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QUANTIZED BOND GRAPHS: AN APPROACH FOR DISCRETE EVENT SIMULATION OF PHYSICAL SYSTEMS.

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ABSTRACT:

This paper introduces a new class of Bond Graph models, called Quantized Bond Graphs (QBG), which can be "exactly" simulated by a Discrete Event System Specification or DEVS structure (Zeigler; 1976, 2000). Standard Bond Graph representing Physical Systems can be approximated by QBG for the purpose of simulation. It is shown that derivative causalities dissapear from this kind of models, turning into causal commutations as in Switched Bond Graphs. Stability properties of the resulting simulation models are also studied. Finally, interesting results obtained from the simulation of a technical system under this approach are shown.

Keywords: *Continuous Systems, Discrete Event Systems, Simulation, Bond Graphs*.

INTRODUCTION

Many difficulties arise when trying to simulate physical systems directly from object–oriented descriptions like bond graphs (BG). Structural singularities, resulting in implicit equations, are probably the most relevant among them (Cellier, 1991). Stiffness is another frequently encountered problem in current mathematical description of continuous dynamic systems, like ODE's.

Different solutions have been proposed to solve these problems, ranging from numerical methods (Press et al., 1986) up to symbolic computation (Cellier et al., 1996). In recent work (Giambiasi, 1998, Zeigler, *et al*., 2000) advantages of simulating stiff problems within a discrete event paradigm have been shown. In (Naamane, *et al*., 1999) an approach to discrete event simulation of BG is developed, which is based on the approximation of the continuous system trajectories associated to the BG by piece-wise polynomial trajectories.

In this paper an alternative treatment is proposed. It consists of quantifying the mathematical relationships of storages and sources of the BG, thus converting it into a quantized simulation model. The signals associated to the sources are approximated by piece-wise constant trajectories. The same is done with the constitutive relations of the storages, with an essential modification: hysteresis is added at the end of each interval of quantization. The description resulting from this procedure is the Quantized Bond Graph.

The paper shows that, in absence of structural singularities, the power variables of QBG are piece-wise constant. From a result in (Zeigler, *et al.*, 2000) it follows that each component of QBG can be simulated by an atomic DEVS model and, consequently, the whole QBG can be simulated by a coupled DEVS structure.

A stability analysis of QBG via Lyapunov-second method is presented, in order to specify the quantization parameter needed in order to preserve the stability of the original system.

An important result presented in the paper concerns the presence of dependent storage elements in the original BG, which implies the appearance of derivative causality. It is shown that derivative causality is not an issue in QBG, then by the process of quantization it turns into causal commutations as in Switched Bond Graphs (Strömberg, 1994). This problem is simpler than handling derivative causality in continuous system with more traditional techniques (numerical methods).

Simulation results will be presented, comparing the performance of DEVS simulation of QBG with that of traditional numerical integration of differential equations

(Press, *et al*., 1986). It is shown that results are obtained, which are similar to those yielded by complex numerical methods, but with much less computational complexity and burden.

QUANTIZED STATE SYSTEMS

The definition of *quantization function with hysteresis* will be introduced next, before presenting the QBG.

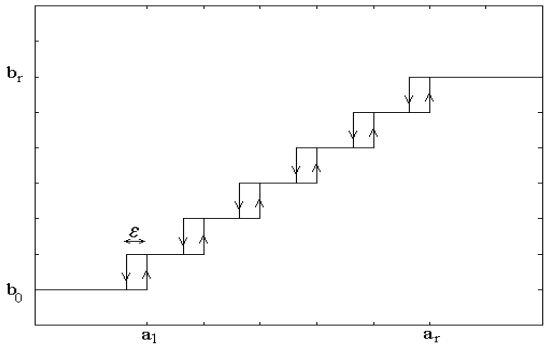

Figure 1. Quantization function with hysteresis

Consider the finite sets:

$$A_c = \{a_0, a_1, ..., a_r, a_{r+1}\},\ B_c = \{b_0, ..., b_r\}, \text{ where:}$$

$$a_0 = -\infty \quad a_{r+1} = \infty;\ a_i < a_{i+1} \quad i = 0, ..., r$$
$$\text{and} \quad b_i < b_{i+1} \quad i = 0, ..., r-1 \tag{1}$$

From these sets, the following piecewise constant function is defined:

$$d(a) = b_i \quad if \quad a \in [a_i, a_{i+1}); \tag{2}$$

Finally, it is defined:

$$B(a,c) = \begin{cases} d(a) & if \quad d(a) = d(a+e) \\ d(c) & otherwise \end{cases} \tag{3}$$

When variable *a* is a trajectory in time and variable *c* is $c = a(t_1^+)$, being t_1 the last instant of change of *d*(*a*), the function *B*(*a*,*c*) will be called quantization function with hysteresis. The relationship *e* >0 must be satisfied, see Fig. 1.

