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Accuracy of sample dimension-dependent pedotransfer functions in estimation of soil saturated hydraulic conductivity

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ABSTRACT

Saturated hydraulic conductivity K_{sat} is a fundamental characteristic in modeling flow and contaminant transport in soils and sediments. Therefore, many models have been developed to estimate K_{sat} from easily measureable parameters, such as textural properties, bulk density, etc. However, K_{sat} is not only affected by textural and structural characteristics, but also by sample dimensions e.g., internal diameter and height. Using the UNSODA database and the contrast pattern aided regression (CPXR) method, we recently developed sample dimensiondependent pedotransfer functions to estimate K_{sat} from textural data, bulk density, and sample dimensions. The main objectives of this technical note were evaluating the proposed pedotransfer functions using a substantially larger database, and comparing them with seven other models. For this purpose, we selected more than nineteen thousand soil samples from all around the United States. Results showed that the sample dimensiondependent pedotransfer functions estimated K_{sat} more accurately, within 1.5 orders of magnitude boundaries of the measurements, than seven other models frequently used in the literature.

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1. Introduction

Saturated hydraulic conductivity K_{sat} is essential in modeling surface and subsurface flow as well as solute transport in soils and sediments. Although many models have been developed to quantify K_{sat} , its estimation is commonly done using pedotransfer functions, i.e. empirical relationships linking K_{sat} to soil basic properties such as textural fraction contents, organic matter content, etc. (Pachepsky and Rawls, 2004). In the literature, pedotransfer functions have been developed using various techniques, such as multiple linear regression, artificial neural networks, genetic programming, decision tree analysis, group method of data handling, support vector machines and contrast pattern aided regression, and compared with experiments. For example, Minasny and McBratney (2000) evaluated eight proposed pedotransfer functions for estimation of saturated hydraulic conductivity K_{sat} using published Australian soil data sets comprised different field and laboratory measurements over large areas and limited predictive variables. They found that the published PTFs of Dane and Puckett (1994), Cosby et al. (1984), and Schaap et al. (1998) gave the best estimations for sandy, loamy, and clayey soils, respectively. Sobieraj et al. (2001) evaluated the performance of nine K_{sat} pedotransfer functions including the

* Corresponding author. *E-mail address*: behzad.ghanbarian@beg.utexas.edu (B. Ghanbarian). in a rainforest catchment and found them inaccurate. At shallow depths e.g., 0–0.1 m, pedotransfer functions, except Jabro (1992), commonly underestimated K_{sat} , while at subsequent depths e.g., 0.1–0.4 m pedotransfer functions, except Campbell and Shiozawa (1994), typically overestimated K_{sat} . Gwenzi et al. (2011) evaluated five PTFs including Cosby et al. (1984), Jabro (1992), Puckett et al. (1985), Dane and Puckett (1994), and Saxton et al. (1986). They showed that all evaluated K_{sat} models underestimated by an order of magnitude, suggesting that application of water balance simulation models based on such PTFs may constitute a bias in model outputs. There were indications that the sample dimension might be a factor effecting that of magnitude of magnitude of magnitude and Williame 1005.

Rosetta model (see their Table 3) in modeling the storm flow generated

There were indications that the sample dimension might be a factor affecting the value of measured K_{sat} (e.g., Vepraskas and Williams 1995; Zobeck et al. 1985). Recently, Ghanbarian et al. (2015) proposed sample dimension-dependent pedotransfer functions to estimate saturated hydraulic conductivity K_{sat} using the contrast pattern aided regression (CPXR) method and soil samples from the UNSODA database. They showed that including sample dimensions i.e., sample internal diameter and height (or length) substantially improved K_{sat} estimations.

Although numerous studies assessed the performance of different pedotransfer functions in estimation of water content and soil water retention curve, such evaluations for K_{sat} pedotransfer functions and estimations based on a remarkably large database including several thousands of samples have not been done so far. The main objective of this study was to use a large database that consists of various soil







samples from all around the United States for (1) evaluating the sample dimension-dependent K_{sat} pedotransfer functions developed by Ghanbarian et al. (2015) and (2) comparing performance of the proposed sample dimension-dependent K_{sat} pedotransfer functions with performance of seven other pedotransfer functions frequently applied in the literature.

2. Materials and methods

Here, we first describe datasets used in this study and then present proposed pedotransfer functions available in the literature for estimating saturated hydraulic conductivity from clay, silt, and sand content as well as bulk density.

