DEVS Representation of Differential Equation Systems: Review of Recent Advances.

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Abstract

In this paper, two recently introduced DEVS-based techniques for simulation of continuous system represented by systems of differential equations are discussed. Their theoretical properties and practical qualities are treated and compared through the introduction of simple examples where some advantages with respect to classical discrete time methods are easily perceived.

1. Introduction:

Digital simulation of continuous systems requires discretization. Classical methods such as Euler, Runge-Kutta, Adams, etc., and their variable step versions are based on the discretization of time [5]. This approximation procedure results in a discrete time simulation model.

Some years ago, different methods allowing continuous system simulation under the DEVS paradigm were developed [1] [2] [7] [8] showing some benefits with respect to the classical discrete time algorithms. The main advantages are related to the reduction of the computational cost and the suitability for distributed implementation.

DEVS offers an alternative way of handling continuous time. Instead of advancing time in discrete steps, we can advance time based on discrete events, where an event is a significant change in an input, state or output variable. The key to handling the continuous variables of is to determine when a significant event occurs in a component. A significant event detector, called a quantizer, monitors its input and uses a logical condition to decide when a significant change, such as crossing a threshold has occurred. The concept of quantization formalizes the operation of significant event detectors. A quantum is measure of how big a change must be to be considered significant. Thus quantization of state variables is a method to obtain a discrete event approximation of a continuous system [7] [8].

The definition of Quantized State Systems (QSS) in [2] adds hysteresis to the quantization and gives tools to show formal properties of the method. In that work, the authors show that a generic system of ordinary differential equations can be modified with the addition of *Quantization Functions with Hysteresis*, transforming it into a QSS, whose behavior can be exactly represented by a DEVS model. It is also shown that the QSS can be thought as a system of differential equations with perturbations, and using this fact, the fundamental properties of convergence and stability of the method are deduced. Although the simulation examples already showed very good results, these theoretical properties constitute a formal proof of the possiblity of applying the method to general systems and they give some clues for the choice of the quantization and hysteresis in order to satisfy certain error bounds.

The introduction of hysteresis in QSS is necessary in order to guarantee DEVS models that perform a finite number of transitions in a finite interval of time (i.e. legitimate DEVS [7]). Another important property of QSS is the capability of decoupling certain classes of structural singularities that are very common in systems of equations derived from object oriented modeling paradigms such as Bond Graphs [3].

In this paper, making use of some simple examples, we discuss and compare the different properties, the performance and the advantages of the use of the quantization-based methods in the simulation of continuous time systems.

After introducing the approaches of [7] and [2] and recalling their main theoretical properties, we present an example of a second order stiff system. The results of simulation of this example using both methods are compared in order to elucidate the differences between the approaches and their advantages over the classical discrete time algorithms [4].

A second example is utilized to make evident the need for using hysteresis. This naturally leads to a discussion of the problem of choosing the hysteresis width. Using some theoretical properties and analyzing the different possibilities within a simple example we conclude that the best selection is setting the hysteresis width equal to the quantum size.

2. Quantized Systems and Quantized State Systems

The Quantized Systems [7] and QSS [2] constitute two similar ways to introduce a formal transformation in a continuous system that allows its representation in the DEVS formalism. This transformation is performed through the addition of quantization and the main difference between both approaches arises in the use of hysteresis in the QSS quantizers.

2.1. Quantized Systems

To simulate systems with a digital computer requires that we approximate a continuous trajectory with a finite number of values in a finite time interval. One way is to discretize the time base to obtain a discrete time approximation. However, rather than discretize the time base we may partition the trajectory into a finite number of segments each of which has a finite computation associated with it. For a quantization of a single real state variable, we let the value set be a finite interval of the reals and let a partition π be given by an integer mesh, {…,-D,0,D,2D,…,nD….} where a block is an interval [nD,n+1D) and D is called the *quantum.* We call such a quantization, *quantum-based*. Given a number *x*, the partition block [*x*] is computed as the integer floor of *x*/D. The latter value can also be designated as the representative of *x*, corresponding to rounding down. Other choices of representative are possible, for example, the half point, floor(*x*/D) + D/2. Quantization of an n-dimensional real valued space can be done in many ways. One way is through the independent quantization of each dimension.