Now consider the following system of state and output equations:

$$\begin{cases} \dot{x}_1 = f_1(x_1, ..., x_n, u_1, ..., u_m) \\ \quad \vdots \\ \dot{x}_n = f_n(x_1, ..., x_n, u_1, ..., u_m) \end{cases}$$
$$\begin{cases} y_1 = g_1(x_1, ..., x_n, u_1, ..., u_m) \\ \quad \vdots \\ y_p = g_p(x_1, ..., x_n, u_1, ..., u_m) \end{cases} \tag{4}$$

Then, the following system is defined as a *Quantized State System* associated to the system given by (4) :

$$\begin{cases} \dot{x}_1 = f_1(q_1, ..., q_n, w_1, ..., w_m) \\ \quad \vdots \\ \dot{x}_n = f_n(q_1, ..., q_n, w_1, ..., w_m) \end{cases}$$
$$\begin{cases} y_1 = g_1(q_1, ..., q_n, w_1, ..., w_m) \\ \quad \vdots \\ y_n = g_n(q_1, ..., q_n, w_1, ..., w_m) \end{cases} \tag{5}$$

with:

$w_j = W_j(u_j)$ where W_j is a bounded piecewise constant function that verifies $w_j = W_j(u_j) \Rightarrow W_j(w_j) = w_j$.

$q_j = B_j(x_j, c)$, where B_j is a quantization function with hysteresis defined by some finite sets X_j y Q_j and the parameter e_j, satisfying $B_j(q_j, c) = q_j \quad \forall c,\ q_j \in Q_j$.

Figure 2 shows a block diagram of the Quantized State System defined by (5).

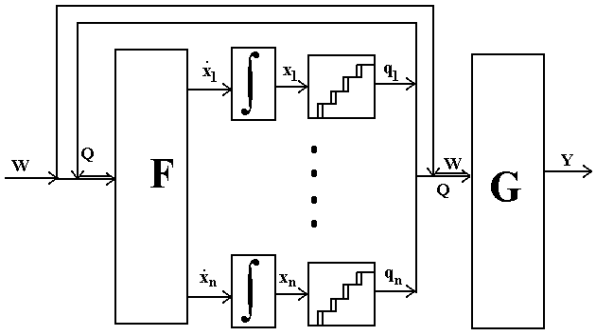

Figure 2. Block diagram of the *Quantized State System*.

SOME PROPERTIES OF QUANTIZED STATE SYSTEMS

Theorem 1: Given the Quantized State System defined in (5), with $f_1, ..., f_n$ continuous and bounded functions in any bounded domain, the trajectories of variables $q_1, ..., q_n$ are piecewise constant in any finite interval of time.

Proof: Let $Q_j = \{q_j^1, ..., q_j^{rj}\}$ y $X_j = \{x_j^0, ..., x_j^{rj}\}$ be finite sets and e_j be the parameters that define a quantization function. From equation (1), it results for an arbitrary quantized variable:

$$q_j^1 \le q_j \le q_j^{r_j} \tag{6}$$

Then, variables $q_1,...,q_n$ have bounded trajectories. Similarly, quantized inputs $(w_1,...,w_m)$ have bounded trajectories. Thus, $f_1,...,f_n$ being bounded, it follows from equation (5) that the derivatives $\dot{x}_1,....,\dot{x}_n$ have bounded trajectories. Then, there exists a positive number *M* such that:

$$-M \le \dot{x}_j \le M \quad \forall j \tag{7}$$

A state variable can be calculated as:

$$x_j = x_j(t_o) + \int_{t_0}^{t} \dot{x}_j(t)dt \tag{8}$$

From (7) and (8) it follows that:

$$x_j(0) - M(t-t_0) \le x_j \le x_j(0) + M(t-t_0) \tag{9}$$

Inequality (9) shows that the state variables have bounded trajectories in any finite time interval. Moreover, from (7) and (8) it follows that the state variables have continuous trajectories.