2.1. Datasets

Soil samples used in this study are from the USKSAT database (Pachepsky and Park, 2015) that consists of 45 datasets and >27,000 lab experiments from the Unites States. We selected those samples whose textural data (i.e., sand, silt and clay contents), bulk density and sample dimensions (i.e., internal diameter and length/height) were available. The entire database used in this study includes 24 datasets and 19,822 soil samples from different states in the nation. The number of samples, K_{sat} measurement method, and average and standard deviation of different parameters for each dataset are summarized in Table 1. The average K_{sat} value varying between 12.3 (Jabro 1992) and 1328.5 cm/day (Romkens et al., 1986), near three orders of magnitude variation, indicates different types of soils with various structures. This is an order of magnitude greater than the average K_{sat} range that Schaap et al. (1998) investigated through 620 soil samples from 30 sources in the United States. In the Results and Discussion section below, we however, show that the measured K_{sat} value actually spans even much >3 orders of magnitude (about 8) representing a very wide range of soils. Fig. 1 showing the percentage of samples in each USDA soil texture class and the distribution of samples on the ternary diagram also implies various types of soils with different textures used in this study.

2.2. Saturated hydraulic conductivity pedotransfer functions

There are tremendous amount of models in the literature estimating the saturated hydraulic conductivity from other properties. However, the focus of our study is $K_{\rm sat}$ estimation from frequently available characteristics, such as soil textural data i.e., sand, silt, and clay content and bulk density for soil mapping purposes at large scales. In Table 2, we list 7 models commonly used in the literature including Brakensiek et al. (1984), Campbell and Shiozawa (1994), Cosby et al. (1984), Jabro (1992), Puckett et al. (1985), Dane and Puckett (1994), and Saxton et al. (1986).

Both Campbell and Shiozawa (1994) and Puckett et al. (1985) used the same database consisting of 42 samples (six Ultisols taken at seven locations in the lower coastal Plain of Alabama). The Cosby et al. (1984) model was proposed based on 1448 samples from Holtan et al. (1968) and Rawls et al. (1976). Jabro (1992) presented his model using 350 soil samples collected from the literature e.g., Southern Cooperation Series Bulletins. Dane and Puckett (1994) developed their model using 577 samples (Ultisols from lower coastal Plain of Alabama). The Saxton et al. (1986) model was developed using a database including 230 selected data points uniformly spaced on the hydraulic conductivity curves for 10 textures classes reported by Rawls et al. (1982).

All these models, except Brakensiek et al. (1984), estimate $K_{\rm sat}$ from sand, silt, and clay content and bulk density ρ_b . The Brakensiek et al. (1984) model, however, requires porosity value as well, which was estimated by $1 - \rho_b/\rho_s$ in which ρ_s , the particle density, was assumed to be approximately equal to 2.65 g/cm³ for all samples. We should point out that we eliminated the Jabro (1992) dataset (see Table 1) from this model evaluation process since the assessment of the Jabro (1992) model using the same dataset used to develop it would be biased and not supported.

Ghanbarian et al. (2015) used a new data mining technique called contrast pattern aided regression (CPXR) and >200 soil samples from the UNSODA database to develop sample-dimension dependent pedotransfer functions to estimate K_{sat} . The 10-fold cross-validation method was applied to investigate the accuracy and reliability of the proposed models. The sample dimension-dependent model of Ghanbarian et al. (2015) presented in Table 3 consists of 14 patterns

Table 1

The average and standard deviation (in parentheses) of salient properties of soil samples (n = 19,822) from various references in the USKSAT database used in this study.

