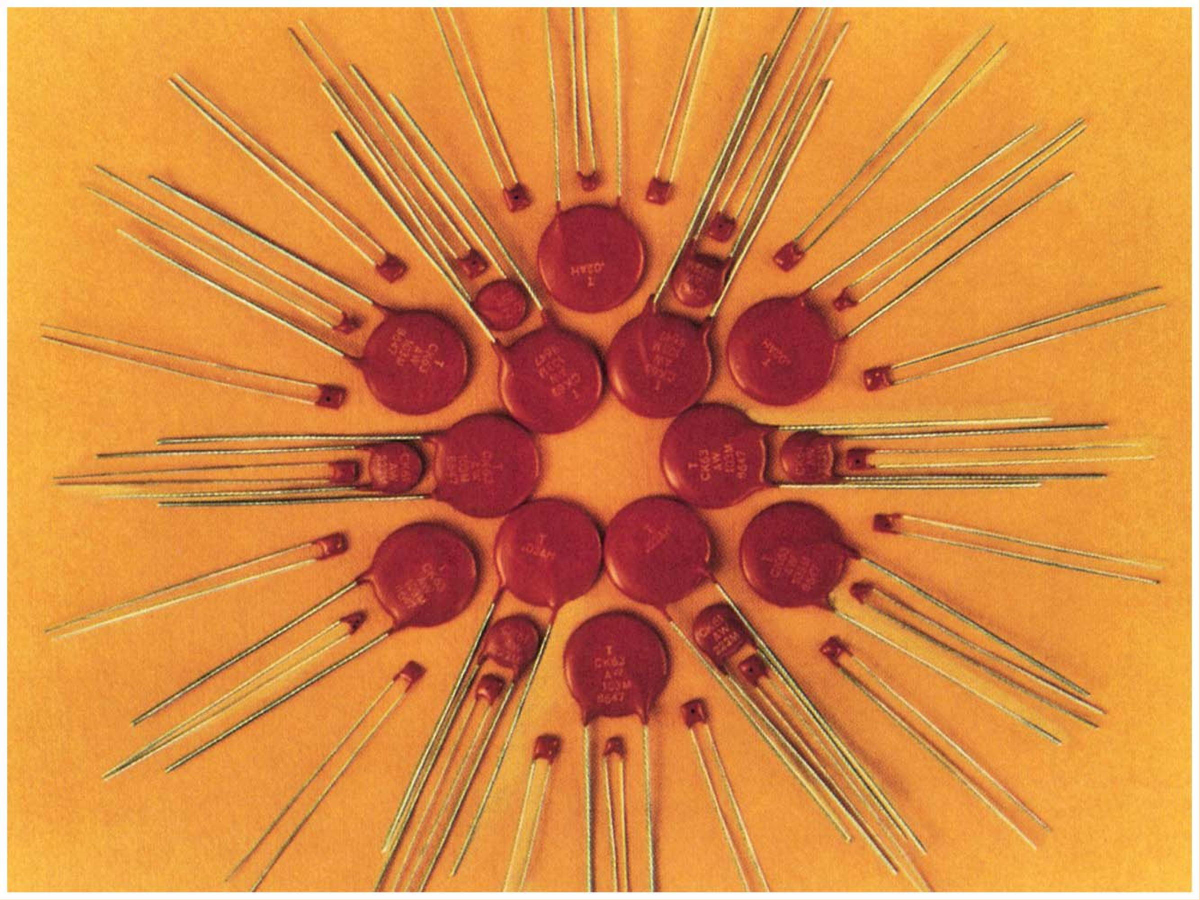
Como: $\vec{E} = -\nabla V$ ⟹ $\nabla \times \mathbf{E} = 0$

