Relax ! This is just a question of rank...

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Abstract. One of the approaches used by mathematicians when they are faced with too difficult to tackle problems, is to "relax" them; this term has several different meanings in mathematics, depending of the fields of application: softening the constraints (when they are too hard), enlarging the underlying (functional) spaces where the solutions will be searched, subsituting a more amenable objective function for the original one, etc. In the present communication, we consider the relaxation of a very bumpy function, occurring often in numerical matricial analysis as well as in a class of specific optimization problems (the so-called "matrix rank optimization problems"), that is: **the rank of a matrix**. A function which is naturally associated with it is

$$x = (x_1, ..., x_d) \in \mathbb{R}^d \longmapsto c(x) := \text{ number of } x_i \text{ which are } \neq 0$$

(appearing in constraints or as an objective function in transmitting data for example). There is an intense research activity (*cf.* the references for some recent entries) around the relaxation of such functions and the use of resulting convex optimization techniques. Our aim here is to explain the convex relaxation forms of functions like rank, *cf.* [3] (they came to us as a surprise), to give new proofs of them, and to suggest further developments (for other types of relaxation arising in variational calculus).

We have done that with LE HAI YEN, a student in Master.

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