Reference	Samples no.	Measurement method	Bulk density	Sand%	Silt%	Clay%	Sample height (cm)	Sample diameter (cm)	K _{sat} (cm/day)
Kool et al. (1954)	74	Falling head	1.41 (0.14)	17.5 (8.1)	53.7 (11.1)	29.1 (16.4)	4 (0)	5.4 (0)	110.5 (207.0)
Hubbard et al. (1985)	41	Constant head	1.71 (0.16)	67.8 (12.4)	8.4 (3.1)	23.8 (11.8)	7.5 (0)	7.6 (0)	89.3 (159.8)
Boul and Southard (1988)	27	Constant head	1.55 (0.08)	44.8 (8.5)	25.3 (9.4)	29.8 (8.0)	7.6 (0)	7.6 (0)	46.5 (106.7)
Robbins (1977)	15	Hydraulic head	1.33 (0.06)	25.2 (6.7)	59.9 (5.0)	14.9 (3.2)	51 (0)	5 (0)	42.3 (12.2)
Bruce et al. (1983)	73	Constant head	1.54 (0.11)	46.8 (13.5)	20.8 (4.3)	32.4 (13.7)	6 (0)	5.4 (0)	145.9 (224.1)
Peele et al. (1970)	257	Constant head	1.47 (0.18)	52.2 (22.1)	23.5 (12.5)	24.3 (15.3)	9.4 (0)	9.74 (0)	325.7 (461.5)
Dane et al. (1983)	120	Constant head	1.54 (0.09)	91.9 (3.9)	4.6 (5.2)	3.8 (1.8)	6.4 (1.9)	6.8 (1.1)	1149.0 (889.3)
Romkens et al. (1986)	72	Constant head	1.46 (0.10)	12.9 (9.2)	64.6 (9.4)	22.5 (7.2)	5.9 (1.1)	7 (0.83)	1328.5 (4583.5)
Bathke and Cassel (1991)	25	Constant-head	1.36 (0.16)	44.9 (17.9)	20.9 (10.0)	28.4 (14.7)	7.6 (0)	7.6 (0)	421.7 (1183.3)
		permeameter							
Dane (1980)	6	Pressure head	1.57 (0.09)	83.6 (1.8)	13.2 (1.0)	3.2 (1.7)	6(0)	5.4 (0)	235.9 (166.6)
Zobeck et al. (1985)	6	Miscellaneous*	1.49 (0.04)	14.8 (1.3)	44.7 (8.3)	40.5 (7.0)	10.5 (8.2)	12.2 (10.0)	129.4 (171.1)
Jabro (1992)	18	Constant head	1.53 (0.07)	17.3 (6.7)	56.9 (8.5)	25.8 (6.1)	7.6 (0)	7.6 (0)	12.3 (8.4)
Bruce (1972)	3	Constant head	1.60 (0.05)	56.7 (18.9)	20.0 (3.6)	23.3 (17.2)	6.2 (0)	6.0 (0)	841.7 (718.1)
Habecker et al. (1990)	14	Falling-head	1.45 (0.32)	35.8 (21.1)	54.2 (21.4)	10.0 (2.1)	7.5 (0)	7.6 (0)	277.1 (522.9)
		permeameter							
Radcliffe et al. (1990)	9	Constant head	1.52 (0.09)	52.0 (12.9)	19.3 (5.1)	28.7 (16.0)	7.6 (0)	7.6 (0)	209.3 (198.4)
Coelho (1974)	175	Falling head	1.45 (0.16)	39.4 (21.8)	35.4 (14.4)	24.5 (11.1)	7.6 (0)	7.6 (0)	90.6 (186.5)
Southard and Boul (1988)	27	Constant head	1.55 (0.08)	44.5 (8.4)	25.4 (9.2)	30.1 (7.4)	7.6 (0)	7.6 (0)	13.9 (23.3)
Rawls et al. (1998)	166	Constant head	1.52 (0.13)	90.3 (9.7)	3.4 (3.3)	6.3 (8.6)	5.4 (0)	3.0 (0)	640.1 (737.2)
Carlisle et al. (1978)	780	Constant head	1.50 (0.25)	83.7 (16.7)	5.7 (5.9)	10.4 (12.8)	5.4 (0)	3.0 (0)	548.5 (823.1)
Carlisle et al. (1981)	17,869	Constant head	1.53 (0.17)	85.8 (15.9)	5.6 (7.3)	8.7 (11.9)	5.4 (0)	3.0 (0)	548.6 (725.6)
Baumhardt et al. (1995)	4	Constant head	1.42 (0.17)	40.4 (24.0)	22.9 (16.3)	36.8 (11.9)	50 (0)	75 (0)	101.6 (75.3)
Romkens et al. (1985)	36	Constant head	1.41 (0.05)	12.8 (10.5)	69.2 (9.5)	18.1 (8.4)	7.5 (0)	7.5 (0)	21.3 (37.3)
Sharratt (1990)	14	Unknown	1.30 (0.23)	36.5 (25.0)	56.7 (23.1)	6.8 (3.9)	2.0 (0)	5.0 (0)	334.9 (1009.8)
Price et al. (2010)	9	Constant head	1.12 (0.24)	56.1 (4.0)	28.0 (2.6)	15.6 (1.7)	7.5 (0)	7.5 (0)	62.4 (66.6)

* Including constant head blocks, constant head cores, and falling head permeameter.





Fig. 1. Percentage of samples in each USDA soil texture class and distribution of samples on the soil texture triangle (n = 19,822).

each of which corresponds to a specific model. The sample dimensiondependent pedotransfer functions of Ghanbarian et al. (2015) estimate the saturated hydraulic conductivity K_{sat} from clay, silt, and sand contents, geometric mean diameter and standard deviation of diameters of soil particles, bulk density, and sample dimensions (i.e., internal diameter and height). The interested reader is referred to the original article for further information about the models proposed by Ghanbarian et al. (2015).

Most patterns given in Table 3 match >10% (i.e., support >10%) of the training data. As Ghanbarian et al. (2015) noted, even patterns with supports less than that still match about 10 samples of the training data. Moreover, patterns having relatively small supports often have large ARR (average residual reduction) values, implying that the matching samples of each such pattern represent an important data group where the baseline model f_0 makes remarkable large prediction errors and those errors are reduced significantly by the local model (see Table 3) corresponding to each pattern (Ghanbarian et al. 2015).