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_o}$$

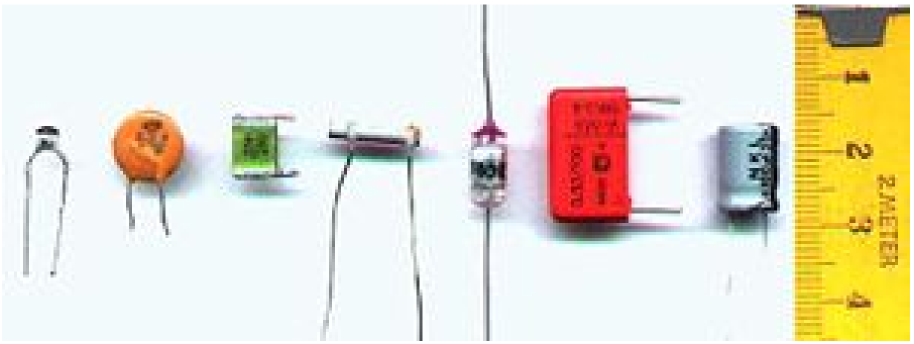
$$\vec{\nabla} \times \vec{E} = 0$$

Ecuaciones de Maxwell para la Electrostática

Capacitores y Dieléctricos



Capacitores → **Dispositivos que almacenan cargas**



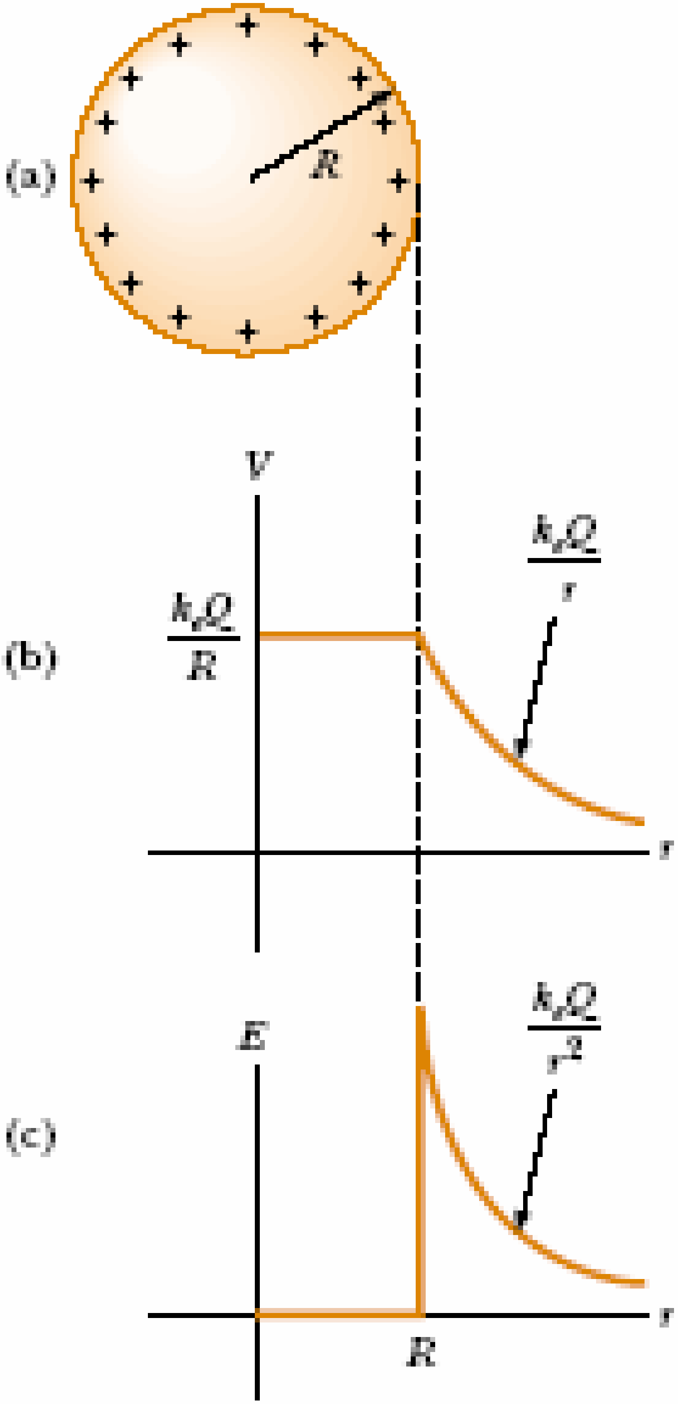
Capacidad de un conductor

En un conductor aislado q es proporcional a V $(V(\infty) = 0)$

$$q = C\,V$$

C: Capacidad

$[C] = C/V = F$ **(Faradio)**

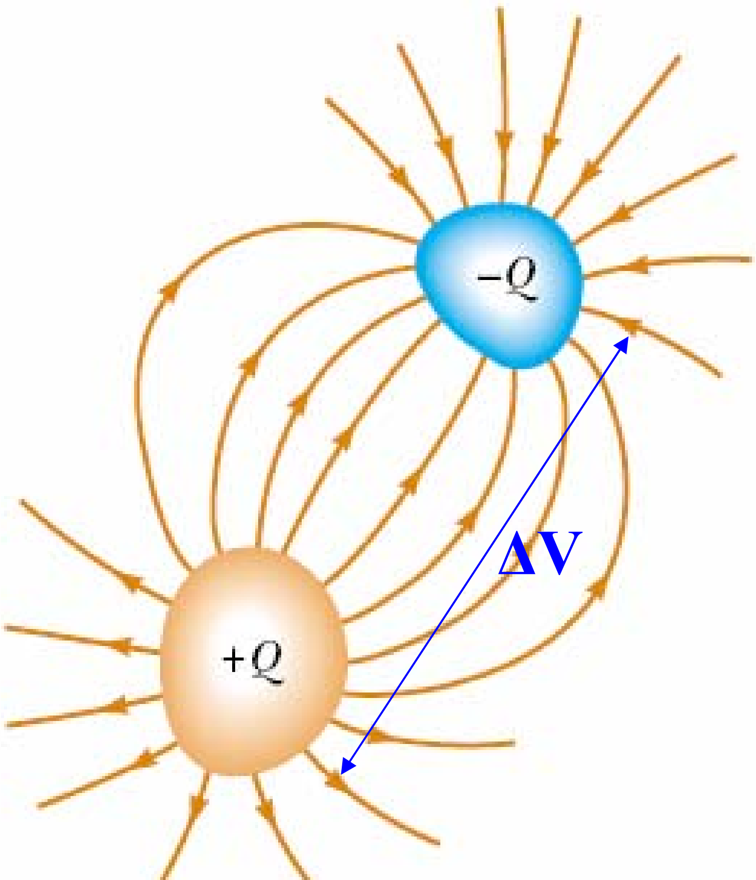


Ejemplo:

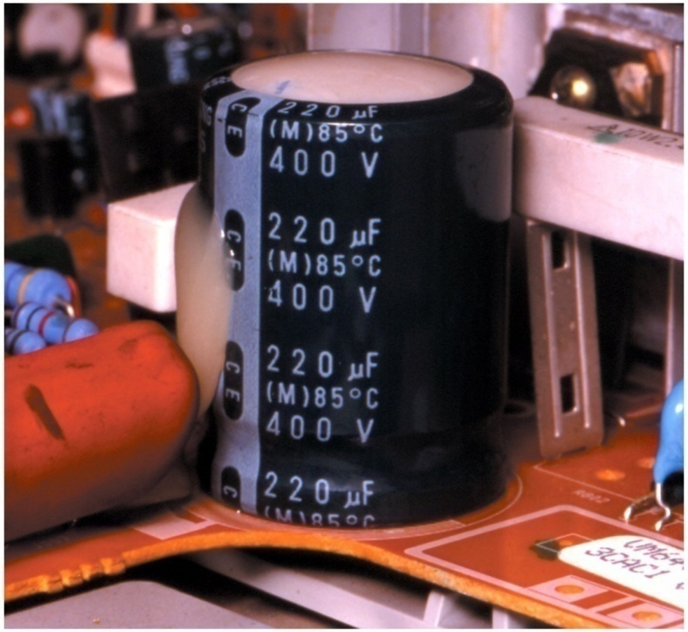
Conductor esférico cargado

$$C = \frac{Q}{\Delta V} = \frac{Q}{k_e Q / R} = \frac{R}{k_e} = 4\pi\epsilon_0 R$$

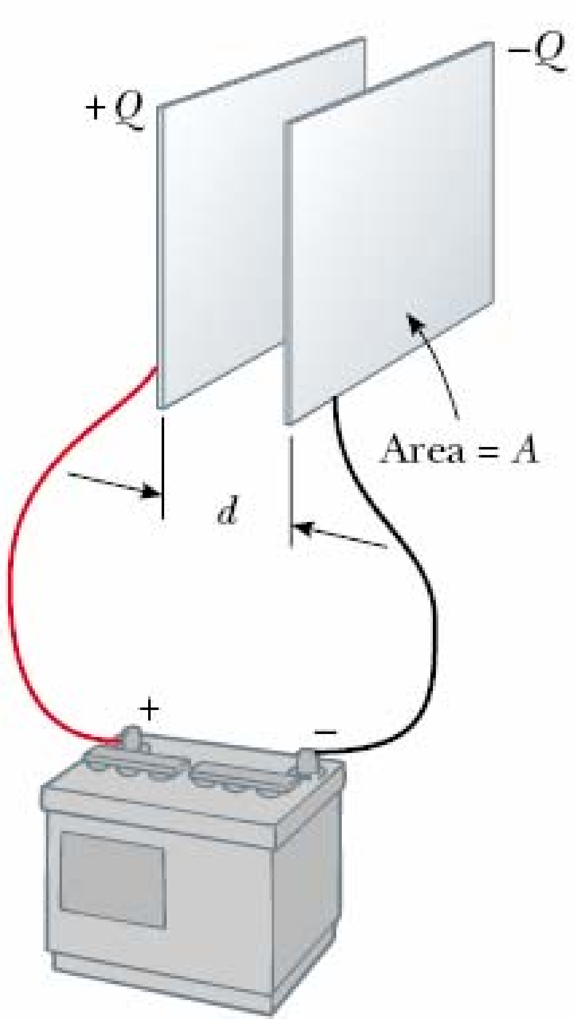
Capacitores



$$C \equiv \frac{Q}{\Delta V}$$



Capacitor de placas paralelas



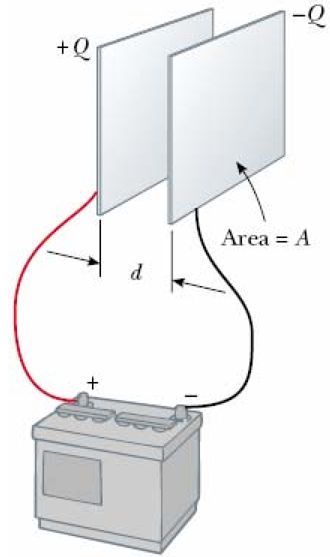
$$E = \frac{\sigma}{\epsilon_0} = \frac{Q}{\epsilon_0 A}$$

$$\Delta V = Ed = \frac{Qd}{\epsilon_0 A}$$

$$C = \frac{Q}{\Delta V} = \frac{Q}{Qd/\epsilon_0 A}$$

$$C = \frac{\epsilon_0 A}{d}$$

Ejemplo:

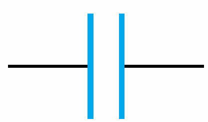


$A = 1$ cm x 1 cm

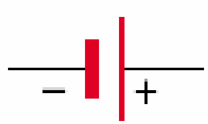
$d = 1$ mm

$C = 0.88110^{-12}$ F = 0.88 pF

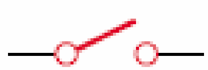
Símbolos



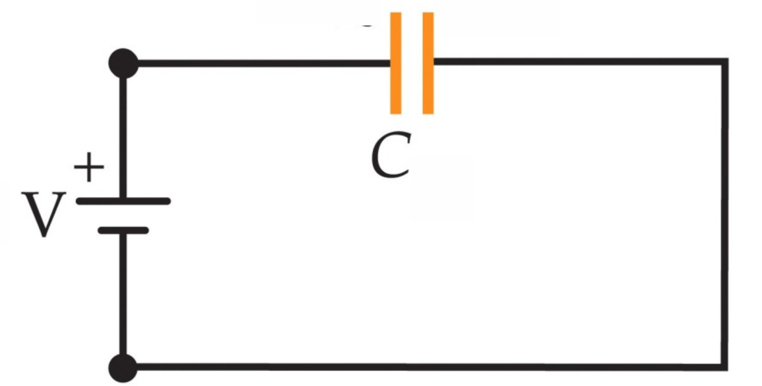
Capacitor



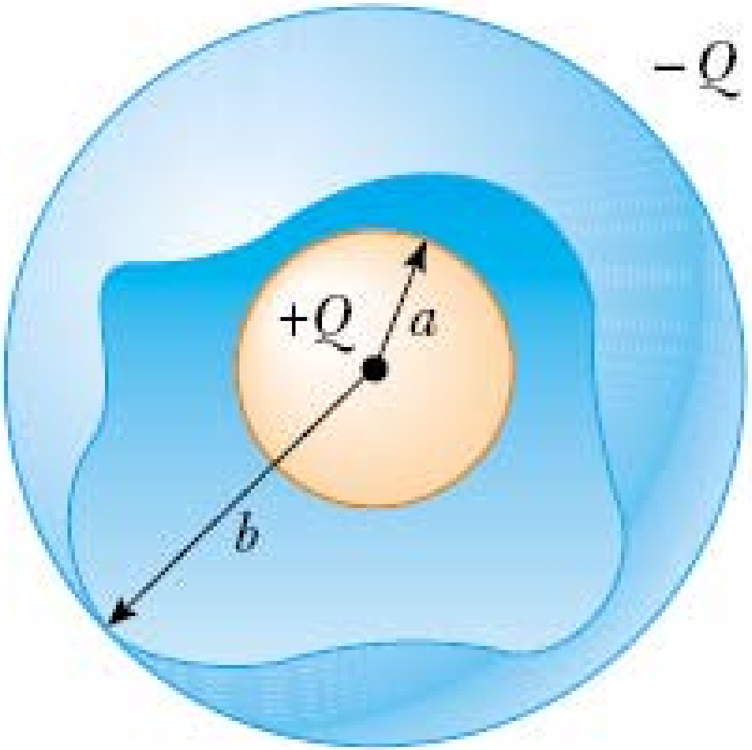
Bateria



Interruptor



Capacitor esférico



$$V_b - V_a = -\int_a^b E_r\,dr = -k_eQ\int_a^b \frac{dr}{r^2} = k_eQ\left[\frac{1}{r}\right]_a^b$$

$$= k_eQ\left(\frac{1}{b} - \frac{1}{a}\right)$$

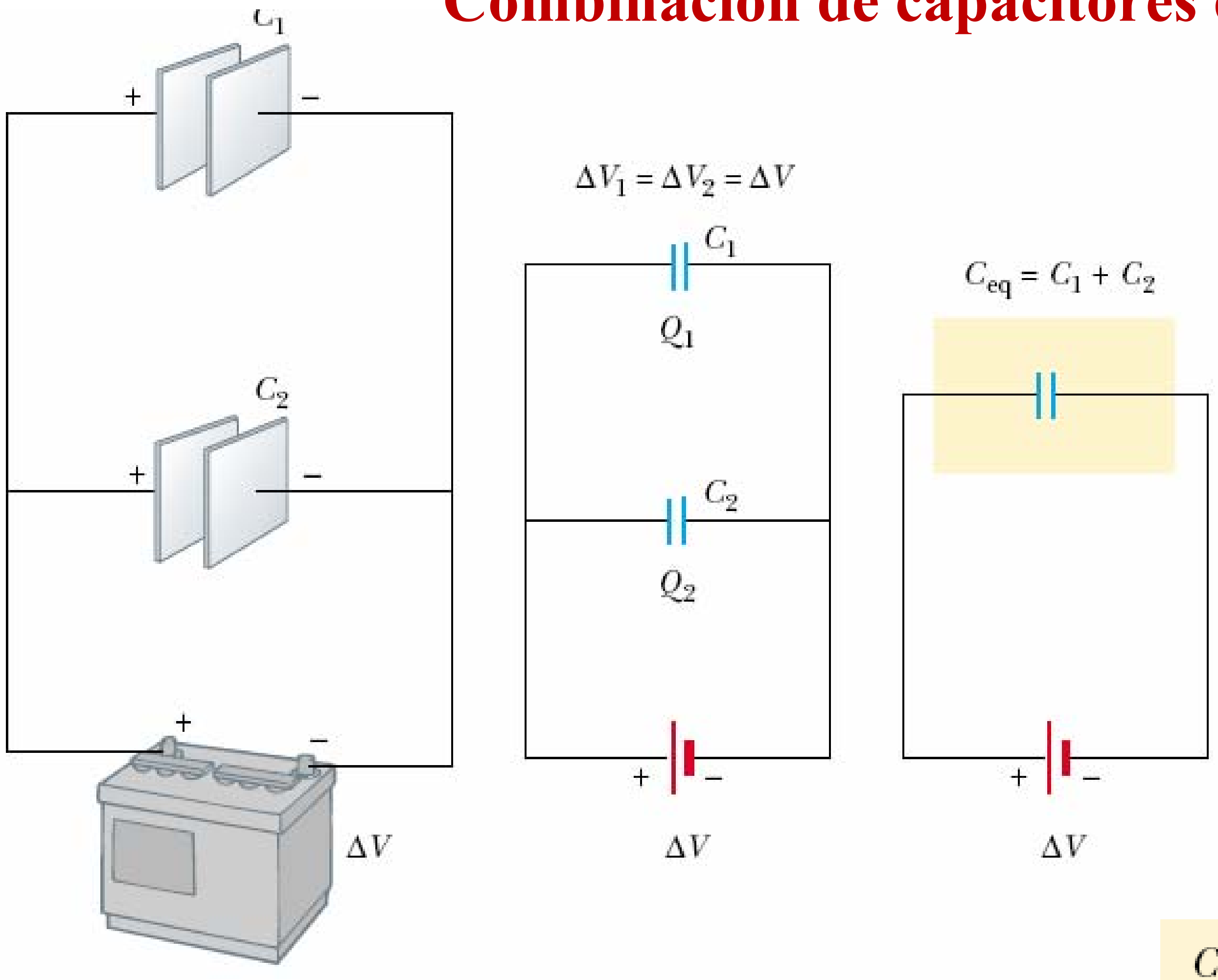
$$\Delta V = |V_b - V_a| = k_eQ\,\frac{(b-a)}{ab}$$

$$C = \frac{Q}{\Delta V} = \frac{ab}{k_e(b-a)}$$

$b \longrightarrow \infty$:

$$C = \lim_{b\to\infty} \frac{ab}{k_e(b-a)} = \frac{ab}{k_e(b)} = \frac{a}{k_e} = 4\pi\epsilon_0 a$$