Thus, it can be easily deduced that the time that a quantized variable needs to change its value twice is greater than:

$$t_{min} = \frac{e_j}{M} \tag{10}$$

It means that a quantized variable can only have a finite number of changes in any finite time interval. That implies that quantized variables have piecewise constant trajectories.

Theorem 2: In a Quantized State System verifying the hypothesis of Theorem 1, the trajectories of the state variable derivatives are piecewise constant.

Proof: It is straightforward from Theorem 1.

Theorem 3: In a Quantized State System verifying the hypothesis of Theorem 1, the state variable trajectories are piecewise linear.

Proof: It is straightforward from Theorem 2.

Continuous systems with piecewise constant input and output trajectories can be simulated by a DEVS model [6]. However, this simulation requires the knowledge of the continuous system solution. Simulating knowing the solution is useless, but it is possible to divide the system into small subsystems, each of them composed by a single integrator and its corresponding quantizer.

If theorems 1 and 2 are satisfied, each subsystem will have piecewise constant input and output trajectories and the continuous solution of the subsystems is straightforward, then the system can be simulated by a coupled DEVS structure.

Remark. Equation (10) shows the need of using hysteresis. If hysteresis were not used, (i.e. e_j were zero) a quantized variable could change its value an infinite number of times and the resulting discrete event model would produce infinite events in a finite time interval, which is impossible to be simulated.

QUANTIZED BOND GRAPHS

A simple example will be presented in order to introduce the definition of Quantized Bond Graphs.

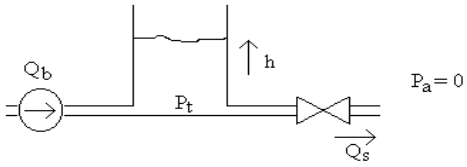

Figure 3. Hydraulic system

The hydraulic system of Figure 3 can be represented by the Bond Graph of Figure 4. Let us assume that the capacitor is linear, then its constitutive relation *volume V - pressure* P_t can be approximated by the law depicted in Fig. 5 (a piece-wise constant function). A "physical" tank with such a V - P_t relationship is shown in Fig. 6, where it is assumed that $\Delta h << h_a$, y Va>>Vb. This system is still represented by the Bond Graph of Fig. 4, now with the law of Fig. 5 as the constitutive relationship of the capacitor. Thus, we are lead to the following (rather) informal:

Definition. A quantized linear capacitor is a capacitor having its *displacement vs. effort* relationship as that shown in Fig. 7. To ensure the simulability property pointed out in the previous *Remark*, hysteresis has been added to the curve in Fig. 5. The extension to the nonlinear case is immediate (see Fig. 8).

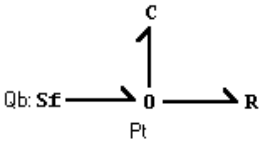

Figure 4. Bond Graph of hydraulic system

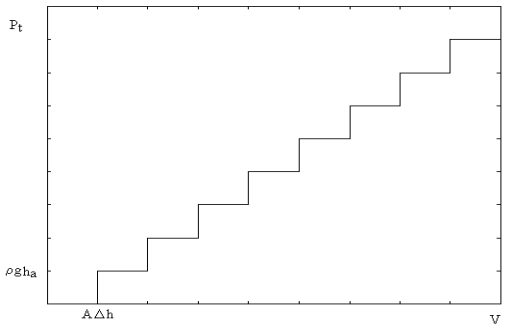

Figure 5. Quantized characteristic of P_t vs. V relationship

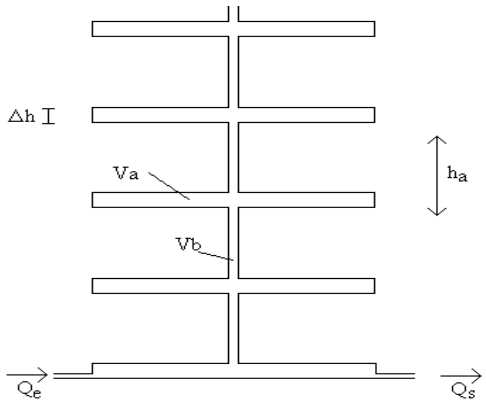

Figure 6. A quantized tank.