To estimate K_{sat} using the Ghanbarian et al. (2015) model, first the relevant pattern that the data match with is found, and then using the corresponding local model given in Table 3 K_{sat} is estimated. One should expect that one soil sample may match more than one pattern. In that case, all relevant patterns should be identified and used to estimate K_{sat} . The weighted average of all K_{sat} values estimated by different patterns and local models then represents the Ghanbarian et al. (2015) model estimated using the baseline model f_0 given in Table 3. The Java code for K_{sat} estimation by the Ghanbarian et al. (2015) model and other models presented in Table 2 is given in the Supplementary material.

In order to compare statistically the accuracy of the discussed models in the estimation of K_{sat} , the root mean square log-transformed error (RMSLE) and mean log-transformed error (MLE) parameters were determined as follows

$$RMSLE = \sqrt{\frac{1}{n} \sum_{i=1}^{n} \left[\log(K_{\text{sat}})_{\text{cal}} - \log(K_{\text{sat}})_{\text{meas}} \right]^2}$$
(1)

$$MLE = \frac{1}{n} \sum_{i=1}^{n} \left[\log(K_{\text{sat}})_{\text{cal}} - \log(K_{\text{sat}})_{\text{meas}} \right]$$
(2)

where *n* is the number of values, and $(K_{sat})_{cal}$ and $(K_{sat})_{meas}$ are the calculated (estimated) and measured saturated hydraulic conductivities, respectively.

3. Results and discussion

Comparison of the performance of the selected models in the estimation of saturated hydraulic conductivity K_{sat} from sand, silt and clay content as well as bulk density is shown in Fig. 2. As can be seen, the Campbell and Shiozawa (1994) and Puckett et al. (1985) models with MLE = -0.96 and -0.41 tend to underestimate K_{sat} considerably over 8 orders of magnitude variation, while the Dane and Puckett (1994), Saxton et al. (1986) and Ghanbarian et al. (2015) models estimates are distributed around the 1:1 line (MLE = 0.09, -0.06, and 0.08, respectively). Interestingly, although the Puckett et al. (1985) and Campbell and Shiozawa (1994) models were developed using the same exponential function and dataset (42 samples from the lower coastal Plain of Alabama) and have similar trend in K_{sat} underestimation, the Puckett et al. (1985) model is almost 25% more accurate than the Campbell and Shiozawa (1994) model (RMSLE = 0.92 vs. 1.27). Such a discrepancy in *K*_{sat} estimation implies that the Puckett et al. (1985) model could more efficiently detect nonlinear interactions between input and output variables than the Campbell and Shiozawa (1994) model. The MLE values for different K_{sat} models and USDA soil texture classes presented in Table 4 also show that both Puckett et al. (1985) and Campbell and Shiozawa (1994) tend to underestimate K_{sat}

Table 2

Some proposed pedotransfer functions frequently used in the literature to estimate soil saturated hydraulic conductivity K_{sat}-

Reference	K _{sat} (cm/day) model*
Brakensiek et al. (1984)	$K_{sat} = 24 \exp[19.52348\phi - 8.96847 - 0.028212Cl + 0.00018107Sa^2 - 0.0094125Cl^2 - 8.395215\phi^2 + 0.077718\phi Sa - 0.00298\phi^2 Sa^2 - 0.019492\phi^2 Cl^2 + 0.0000173Sa^2 Cl + 0.02733\phi Cl^2 + 0.001434\phi Sa^2 - 0.0000035Cl^2 Sa]$
Campbell and Shiozawa (1994)	$K_{\rm sat} = 129.6 \exp(-0.07 \text{Si} - 0.167 \text{Cl})$
Cosby et al. (1984)	$K_{\text{sat}} = 60.96 \times 10^{(-0.6+0.0126\text{Sa}-0.0064\text{Cl})}$
Jabro (1992)	$K_{\text{sat}} = 24 \times [9.56 - 0.81\log(\text{Si}) - 1.09\log(\text{Cl}) - 4.64\rho_{\text{b}}]$
Puckett et al. (1985)	$K_{\rm sat} = 376.7 \exp(-0.1975 \text{Cl})$
Dane and Puckett (1994)	$K_{\rm sat} = 729.22 \exp(-0.144 \text{Cl})$
Saxton et al. (1986)	$K_{\rm sat} = 24\exp(12.012 - 0.0755Sa)$

*Sa: sand (%), Si: silt (%), Cl: clay (%), ρ_b : bulk density (g cm⁻³), and ϕ : porosity.

Table 3

Patterns, criteria, baseline regression model f_0 , and pedotransfer functions for the CPXR model developed by Ghanbarian et al. (2015) to estimate soil saturated hydraulic conductivity from sand, silt, and clay content, geometric mean and geometric standard deviation of particles, bulk density, and sample dimensions i.e., length and internal diameter. See the context for more detail about K_{sat} estimation using the baseline, criteria, and local models.