Combinación de capacitores en paralelo



$$C_{eq} = C_1 + C_2 + C_3 + \cdots$$

$$Q = C_{eq}\,\Delta V$$

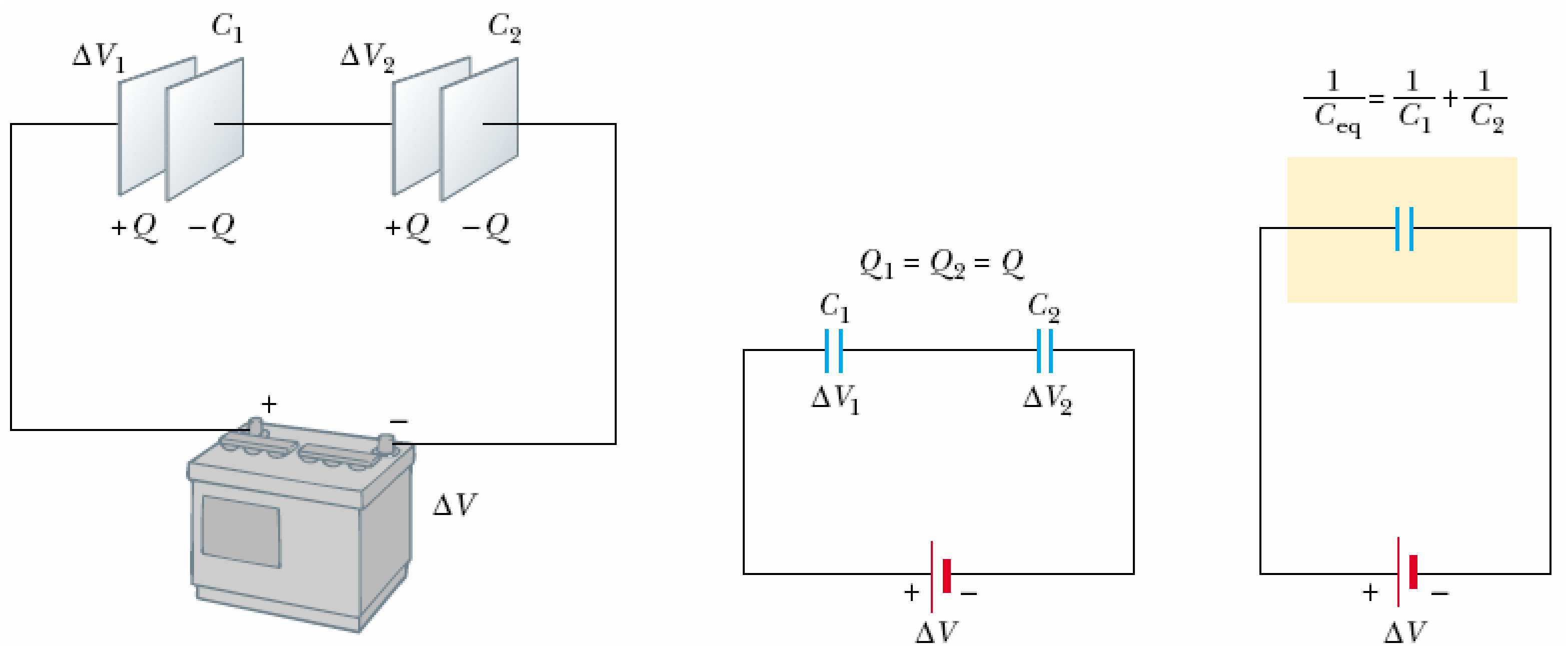
$$Q = Q_1 + Q_2$$

$$C_{eq}\,\Delta V = C_1\,\Delta V + C_2\,\Delta V$$

$$Q_1 = C_1\,\Delta V \qquad Q_2 = C_2\,\Delta V$$

$$C_{eq} = C_1 + C_2$$

Combinación en serie

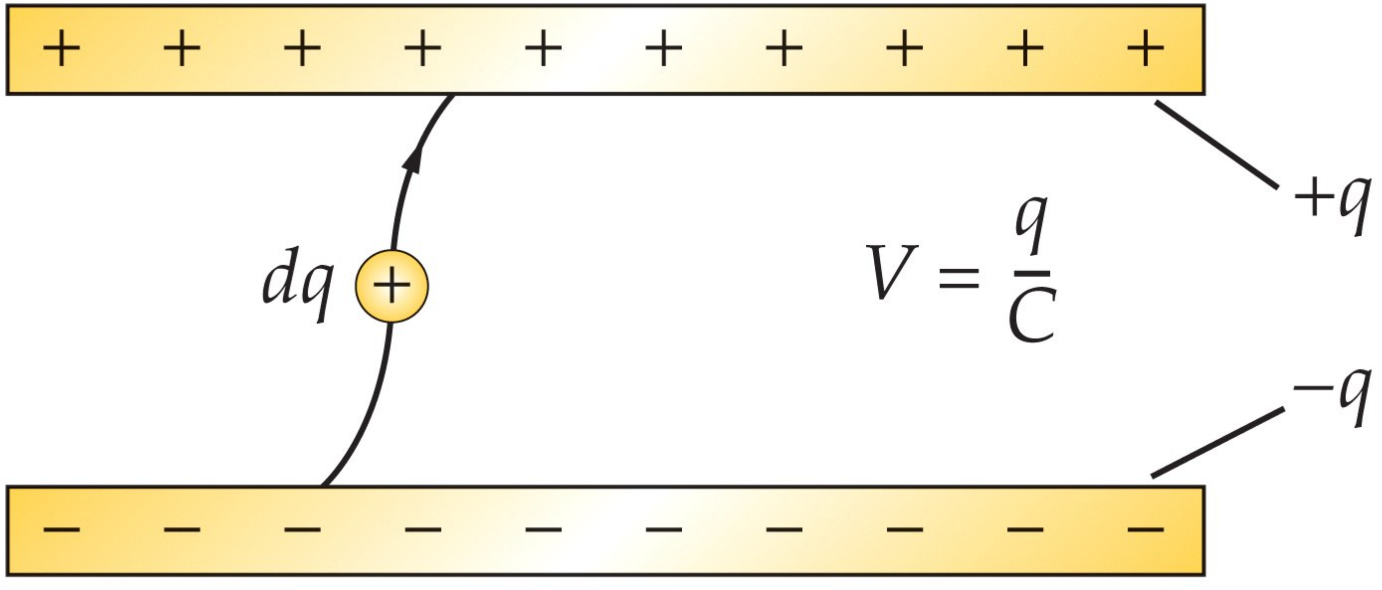


$$\Delta V = \Delta V_1 + \Delta V_2 \qquad \Delta V_1 = \frac{Q}{C_1} \qquad \Delta V_2 = \frac{Q}{C_2}$$

$$\Delta V = \frac{Q}{C_{eq}} \qquad \frac{Q}{C_{eq}} = \frac{Q}{C_1} + \frac{Q}{C_2} \qquad \frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots$$

Energía almacenada en un capacitor



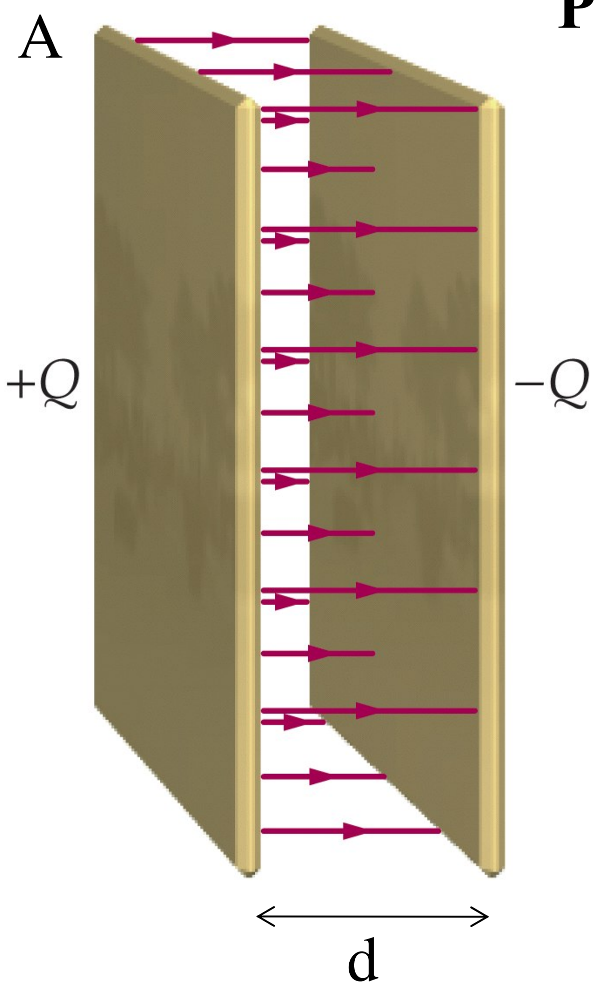
$$dW = \Delta V\, dq = \frac{q}{C}\, dq$$

$$W = \int_0^Q \frac{q}{C}\, dq = \frac{1}{C}\int_0^Q q\, dq = \frac{Q^2}{2C}$$

$$U = \frac{Q^2}{2C} = \tfrac{1}{2}Q\,\Delta V = \tfrac{1}{2}C(\Delta V)^2$$

Energía potencial almacenada en el capacitor

Para un capacitor de placas paralelas



$$\Delta V = Ed.$$

$$C = \epsilon_0 A/d$$

$$U = \frac{1}{2}\frac{\epsilon_0 A}{d}(E^2 d^2) = \frac{1}{2}(\epsilon_0 A d)E^2$$