To study representation of a DESS (Differential Equation System Specification) by a DEVS we introduce quantized systems as in 0a). A *quantized system* can be decomposed into, hence has the same behavior as, a system with input and output quantizers, as 0 b). A *quantizer* is a system defined at the I/O function level, with its only function, Q_π defined by $Q_\pi(\omega) = \varpi$. We call the system sandwiched between the quantizers, the *internal system.* We denote a quantized system by $< Q_{\pi1}, S, Q_{\pi2} >$, the sequence of the input quantizer, internal system, and output quantizer, respectively.

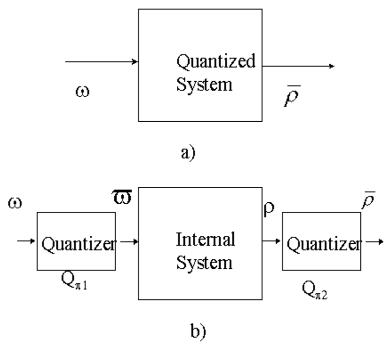

Figure 1: Quantized Input/Output Systems

The output segments of quantized system are clearly piecewise constant. A quantized system was shown to be realizable as a DEV&DESS (the combined DEVS and DESS) where the crossings of the partition block boundaries are implemented as state events[8].

2.2. Quantized State Systems

As it was mentioned above, the quantization in QSS is performed using hysteresis. Before defining QSS, the concept of *Quantization Function with Hysteresis* is given.

Let $D = \{d_0, d_1, \cdots, d_n\}$ be a set of real numbers where $d_{i-1} < d_i$. Let $x \in \Omega$ be a continuous trajectory where $x : \Re \rightarrow \Re$. Let $b : \Omega \times \Re \rightarrow \Omega$ be a mapping and assume that $q = b(x, t_0)$ satisfies:

$$q(t) = \begin{cases} d_m & \text{if } t = t_0 \\ d_{i+1} & \text{if } x(t) = d_{i+1} \wedge q(t^-) = d_i \wedge i < r \\ d_{i-1} & \text{if } x(t) = d_i - e \wedge q(t^-) = d_i \wedge i > r \\ q(t^-) & \textit{otherwise} \end{cases}$$

where $m = \begin{cases} 0 & \text{if } x(t_0) < d_0 \\ r & \text{if } x(t_0) \geq d_r \\ j & \text{if } d_j \leq x(t_0) < d_{j+1} \end{cases}$

Then, the map b is a *Quantization Function with Hysteresis*. The hysteresis width is ε and the parameters d_0 and d_r are the lower and upper saturation values (see Fig. 2).

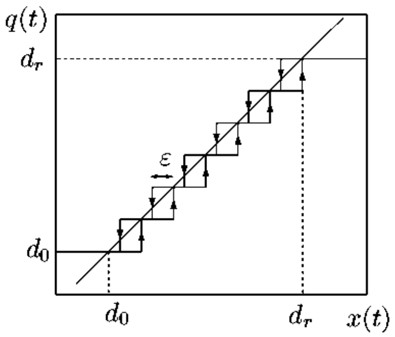

Figure 1. Quantization Function with hysteresis

Consider now the following State Equation System (SES):

$$\begin{cases} \dot{x} = f(x(t), u(t)) \\ y(t) = g(x(t), u(t)) \end{cases}$$

This is a typical representation of Differential Equation Systems, where the components of the vectors x, u and y are called *state*, *input* and *output variables*, respectively. This kind of equations allows the representation of most continuous systems.[1] Associated to the SES, we define the QSS according to:

$$\begin{cases} \dot{x} = f(q(t), u(t)) \\ y(t) = g(q(t), u(t)) \end{cases}$$

where q and x are related componentwise by quantization functions with hysteresis. The components of q are called *quantized variables*. Fig. 2 shows a Block Diagram representation of a generic QSS. Provided that the function f is bounded in any bounded domain, when the input u is piecewise constant it can be assured that the quantized and state variables have piecewise constant and piecewise linear trajectories respectively. Fig. 3 shows typical trajectories in a QSS. The particular form of the trajectories allows the simulation of generic QSS by DEVS models

[1] In this paper we distinguish the SES from the DESS that is its formal representation in mathematical systems theory [8].