The definition of the remaining basic constituents of Quantized Bond Graphs follows. A *quantized inertia element* is defined –analogously to a quantized capacitor– as an inertia element whose *flow vs. momentum* constitutive relationship is a *quantization function with hysteresis*. A *quantized source* is a source where the trajectory of its independent variable *(effort / flow)* is a piece-wise constant and bounded function of time. Finally, a *Quantized Bond Graph* is a Bond Graph where all its storages and sources are quantized elements.

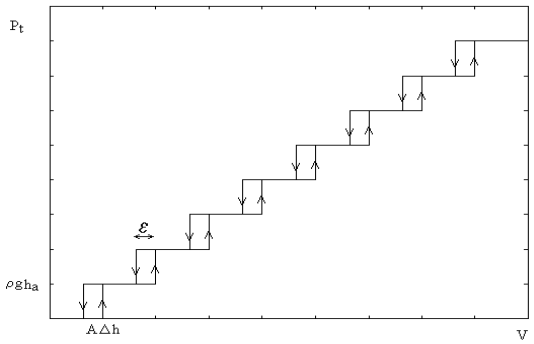

Figure 7. Quantization P_t vs. V function with hysteresis.

It is important to note that QBG are not direct models of real systems, but that they can be formulated as (accurate enough) approximations of standard, continuous variable Bond Graphs, via the quantization of their storages and sources.

Fig. 8 shows both the *continuous constitutive relationship of a nonlinear storage*, and its *associated* quantization function with hysteresis. The *formal definition of the quantized constitutive relation with hysteresis for a storage* is the following:

Definition: Consider that the quantization function with hysteresis defined by (1), (2) y (3) also satisfies (11):

$$B(b,c) = b \quad \forall c,\ b \in B_c \tag{11}$$

Then, the quantized constitutive relation with hysteresis of the storage is given by relation (12) below:

$$h(b,c) = v[B(b,c)] \tag{12}$$

where $v(x)$ is the function relating the energy and power variables of the storage (in the continuous variable representation), and the function h is the resulting characteristic of the associated quantized storage element.

Equation (13) defines q_j , the corresponding *quantized energy variable*, where x_j is the original energy variable.

$$q_j = B(x_j, c) \tag{13}$$

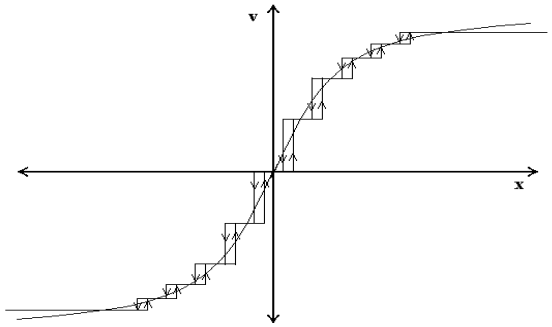

Figure 8. Nonlinear, continuous constitutive relation of a storage element, and associated quantization function with hysteresis.

SOME PROPERTIES OF QUANTIZED BOND GRAPHS

Theorem 4: Consider a Quantized Bond Graph without coupled storages, where all the passive and structural component are defined by continuous and bounded relations. Then the trajectories of all power variables are piece-wise constant.

Proof: Under the assumptions made, the application to the QBG of the standard procedure for the derivation of state equations (Karnopp and Rosenberg, 1983), yields a QSS of the form (14):

$$\begin{cases} \dot{x}_1 = f_1(q_1,...,q_n, w_1,..., w_m) \\ \quad \vdots \\ \dot{x}_n = f_n(q_1,...,q_n, w_1,..., w_m) \end{cases} \tag{14}$$

where, because of the assumptions, the functions f_i are bounded and continuous in any compact domain. This property, along with Theorem 1, guarantees that the trajectories of the q_i's variables (the *quantized* energy variables) are piece-wise constant. The *quantized* power variables are consequentely also piece-wise constant, because they are computed from the former variables via static relationships.

Theorem 5: Consider again a QBG satisfying the assumptions of Theorem 4. Then the trajectories of the *energy variables* x_j are piece-wise linear.

Proof: follows immediately from Theorem 4.

Corollary (of Theorems 4 y 5): QBG are exactly representable by a DEVS structure. This a direct consequence of a result in (Zeigler, *et al*., 2000), stating that a DEVS structure can exactly model systems whose input and output trajectories are piece-wise constant.