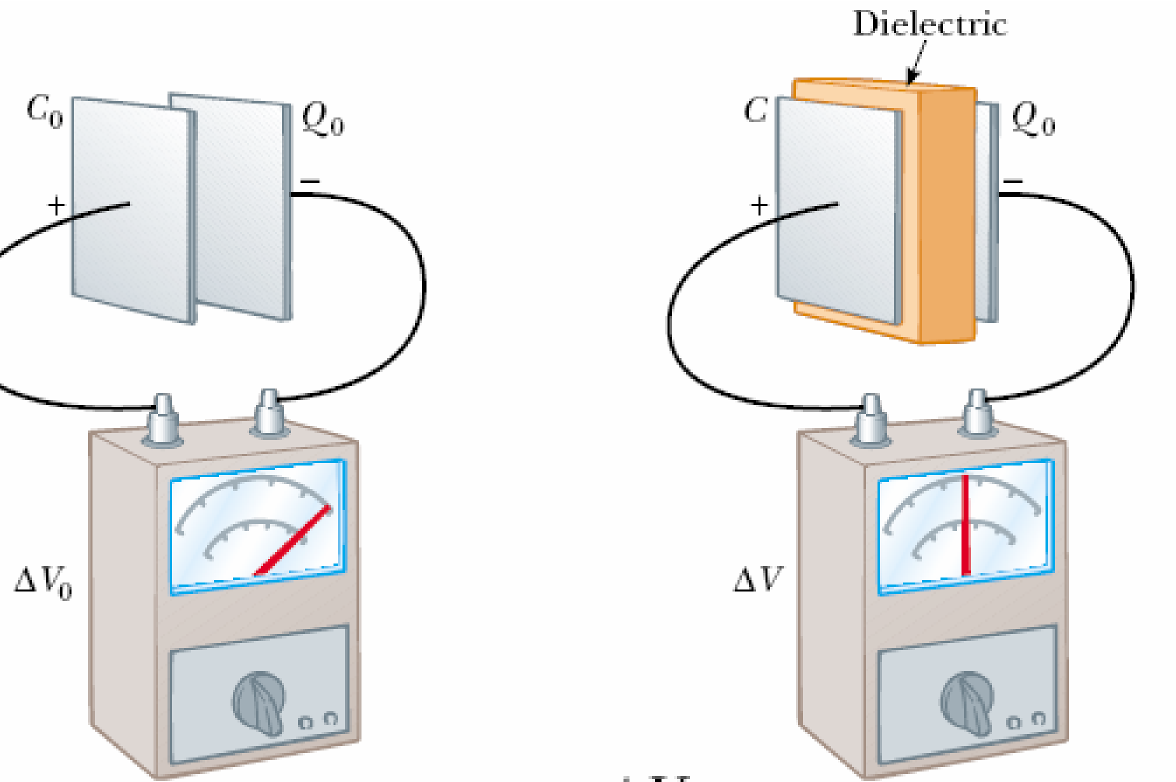
Ad : volumen comprendido entre las placas

$$u_E = \frac{1}{2}\epsilon_0 E^2$$

Densidad de energía

Energía por unidad de volumen almacenada en el campo eléctrico

Capacitores con dieléctrico



$$\Delta V = \frac{\Delta V_0}{\kappa}$$

$$C = \frac{Q_0}{\Delta V} = \frac{Q_0}{\Delta V_0/\kappa} = \kappa \frac{Q_0}{\Delta V_0}$$

$$C = \kappa C_0$$

Para un capacitor de placas paralelas:

$$C = \kappa \frac{\epsilon_0 A}{d}$$

κ : constante dieléctrica

V y d están limitadas

E = V/d   cuando E supera cierto valor se produce una descarga eléctrica y el medio empieza a conducir

Ruptura Dieléctrica

Approximate Dielectric Constants and Dielectric Strengths of Various Materials at Room Temperature

Material	Dielectric Constant κ	Dielectric Strength[a] (10^6 V/m)
Air (dry)	1.000 59	3
Bakelite	4.9	24
Fused quartz	3.78	8
Mylar	3.2	7
Neoprene rubber	6.7	12
Nylon	3.4	14
Paper	3.7	16
Paraffin-impregnated paper	3.5	11
Polystyrene	2.56	24
Polyvinyl chloride	3.4	40
Porcelain	6	12
Pyrex glass	5.6	14
Silicone oil	2.5	15
Strontium titanate	233	8
Teflon	2.1	60
Vacuum	1.000 00	—
Water	80	—

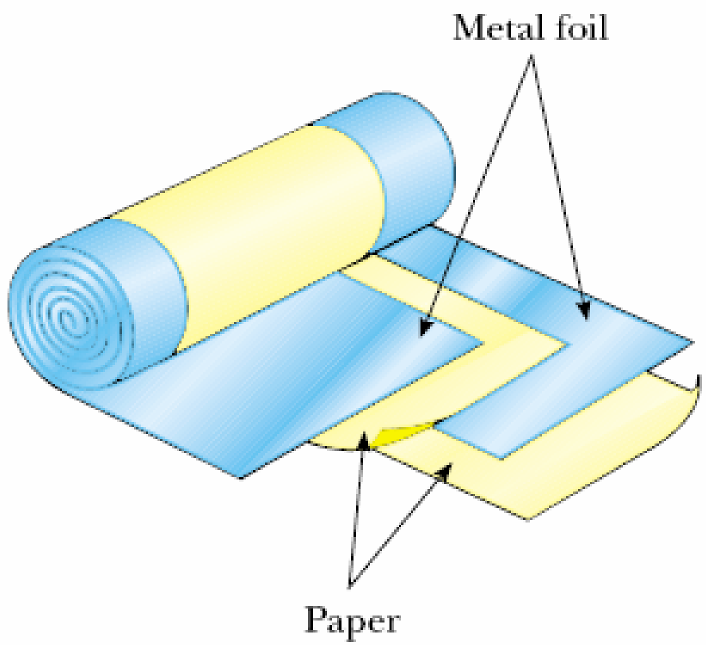
El dieléctrico brinda las siguiente ventajas:

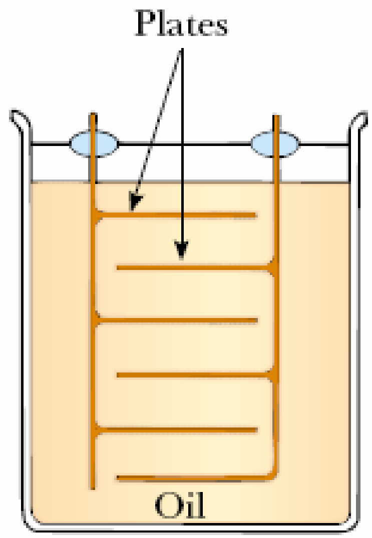
-Aumenta la capacidad

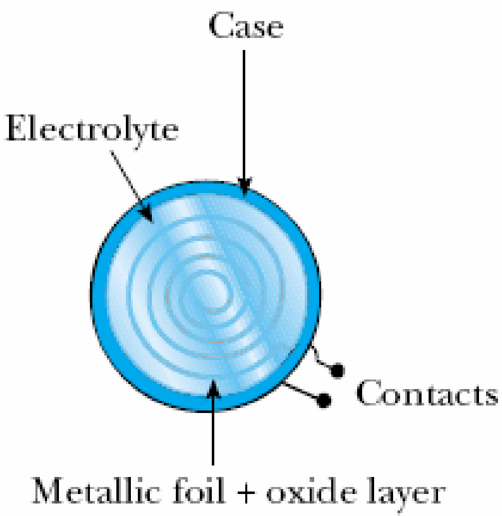
- Aumenta el voltaje de operación máximo

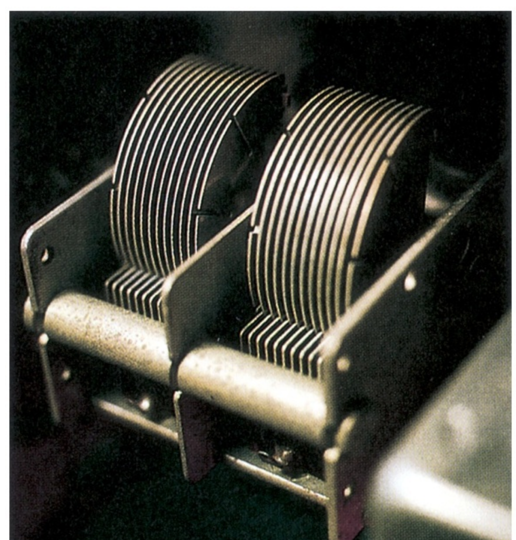
- Proporciona soporte mecanico entre placas

