Apéndice 1
–Operaciones sobre Listas–

Prof. Responsable: Natalia Colussi

1. Operaciones sobre Listas

1.1. Básicas

i)
$$\begin{array}{lll} +\!\!+ : [\mathsf{A}] \to [\mathsf{A}] & & \\ [\,] +\!\!+ ys & \doteq & ys \\ (x \triangleright xs) +\!\!+ ys & \doteq & x \triangleright (xs +\!\!+ ys) \end{array}$$

ii)
$$\begin{array}{lll} \# : [\mathsf{A}] \to [\mathsf{A}] & & \\ \#\,[\,] & \doteq & 0\,. \\ \#\,(x \triangleright xs) & \doteq & 1 + \#\,xs\,. \end{array}$$

iii)
$$\begin{array}{lll} \uparrow : [\mathsf{A}] \to \mathsf{Nat} \to [\mathsf{A}] & & \\ [\,] \uparrow n & \doteq & [\,]\,. \\ (x \triangleright xs) \uparrow 0 & \doteq & [\,]\,. \\ (x \triangleright xs) \uparrow (n+1) & \doteq & x \triangleright (xs \uparrow n)\,. \end{array}$$

iv)
$$\begin{array}{lll} \downarrow : [\mathsf{A}] \to \mathsf{Nat} \to [\mathsf{A}] & & \\ [\,] \downarrow n & \doteq & [\,]\,. \\ (x \triangleright xs) \downarrow 0 & \doteq & x \triangleright xs\,. \\ (x \triangleright xs) \downarrow (n+1) & \doteq & x \triangleright (xs \downarrow n)\,. \end{array}$$

v)
$$\begin{array}{lll} . : [\mathsf{A}] \to \mathsf{Nat} \to \mathsf{A} & & \\ (x \triangleright xs)\,.\,0 & \doteq & x\,. \\ (x \triangleright xs)\,.\,(n+1) & \doteq & xs\,.\,n\,. \end{array}$$

1.2. Complementarias

i) $hd : [\mathsf{A}] \to \mathsf{A}$
$hd.(x \triangleright xs) \doteq x$

ii) $tl : [\mathsf{A}] \to [\mathsf{A}]$
$tl.(x \triangleright xs) \doteq xs$

iii)
$$\begin{array}{lll} init : [\mathsf{A}] \to [\mathsf{A}] & & \\ init.[x] & \doteq & [\,] \\ init.(x \triangleright x' \triangleright xs) & \doteq & x \triangleright init.(x' \triangleright xs) \end{array}$$

iv)
$$\begin{array}{lll} last : [\mathsf{A}] \to \mathsf{A} & & \\ last.[x] & \doteq & x \\ last.(x \triangleright x' \triangleright xs) & \doteq & last.(x' \triangleright xs) \end{array}$$

2. Propiedades sobre Listas

1. Propiedad *Asociatividad* de $+\!\!+$: $(xs +\!\!+ ys) +\!\!+ zs = xs +\!\!+ (ys +\!\!+ zs)$.
2. Propiedad Elemento Neutro a Der. de $+\!\!+$: $xs +\!\!+ [\,] = xs$
3. Propiedad Lista Vacía y $+\!\!+$: $(xs +\!\!+ ys) = [\,] \equiv xs = [\,] \wedge ys = [\,]$
4. Propiedad *init* y $\uparrow$: $init.xs = xs \uparrow (\#\,xs - 1)$.
5. Propiedad *init* y $+\!\!+$: $init.(xs +\!\!+ [x]) = xs$.
6. Propiedad *last* y $+\!\!+$: $last.(xs +\!\!+ [x]) = x$.
7. Propiedades $\uparrow$ y $\downarrow$ con $n \geq 0$ y $m \geq 0$:
   a) $(xs \uparrow n) +\!\!+ (xs \downarrow n) = xs$.
   b) $(xs \downarrow n) \uparrow m = (xs \uparrow (m+n)) \downarrow n$.

c) $(xs \downarrow n) \downarrow m = xs \downarrow (m + n)$.

8. Propiedades . y $+\!\!+$

   a) $(\forall\, i : 0 \leq i < \#xs : (xs +\!\!+ ys)\,.\, i = xs\,.\, i)$

   b) $(\forall\, i : \#xs \leq i < \#(xs +\!\!+ ys) : (xs +\!\!+ ys)\,.\, i = ys\,.\, (i - \#xs))$

9. Propiedad $\#$ y $+\!\!+$: $\#(xs +\!\!+ ys) = \#\, xs + \#\, ys$.

10. Propiedad $\#$ y $\uparrow$ con $n \geq 0$:

$$\begin{array}{lcl} \#(xs \uparrow n) & = & (\quad n \leq \#xs \;\rightarrow\; n \\ & & \Box \quad n > \#xs \;\rightarrow\; \#xs \\ & &) \end{array}$$

11. Propiedad $\#$ y $\downarrow$ con $n \geq 0$:

$$\begin{array}{lcl} \#(xs \downarrow n) & = & (\quad n \leq \#xs \;\rightarrow\; \#xs - n \\ & & \Box \quad n > \#xs \;\rightarrow\; 0 \\ & &) \end{array}$$