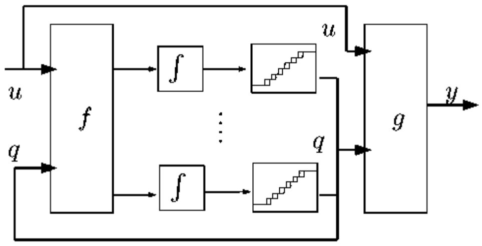

Figure 2. Block Diagram representation of a QSS

through the representation of the changes in the piecewise constant trajectories by events.

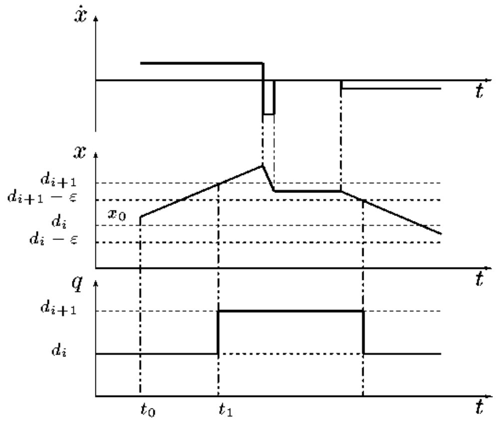

Figure 3. Typical trajectories in QSS

Although it can be assured that a QSS can be simulated by a DEVS model, this does not guarantee that the QSS constitutes a good approximation to the SES. In order to demonstrate that the trajectories obtained with the QSS are *similar* to the exact trajectories in the SES, the following properties were proven:

- If the SES is stable, then a quantization can be found so that the trajectories of the resulting QSS converge to small regions around the equilibrium points of the SES (stability of the method).

- If the function f satisfies some reasonable conditions, the trajectories of the QSS converges to the trajectories of the SES when the quantization goes to zero. (convergence of the method)

The first property allows finding the quantization that assures a bound in the final error of the simulation. The second one tells that an arbitrary small error can be achieved along the whole simulation by taking small values for the quantum size. *Both properties imply that the approximation of a SES by a QSS is a well posed simulation method.*

The proof of the mentioned properties and the DEVS model associated to a QSS can be found in [2].

3. QS and QSS in an example

Consider the following second order system corresponding to a serial RLC circuit

$$\begin{cases} \dot{x}_1 = \dfrac{1}{L} x_2 \\ \dot{x}_2 = U - \dfrac{1}{C} x_1 - \dfrac{R}{L} x_2 \\ y = \dfrac{1}{L} x_2 \end{cases} \qquad (1)$$

With the parameters $R = 100.01$, $L = 0.01$, and $C = 0.01$ a stiff system is obtained since the eigenvalues are -1 and -10000. The analytical solution of the second order stiff system is given by:

$$y(t) = \frac{10000}{9999}(e^{-t} - e^{-10000t})$$

Both methods, Quantized Systems and QSS, were used in the simulation of the model. The simulation results for an input trajectory $U = 100$ using quantum sizes of 1×10^{-2} and 1×10^{-4} for x_1 and x_2 respectively is shown in Fig.4. The difference between the trajectories obtained using the different methods is negligible.

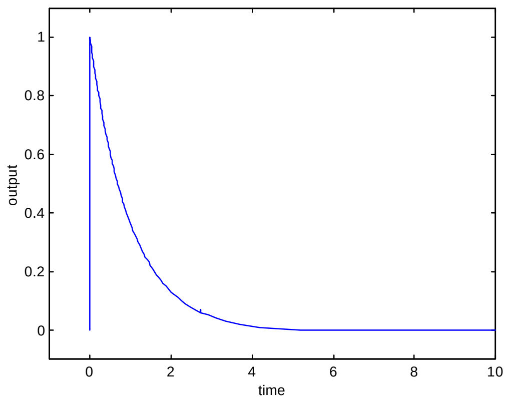

Figure 4 Output trajectory of the second order stiff system

The comparision of the output trajectory with the exact solution shows that the error in both cases is bounded by 10^{-2} (1.0 % of the maximum value). The reason for the similar results is related to the way in which the Quantized System was implemented. Although the hysteresis is not formally present there, the implementation in fact employs a similar approach to assure legitimacy of the resulting DEVS model. However, a difference can be observed in Figure 5, where the number of internal transitions (related to the number of computations) performed by the QS and QSS is compared for different quantum sizes. When small quantum sizes are used (higher accuracy), the QSS is more efficient than the QS since it employs one transition rather than two during hysteresis.