$Log(K_{sat})$ [cm/day]										
$\textbf{Baseline model } \textit{f}_{0}\textbf{:} - 1180 + 11.89 \times \textbf{Sa} + 11.90 \times \textbf{Si} + 11.85 \times \textbf{Cl} + 5.25 \times \textbf{d}_{g} + 0.028 \times \sigma_{g} - 3.86 \times \rho_{b} - 0.039 \times \textbf{ID}$										
Pattern	Criteria	ARR	Support (%)							
1	$0.495 \leq d_g < 0.74, Cl < 15.8, \ 1.55 \leq \sigma_g < 6.96, \ 0.2 \leq Si < 20. \ 3, \ 74.4 \leq Sa < 99.1, \ 1.23 \leq \rho_b < 1.6$	1.5	16.0							
2	$49.6 \le \text{Sa} < 74.4, 2.5 \le L < 36.9, 3.2 \le \text{ID} < 7.5$	2.3	14.6							
3	$40.5 \le Si < 60.6, 0.004 \le d_g < 0.25, 6.96 \le \sigma_g < 12.38, 1.23 \le \rho_b < 1.6$	3.1	8.0							
4	$0.2 \le Si < 20.3$, $Cl < 15.8$, $1.55 \le \sigma_g < 6.96$, $74.4 \le Sa < 99.1$	0.9	32.4							
5	$0.2 \le \text{Si} < 20.3, \ 0.004 \le \text{d}_{\text{g}} < 0.25, \ 2.5 \le L < 36.9$	6.6	5.2							
6	$6.96 \le \sigma_{\rm g} < 12.38, {\rm Cl} < 15.8, 2.5 \le L < 36.9, 3.2 \le {\rm ID} < 7.5, 20.3 \le {\rm Si} < 40.5, 49.6 \le {\rm Sa} < 74.4, 0.004 \le d_{\rm g} < 0.25$	1.6	6.1							
7	$60.6 \le Si < 80.7, 2.5 \le L < 36.9, 1.23 \le \rho_b < 1.6, 0.004 \le dg < 0.25, 0.1 \le Sa < 24.9$	3.4	14.1							
8	$49.6 \le \text{Sa} < 74.4, 2.5 \le L < 36.9, 20.3 \le \text{Si} < 40.5, 3.2 \le \text{ID} < 7.5$	2.1	12.7							
9	$15.8 \le Cl < 31.5, 2.5 \le L < 36.9, 0.004 \le d_g < 0.25, 1.23 \le \rho_b < 1.6$	2.4	14.6							
10	$49.6 \le \text{Sa} < 74.4, 2.5 \le L < 36.9, 1.23 \le \rho_b < 1.6, 3.2 \le \text{ID} < 7.5$	4.3	8.0							
11	$20.3 \le \text{Si} < 40.5, 2.5 \le L < 36.9, \text{Cl} < 15.8, 49.6 \le \text{Sa} < 74.4$	0.9	15.0							
12	$1.55 \le \sigma_{\rm g} < 6.96, {\rm Cl} < 15.8$	0.9	36.6							
13	$Cl < 15.8, 0.004 \le d_g < 0.25, 2.5 \le L < 36.9$	0.5	25.4							
14	$Cl < 15.8, 2.5 \le L < 36.9$	0.4	59.2							
Pattern	Local model corresponding to each pattern									
1	$-1355.1\ +\ 14.07\times\text{Sa}\ +\ 9.91\times\text{Si}\ -\ 50.2\times\text{dg}\ +\ 18.39\times\sigma_g\ -\ 12.99\times\rho_b\ -\ 0.24\times\text{ID}\ -\ 0.014\times\text{L}$									
2	$70.74 - 0.52 \times Sa - 0.64 \times Si - 30.4 \times d_g - 1.73 \times \sigma_g - 4.66 \times \rho_b + 2.67 \times ID - 0.025 \times L$									
3	$52.1 + 0.58 \times Sa - 0.56 \times Si - 230.6 \times dg - 2.68 \times \sigma_g - 1.18 \times \rho_b + 0.35 \times ID - 0.064 \times L$									
4	$-1033.7 + 10.4 \times Sa + 9.7 \times Si + 7.6 \times Cl + 4.56 \times dg + 4.54 \times \sigma_g - 5.14 \times \rho_b - 0.2 \times ID - 0.0006 \times L$									
5	$53.5 + 0.095 \times Sa - 0.54 \times Si - 73.4 \times d_g - 1.58 \times \sigma_g - 7.77 \times \rho_b - 0.36 \times ID + 0.42 \times L$									
6	$762.4 - 2.28 \times Sa - 7.1 \times Si - 770.8 \times dg - 22.7 \times \sigma_{g} - 2.07 \times \rho_{b} - 3.88 \times ID - 1.72 \times L$									
7	$-50.5 - 1.59 \times \text{Sa} + 0.56 \times \text{Si} + 587.4 \times d_g + 7.4 \times \sigma_g - 20.55 \times \rho_b + 0.43 \times \text{ID} - 0.18 \times L$									
8	$-243.4 + 1.07 \times \text{Sa} + 2.16 \times \text{Si} + 156.74 \times \text{dg} + 5.04 \times \sigma_g - 1.42 \times \rho_b + 6.12 \times \text{ID} + 1.14 \times \text{L}$									
9	$13.7 + 0.042 \times Sa - 0.104 \times Si - 60.1 \times dg - 0.035 \times \sigma_g - 2.84 \times \rho_b + 0.16 \times ID - 0.011 \times L$									
10	$281.9 - 1.395 \times Sa - 2.37 \times Si - 142.4 \times d_g - 5.28 \times \sigma_g - 22.61 \times \rho_b + 0.46 \times ID - 2.86 \times L$									
11	$148.4 - 0.45 \times \text{Sa} - 1.3 \times \text{Si} - 145.3 \times \text{d}_{\text{g}} - 4.2 \times \sigma_{\text{g}} - 4.3 \times \rho_{\text{b}} - 0.053 \times \text{ID} - 0.086 \times L$									
12	$-896.5 + 9.2 \times Sa + 9.1 \times Si + 9.72 \times Cl - 5.4 \times dg - 1.37 \times \sigma_g - 4.7 \times \rho_b - 0.18 \times ID - 0.0005 \times L$									
13	$-76.04 + 0.146 \times Sa - 0.66 \times Si - 144.7 \times d_g - 3.13 \times \sigma_g - 1.32 \times \rho_b + 0.03 \times ID - 0.0606 \times L^{-1}$									
14	$-204.9 + 2.17 \times Sa + 2.15 \times Si + 2.4 \times Cl + 2.54 \times dg - 0.32 \times \sigma_g - 3.79 \times \rho_b - 0.2 \times ID + 0.086 \times L$									

