Approximation of Dirichlet Boundary Control Problems on Curved Domains

Eduardo Casas
Universidad de Cantabria, Santander, Spain
e-mail: eduardo.casas@unican.es

ABSTRACT

In this talk we consider the following optimal control problem

$$(\mathrm{P}) \left\{ \begin{array}{l} \min J(u) = \displaystyle\int_\Omega L(x, y_u(x))\, dx + \frac{N}{2} \int_\Gamma u^2(x)\, d\sigma(x) \\ \text{subject to } \ (y_u, u) \in (L^\infty(\Omega) \cap H^1(\Omega)) \times L^2(\Gamma), \\ \alpha \leq u(x) \leq \beta \quad \text{for a.e. } \ x \in \Gamma, \end{array} \right.$$

where Γ is a smooth manifold, y_u is the state associated to the control u, given by a solution of the Dirichlet problem

$$\left\{ \begin{array}{rcll} -\Delta y + a(x, y) & = & 0 & \text{in } \Omega, \\ y & = & u & \text{on } \Gamma. \end{array} \right. \tag{1}$$

To solve numerically this problem it is necessary to approximate Ω by a new domain (typically polygonal) Ω_h. Our goal is to analyze the effect of this change on the optimal control. More precisely, if we define a new optimal control problem in Ω_h, denoted by (P_h), we study the convergence of global or local solutions of problems (P_h) to the corresponding local or global solutions of (P) when the parameter h tends to zero. We also derive some error estimates. In principle, we do not consider the numerical discretization of (P), we just consider a family of infinity dimensional control problems (P_h) defined in Ω_h and we compare the solutions of these problems with the solutions of (P). In this way, we are studying the influence of a small change in the domain on the solutions of the control problem. Finally, we prove that our estimates are sharp and we compare them with the ones proved by Deckelnick, Günther and Hinze (SICON 48, 2009) for the numerical discretization of (P).