Tipos de capacitores



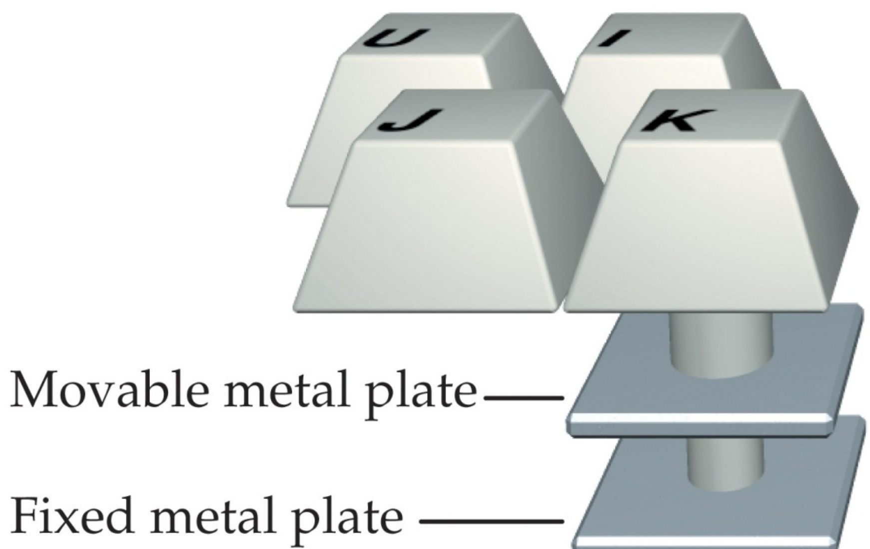




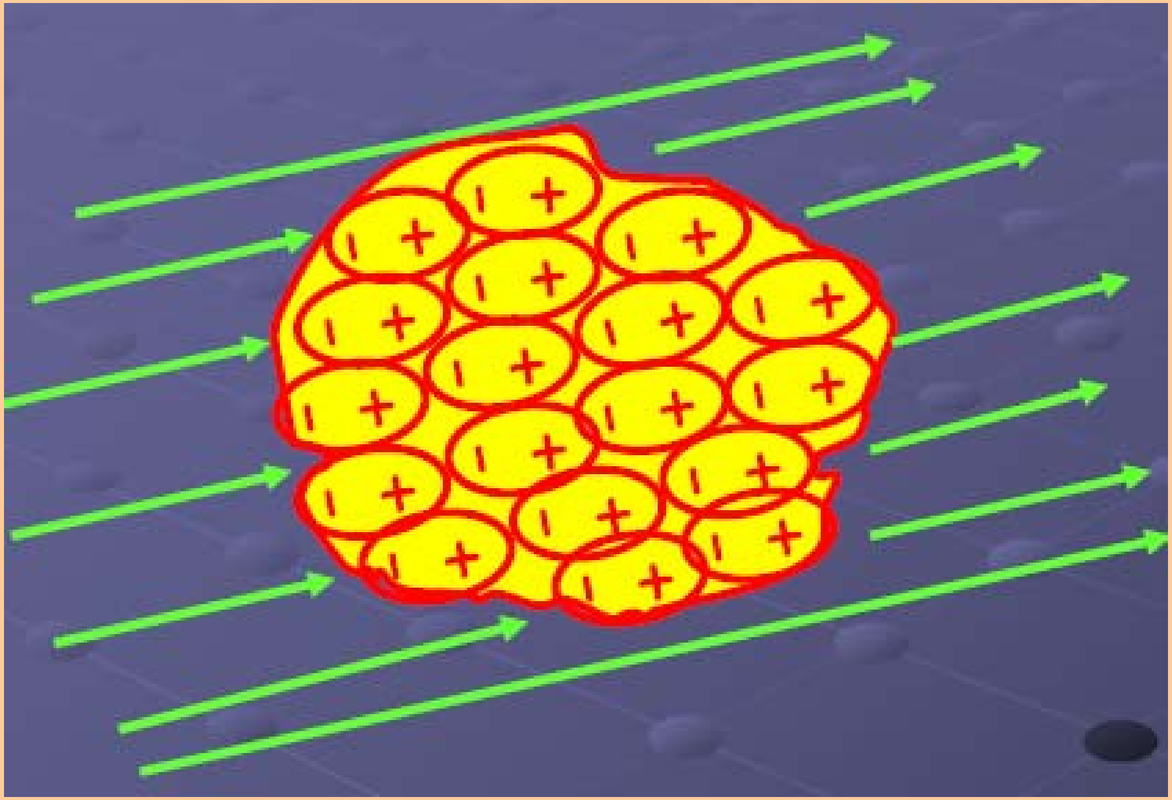



.68 MFD
200 VDC
POS +
MAX SURGE 250VDC

Algunas aplicaciones

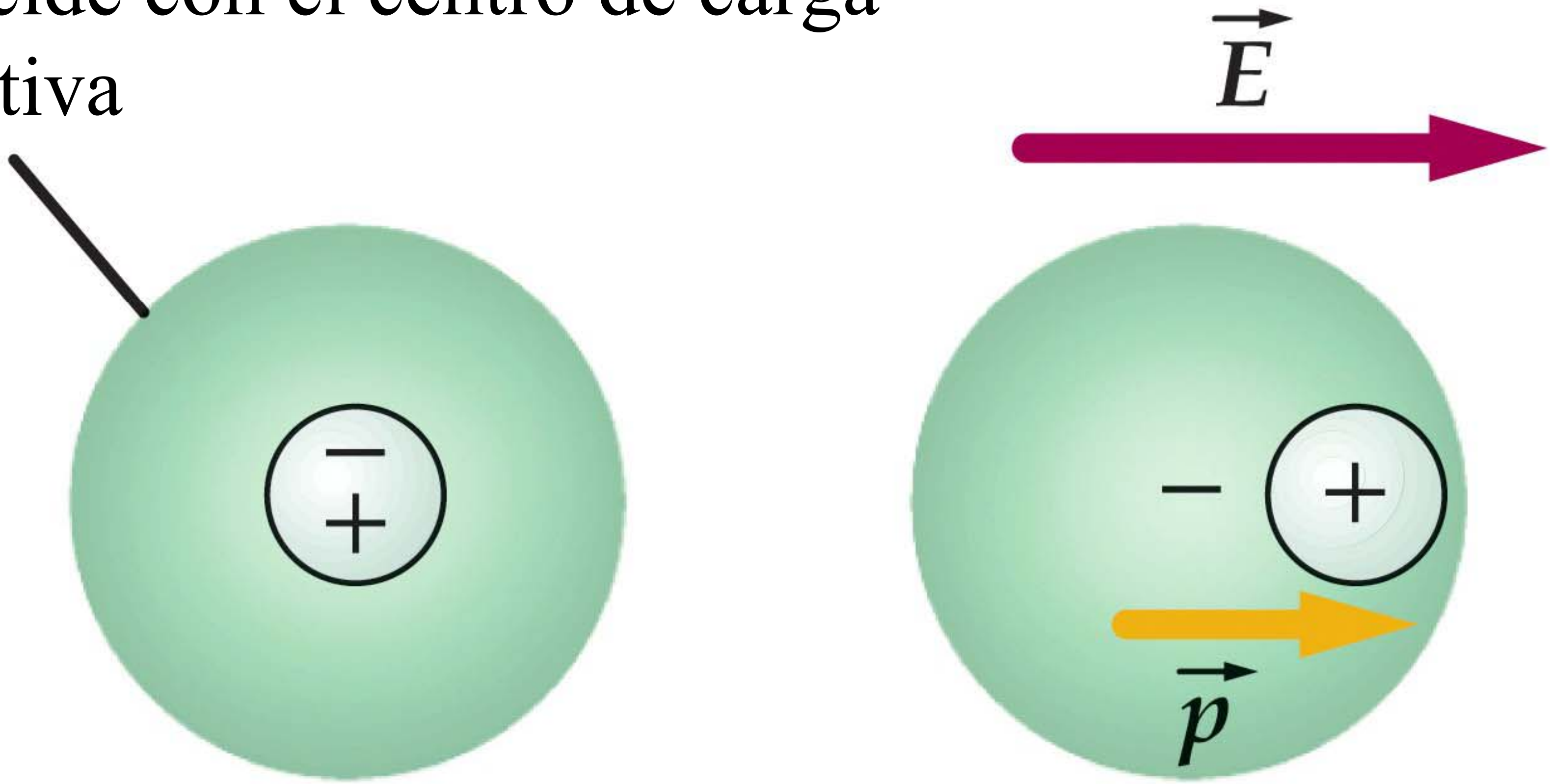


Polarización de la materia