Sa: sand (%), Si: silt (%), Cl: clay (%), ρ_b : bulk density (g cm⁻³), d_g : geometric mean diameter (mm), σ_g : geometric standard deviation (mm), *L*: sample height (or length), ID: internal diameter, ARR: average residual reduction. Other CPXR-based pedotransfer functions estimating soil saturated hydraulic conductivity (SHC or K_{sat}) from other soil properties are available online at http://www.knoesis.org/resources/researchers/vahid/behzad.html

for all soil texture classes, particularly intermediate to fine textured soils, while the Jabro (1992) model has a tendency to overestimate K_{sar} .

As can be observed in Fig. 2, although the value of mean log-transformed error of the Cosby et al. (1984) model is near zero (MLE = -0.001), the K_{sat} estimations are biased since the model overestimates at lower K_{sat} (fine-textured soils) and underestimates at higher K_{sat} values (coarse-textured soils). In this model, the measured K_{sat} spans around 8 orders of magnitude (log(K_{sat}) varies between -2to near 6), while the estimated $log(K_{sat})$ value is restricted to a much narrower range approximately between 0 and 3. The obtained results are, however, consistent with the results of Minasny and McBratney (2000) who found that the Cosby et al. (1984) model gave the best K_{sat} estimations for loamy soils with intermediate K_{sat}. Using 462 soil samples from Australia, Minasny and McBratney (2000) reported RMSLE = 6.41, 2.58, 2.71, 3.08, and 3.53 [ln(mm/h)] for the Campbell and Shiozawa (1994), Cosby et al. (1984), Dane and Puckett (1994), Brakensiek et al. (1984), and Saxton et al. (1986) models, respectively, meaning the Cosby et al. (1984) model was the most accurate. These RMSLE values, however, are not comparable with those we present in Fig. 2 because (1) the K_{sat} units is different (mm/h vs. cm/day), (2) the statistical parameter calculated in Minasny and McBratney (2000) is the root mean square natural log-transformed error, while that we report is root mean square log-transformed error, and (3) the number of samples in Minasny and McBratney (2000) is 462, whereas n in our study is 19,822, resulted into RMSLE values much smaller than those reported by Minasny and McBratney (2000).

Fig. 2 also shows that the Jabro (1992) model tends to overestimate K_{sat} substantially (MLE = 0.57). Although our results obtained at large

scale might be in contrast with the results of Gwenzi et al. (2011) who reported that the Jabro (1992) model underestimated K_{sat} by an order of magnitude, the number of samples in our study (n = 19,822) compared to that (n = 60) in Gwenzi et al. (2011) is much larger and one thus expect the obtained results in this study to be more comprehensive and general.