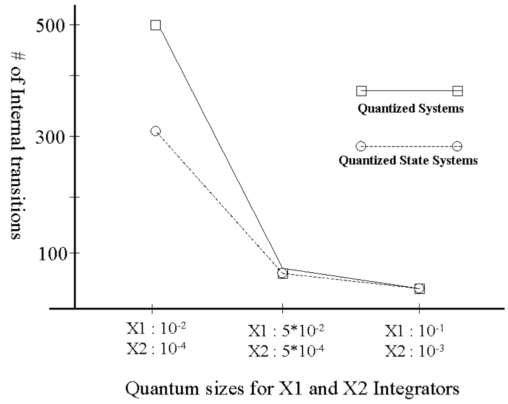

Figure 5: Number of internal transitions in QS and the QSS simulation.

When these results are compared against the results obtained with classic discrete time algorithms achieving the same accuracy (Table 1), the DEVS-based methods appear as a very attractive alternative even for simulation on a single machine.

Method	Features	Number of steps
Euler	Fixed step, 1st order, explicit	>150000
Runge-Kutta	Fixed step, 4th order, explicit	>60000
ode45 (Matlab)	Variable step, 4th order, explicit	>30000
ode113 (Matlab)	Variable step, 4th order, explicit	>60000
ode15s (Matlab)	Variable step, 5th order, implicit	81
Quantization of States	**Discrete events, 1st order, explicit**	**366**

Table 1. Number of steps performed by different methods in the simulation of (2) for the same output error (0.01).

Moreover, stochastic analysis of quantization provides an alternative approach to the deterministic analysis just discussed and demonstrates significant reduction in message passing afforded by the DEVS approximation in distributed (multiprocessor) simulation [9].

4. An illustrative example about hysteresis.

As was discussed above, the introduction of hysteresis was necessary in order to avoid illegitimacy in the DEVS representation of quantized systems. In [2] we showed that the minimum time between internal events in an integrator is bounded by the inverse of the hysteresis width. Also, the final error in the simulation is bounded by a value which is proportional to the larger of the hysteresis width and the quantum size. Thus, if the hysteresis width is taken equal to the quantum size, the time between internal events will be reduced without increasing the error bound.

The following example shows clearly this fact and demonstrates the need for using hysteresis.

Consider the first order system: $\dot{x} = -x + u$ The exact response of the system for the input $u(t) = 9.5$ and the initial condition $x(t=0) = 0$ can be calculated as $x(t) = 9.5 \cdot (1 - e^{-t})$. The simulation with QSS using a quantum size $\Delta q = 1$ and hysteresis width of $e = 1$, $e = 0.6$ and $e = 0.1$ respectively gives the results of Fig. 4

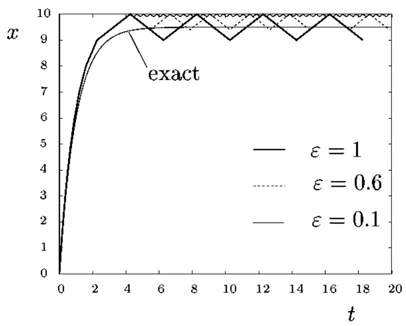

Figure 4. QSS simulation with different values of *e*.

In all the cases the maximum absolute error is bounded by the same value (0.5), but the frequency of the oscillations (and the number of steps) increases as well as the hysteresis becomes smaller. The frequency can be calculated as $f = \frac{1}{2e}$. It is clear that the frequency goes to infinity when e goes to zero. The latter setting represents the situation when hysteresis is not used and we see how the associated DEVS model is illegitimate and the simulation "gets stuck" at x=10.

5. Conclusions

Our objective was to illustrate quantization and hysteresis as the basis for recently developed DEVS-based techniques for simulation of continuous system represented by systems of differential equations. Formal proof of their theoretical properties is given in the references. The practical advantages with respect to classical discrete time methods were demonstrated on stiff systems, the bane of classical numerical techniques. The results should stimulate interest in further study and refinement of DEVS methods as alternatives to conventional numerical simulation.

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