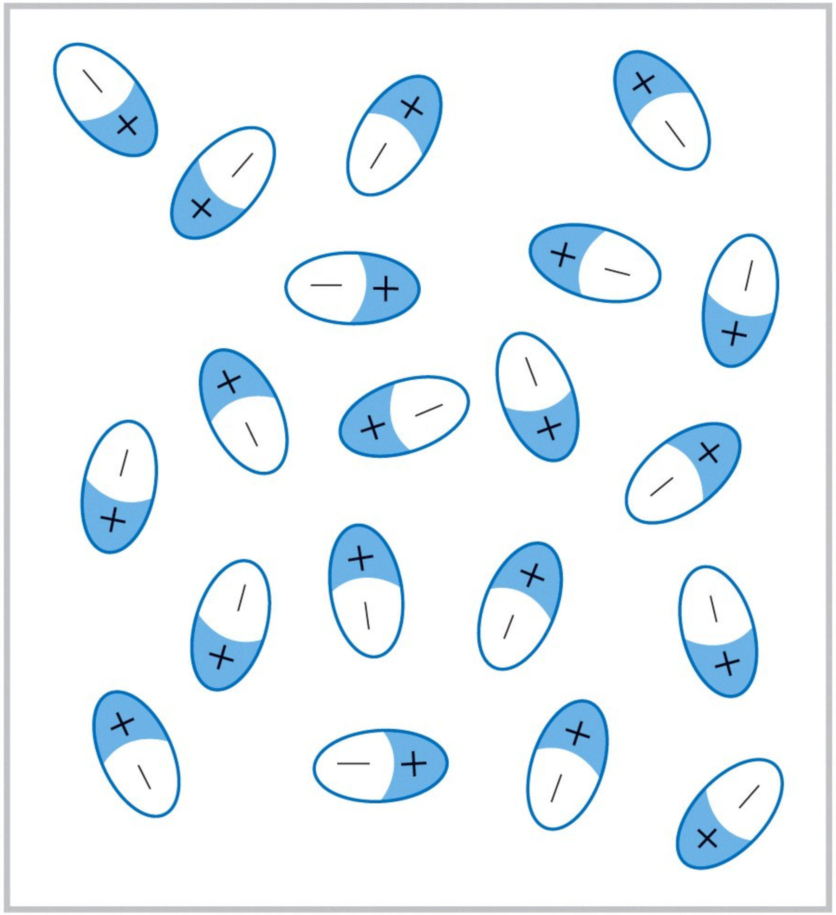
Átomos y Moléculas

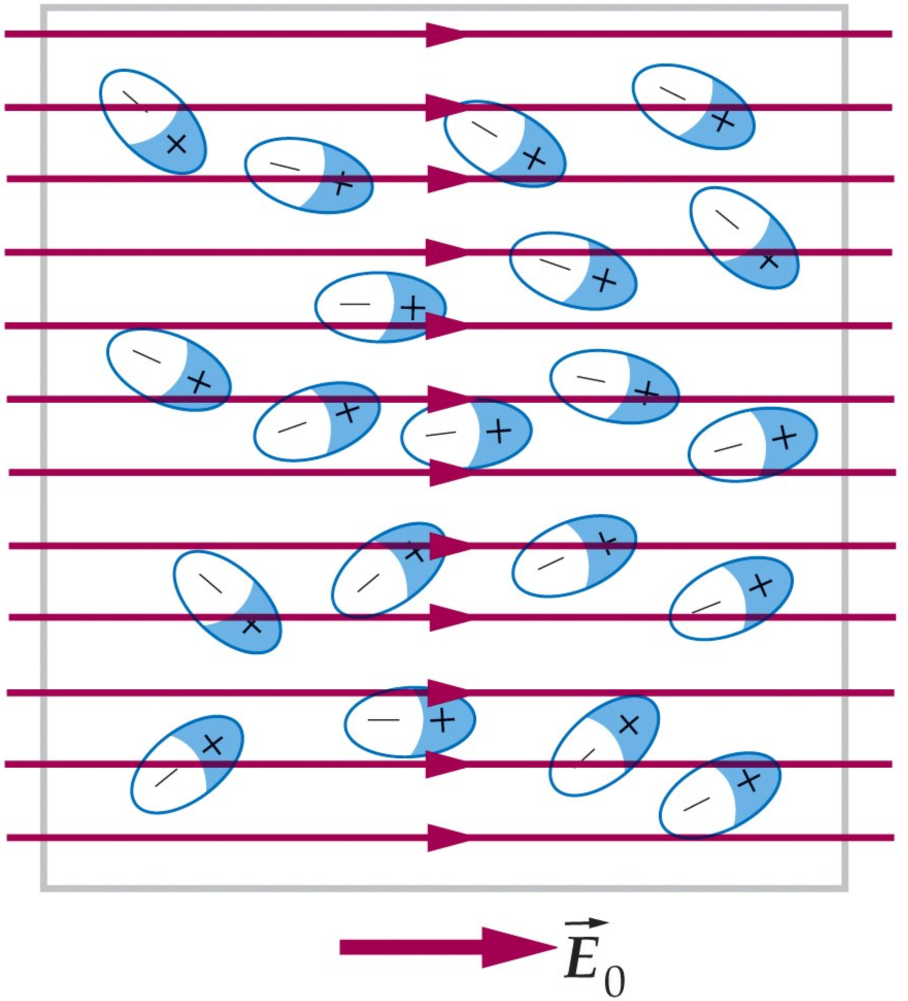
El centro de cargas positiva coincide con el centro de carga negativa

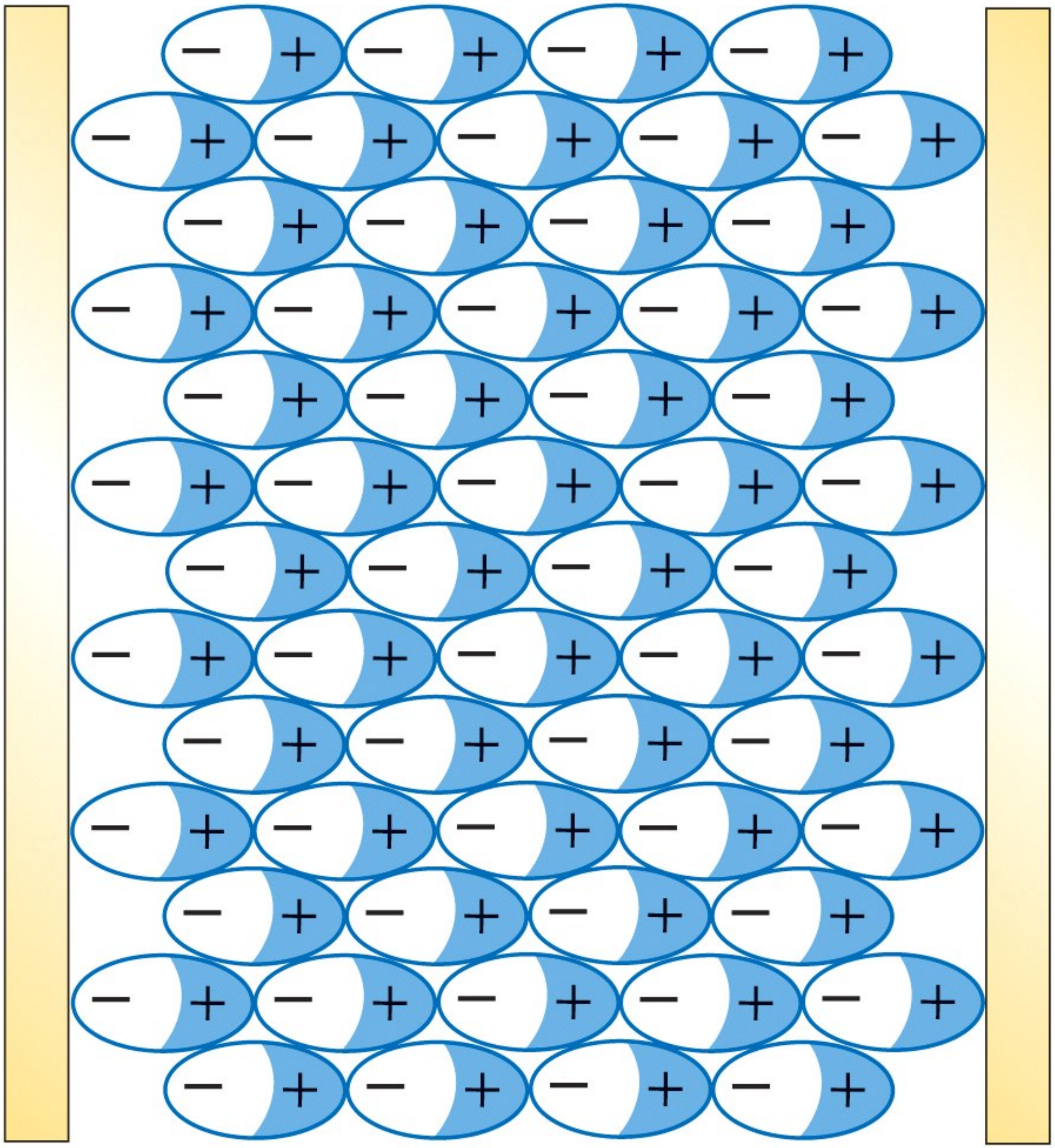


$\vec{P} = \alpha \vec{E}$ (α polarizabilidad electrónica)