The RMSLE values for different K_{sat} models and USDA soil texture classes are given in Table 5. As the bold RMSLE values in Table 5 indicate the most accurate model estimating K_{sat} for different soil texture classes precisely is Ghanbarian et al. (2015). Among all the models evaluated in this study and shown in Fig. 2, the estimations of the Ghanbarian et al. (2015) model are more appropriately and uniformly distributed around the 1:1 line indicating the most accurate model. Comparison of the RMSLE parameter calculated for each model shows that the value of RMSLE = 0.56 for the Ghanbarian et al. (2015) model is considerably smaller than that determined for other models. After Ghanbarian et al. (2015), the Dane and Puckett (1994) model estimates K_{sat} slightly better than the Saxton et al. (1986) and Cosby et al. (1984) models. Interestingly, both Dane and Puckett (1994) and Puckett et al. (1985) models have the same exponential function and input variable i.e., clay. However, the Dane and Puckett (1994) model with RMSLE = 0.73 estimated K_{sat} more accurately than the Puckett et al. (1985) model with RMSLE = 0.92. The main reason probably is in the former 577 soil samples were used to construct a relationship between K_{sat} and clay content, while in the latter only 42 samples. As a consequence, the larger the training database, the more reliable the validation results are. Fig. 2 also shows that the Brakensiek et al. (1984) model estimated K_{sat} more accurately than the Puckett et al. (1985) and Campbell and

Shiozawa (1994) models. One should note that the calculated RMSLE value for the Jabro (1992) model is not directly comparable with those determined for other models since this model uses logarithms of clay and silt contents and, therefore, the number of samples suitable to evaluate this model is 342 less than that used to assess others. Results of our study confirm that the sample dimension-dependent Ghanbarian et al. (2015) model is the most accurate in the estimation of the soil saturated hydraulic conductivity K_{sat} . The hypothetical main reasons for that are discussed below.

First, the model proposed by Ghanbarian et al. (2015) is sample dimension-dependent, meaning that it requires more input variables such as sample height (or length) and internal diameter to estimate K_{sat} . If sample dimensions information are not available, the practical applications of the Ghanbarian et al. (2015) model would be restricted.

The influence of sample dimension on K_{sat} has been well documented in the literature. For example, recently Pachepsky et al. (2014) showed the similarity of scale (sample dimension) dependencies in



Fig. 2. Comparison of the saturated hydraulic conductivity K_{sat} estimated using eight different methods with the measured ones for all the data used in this study (n = 19,822). Note that the number of samples in the elavation of Jabro (1992) model is 19,480 because some samples with either zero silt or clay content were removed from the entire database. The red solid and dashed lines represent the 1:1 line and the factor of 30 (1.5 orders of magniftude) boundaries, respectively.

Table	4
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Mean log-transformed error (MLE) of different models used in this study to estimate K_{sat} for 12 USDA soil texture classes.

K _{eet} model	USDA soil texture class											
Sat model	Sa* MLE	LSa	SaL	L	SiL	Si	SaCL	CL	SiCL	SaC	SiC	С
Brakensiek et al. (1984) Campbell and Shiozawa (1994) Cosby et al. (1984) Jabro (1992) Puckett et al. (1985) Dane and Puckett (1994) Saxton et al. (1986)	0.18 - 0.82 - 0.32 0.77 - 0.29 0.05 0.15	0.59 - 0.66 0.23 0.17 - 0.06 0.41 0.13	0.76 - 0.70 - 0.62 - 0.16 - 0.13 - 0.51 - 0.19	-0.09 -1.80 0.28 0.36 -0.49 0.27 -0.18	-0.74 -2.63 -0.21 0.41 -0.36 0.32 0.17	-0.73 -3.11 -0.79 1.52 -0.11 0.37 1.15	0.49 - 1.10 0.76 0.21 - 0.75 0.14 - 0.86	-0.77 -2.35 0.33 0.04 -1.41 -0.34 -1.07	-1.03 -3.23 -0.18 0.42 -1.48 -0.45 -0.49	-0.32 -1.96 0.63 0.42 -1.80 -0.59 -1.33	- 1.87 - 3.97 - 0.34 0.04 - 2.75 - 1.38 - 1.25	-1.08 -3.11 0.42 0.93 -2.85 -1.30 -0.82
Ghanbarian et al. (2015)	0.03	0.30	0.39	0.16	0.30	0.36	0.06	-0.25	-0.17	-0.46	-0.70	-0.40

*Sa: Sand, LSa: Loamy sand, SaL: Sandy loam, L: Loam, SiL: Silt loam, Si: Silt, SaCL: Sandy clay loam, CL: Clay loam, SiCL: Silty clay loam, SaC: Sandy clay, SiC: Silty clay, and C: Clay.

soils and sediments. They observed that as the characteristic size increased, K_{sat} value first increased by one or two orders of magnitude and then stabilized. More recently, Ghanbarian et al. (2015) showed that the influence of sample dimensions (internal diameter and height) and soil water retention curve on the estimation of the saturated hydraulic conductivity was comparable in both CPXR and multiple linear regression (MLR) methods (for more evidence comprehensive results see Ghanbarian et al. (2015) and references therein).

Second, the Ghanbarian et al. (2015) model was developed using an efficient data mining technique, introduced recently by Dong and Taslimitehrani (Taslimitehrani and Dong, 2014; Dong and Taslimitehrani, 2015) and called contrast pattern aided regression (CPXR), while all other models were proposed using simple algorithms such as multiple linear regressions (MLRs). Using 213 laboratory samples from the UNSODA database, Ghanbarian et al. (2015) compared CPXR-based pedotransfer functions with MLR-based ones in estimation of K_{sat} and demonstrated that under all circumstances the CPXR method performed better than the MLR technique. The main advantage of the CPXR method compared to multiple linear regression-based models (such as those given in Table 2) is that CPXR is capable to detect complex and nonlinear interactions between input variables that define different subgroups of data with highly distinct predictor-response relationships, and to extract nonlinear patterns among input and output variables. This is confirmed by RMSLE values reported in Fig. 2 and Table 5 as well as the 1.5 orders of magnitude boundaries shown in Fig. 2. As can be observed, those models developed based upon multiple linear regression technique do not estimate K_{sat} as accurately as the Ghanbarian et al. (2015) model developed using the CPXR method. One more advantage of the CPXR technique is that the model can be reported in the readable and transparent form (Table 3) and, unlike results of artificial neural networks or support vector machine application, can be visually inspected and analyzed.

4. Conclusion

In this study, we evaluated various models available in the literature to estimate soil saturated hydraulic conductivity K_{sat}. Using a remarkably large database including 19,822 soil samples from all round the United States we compared the reliability of Brakensiek et al. (1984), Campbell and Shiozawa (1994), Cosby et al. (1984), Jabro (1992), Puckett et al. (1985), Dane and Puckett (1994), and Saxton et al. (1986) models. In addition to these models, the Ghanbarian et al. (2015) model estimating K_{sat} from textural data, bulk density as well as sample dimensions e.g., height (or length) and internal diameter was also assessed. As we emphasized before, the assessment of models estimating K_{sat} from other soil properties using a large database including thousands of samples is limited in the literature. In addition, the existing evaluations are mainly restricted to fairly small databases and small scales, and therefore the obtained results reported in the literature might be in contradiction. Our results based on 19,822 measurements including a very wide range of soils from all around the nation are unique and general. We conclusively showed that the proposed sample dimension-dependent pedotransfer functions of Ghanbarian et al. (2015) estimated K_{sat} substantially more accurately than seven other frequently used previously published models.

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Table 5

Root mean square log-transformed error (RMSLE) of different models used in this study to estimate K_{sat} for 12 USDA soil texture classes.

I/ model	USDA soil texture class											
K _{sat} model	Sa ^a RMSLE	LSa	SaL	L	SiL	Si	SaCL	CL	SiCL	SaC	SiC	С
Brakensiek et al. (1984)	0.62	0.92	1.10	0.82 ^b	1.21	1.02	1.02	1.29	1.40	1.05	2.21	1.87
Campbell and Shiozawa (1994)	0.94	0.96	1.11	2.04	2.84	3.22	1.44	2.57	3.40	2.16	4.06	3.35
Cosby et al. (1984)	0.58	0.77	1.04	0.95	1.02	1.10	1.15	1.07	1.04	1.10	0.94	1.12
Jabro (1992)	1.10	0.93	0.95	1.02	1.14	1.71	0.98	1.16	1.10	1.12	1.18	1.59
Puckett et al. (1985)	0.55	0.68	0.82	1.10	1.09	0.86	1.18	1.71	1.83	2.03	2.88	3.15
Dane and Puckett (1994)	0.48	0.80	0.95	0.98	1.04	0.90	0.89	1.03	1.14	1.09	1.62	1.77
Saxton et al. (1986)	0.46	0.65	1.06	0.88	1.06	1.37	1.24	1.51	1.10	1.65	1.72	1.51
Ghanbarian et al. (2015)	0.42	0.64	0.77	0.82	0.83	0.87	0.70	0.74	0.80	0.80	1.01	0.82

^a Sa: Sand, LSa: Loamy sand, SaL: Sandy loam, L: Loam, SiL: Silt loam, Si: Silt, SaCL: Sandy clay loam, CL: Clay loam, SiCL: Silty clay loam, SaC: Sandy clay, SiC: Silty clay, and C: Clay. ^b Bold numbers denote the least RMSLE values and most accurate model in each soil texture class.

Appendix A. Supplementary data

Supplementary data to this article can be found online at http://dx. doi.org/10.1016/j.catena.2016.10.015.

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