Moléculas polares

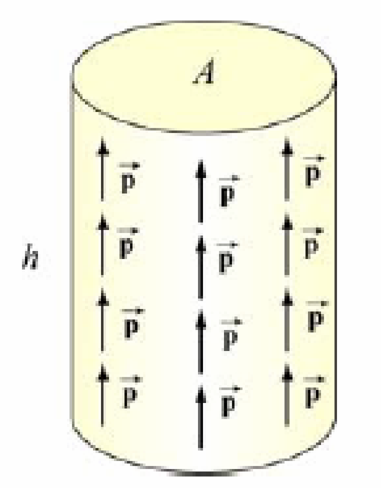






Definimos el vector densidad de polarización:

$$\vec{P} = \frac{\sum_{dv} \vec{p}}{dv} = N\vec{p} \qquad (C/m^2).$$



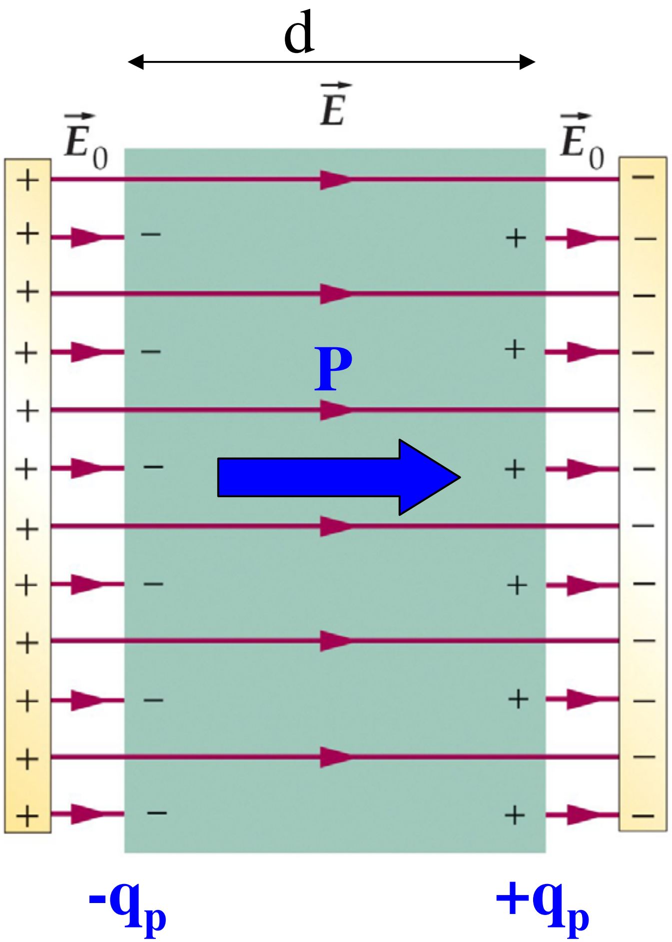
Momento dipolar por unidad de volumen

$$d\vec{p} = \vec{P}\, dv$$

Si el campo eléctrico externo no es muy intenso $\Longrightarrow$ $\vec{P} \,\alpha\, \vec{E}$

$$\boxed{\vec{P} = \chi_e \epsilon_0 \vec{E}}$$

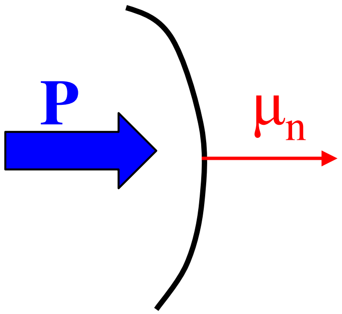
χ_e susceptibilidad eléctrica del material



$$p = P\,A\,d$$

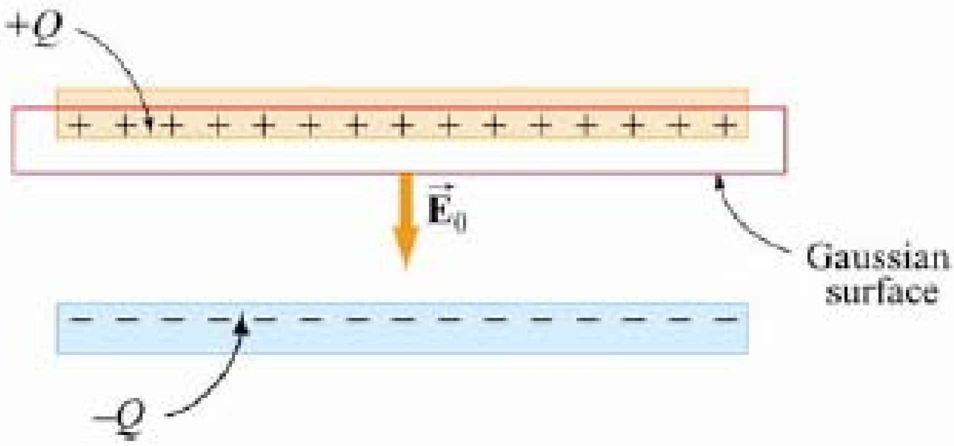
$$p = q_p\,d = \sigma_p\,A\,d$$

$$\boldsymbol{\sigma_p = P}$$

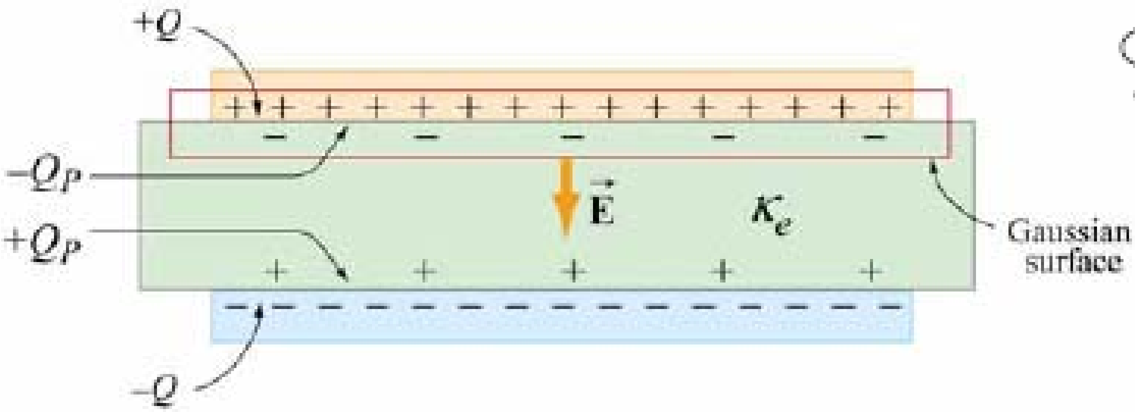


$$\boldsymbol{\sigma_p = \vec{P}\cdot\vec{\mu}_n}$$

Generalización de la ley de Gauss



$$\oiint_S \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = E_0 A = \frac{Q}{\varepsilon_0}, \quad \Rightarrow \quad E_0 = \frac{\sigma}{\varepsilon_0}$$



$$\oiint_S \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = EA = \frac{Q - Q_P}{\varepsilon_0}$$

$$\boxed{E = \frac{Q - Q_P}{\varepsilon_0 A}}$$

$$E=\frac{E_0}{\kappa_e}=\frac{Q}{\kappa_e\varepsilon_0 A}=\frac{Q-Q_P}{\varepsilon_0 A}$$

$$Q_P=Q\left(1-\frac{1}{\kappa_e}\right)$$

$$\sigma_P=\sigma\left(1-\frac{1}{\kappa_e}\right)$$

Definimos:

$\varepsilon=\kappa_e\varepsilon_0$ Permeabilidad del material

$$\oint\!\!\!\oint_S \vec{\mathbf{E}}\cdot d\vec{\mathbf{A}}=EA=\frac{Q-Q_P}{\varepsilon_0}$$

$$\oint\!\!\!\oint_S \vec{\mathbf{E}}\cdot d\vec{\mathbf{A}}=\frac{Q}{\kappa_e\varepsilon_0}=\frac{Q}{\varepsilon}$$

Definimos: $\vec{\mathbf{D}}=\varepsilon_0\kappa\vec{\mathbf{E}}$

Desplazamiento eléctrico

$$\oint\!\!\!\oint_S \vec{\mathbf{D}}\cdot d\vec{\mathbf{A}}=Q$$

Ley de Gauss Generalizada

Forma diferencial:

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon}$$

$$\vec{\nabla} \cdot \vec{D} = \rho$$