DEVS MODEL ASSOCIATED TO A QUANTIZED BOND GRAPH

The simulation using DEVS of systems with piece-wise constant input and output trajectories requires these trajectories being represented via event trajectories, in order to associate an event to each of their step changes. The corresponding DEVS model (Zeigler, 1976) will be next defined as the coupling of atomic DEVS each associated to an elementary component of a QBG (quantized storages, resistors and structural components, i.e., transformers, gyrators, and zero and one junctions).

DEVS model associated to a quantized capacitor

Consider a quantized capacitor. Its associated DEVS model is specified as follows:

$C_d = \langle X, Y, S, d_{\text{int}}, d_{ext}, l, ta \rangle$ where:

X={("in$_1$", f_d)} (set of input ports and values); $f_d \in \Re$

Y={("out$_1$",e_d)} (set of output ports and values); $e_d \in \Re$

S={(w, x, s)} (set of state values)

with x , $w \in \Re$; $s \in \Re_0^+ \infty$

d_{int} (w, x, s) = (w, $x_+ s w$, s'), (internal transition function), with:

$$s' = \begin{cases} \dfrac{x^{i+1} - (x + s \cdot w)}{w} & \text{if} \quad w > 0 \\ \dfrac{e}{|w|} & \text{otherwise} \end{cases}$$

where x^{i+1} is the next quantized value of $x' = x^i$ in this case, and e is the width of the hysteresis window.

d_{ext} [(w, x, s), f_d, e] = (f_d, x', s') (external transition function), with

$$x' = x + w \cdot e \text{ ; and } s' = \begin{cases} \dfrac{x^{i+1} - x'}{f_d} & \text{if} \quad f_d > 0 \\ \dfrac{x' - x^i + e}{|f_d|} & \text{if} \quad f_d < 0 \\ \infty \quad \text{otherwise} \end{cases}$$

l(w, x, s) = ("out$_1$",v(x_+ s w)) (output function)

ta (w, x, s) = s (time advance function)

Provided the flow trajectory is piece-wise constant, and the causality is integral, this DEVS structure exactly models the functioning of a quantized capacitor.

DEVS associated to a Resistor

Consider a resistor causalized in order to compute the effort via the relationship given by a function E. Its associated DEVS model is:

$R = \langle X, Y, S, d_{\text{int}}, d_{ext}, l, ta \rangle$, where

X e Y are defined as in the previous element.

S={(w, s)} with $w \in \Re$; $s \in \Re_0^+$

d_{int}(w, s) = (w,∞)

d_{ext} [(w, s), ("in$_1$", f_d), e] = (f_d, 0)

l(w, s) = ("out$_1$",E (w))

ta (w, s) = s

For the opposite causal situation the definition is similar.

DEVS associated to a one-junction

Consider a one–junction with N bonds attached to it, the flow coming into the junction from the *j-th* bond. Its associated DEVS model is:

$U = \langle X, Y, S, d_{\text{int}}, d_{ext}, l, ta \rangle$ where

X = {("in$_i$",x_i)}, i=1...N,

Y ={("out$_j$",y_j), ("out$f$",yf)}

S = {s_1,..., s_N , s, *port* }

s_i , x_i , $y_i \in \Re$; $s_d \in$ {0, ∞} *port*∈ {1,...,N}

d_{int} (s_1,..., s_N , s, *port*) = (s_1,..., s_N , ∞, *port*)

d_{ext} ((s_1,..., s_N , s, *port*), ("in$_k$",x_k), e) =
(s_1,..., s'_k,..., s_N, 0, k)

where $s'_k = x_k$

l(s_1,..., s_N , s, j) = ("out$f$", s_j)

l(s_1,..., s_N , s, *port*¹j) = ("out$_j$", $\sum_{i=1, i \neq j}^{N} \pm s_i$)

where the signs inside the summation symbols are positive or negative according to the power flow (incoming power flow implies positive sign).

$ta(s_1,..., s_N, s, port) = s$

The DEVS representation of the inertia element is formally the same as that for the capacitor, the only difference being the dualization of the positions of effort e_d and flows f_d in the descriptions. The remaining structural components (*0*–junction, transformer and gyrator) have representations similar to that one of the *1*–junction. The representation of the sources is trivial. Next, the definition of the coupling will be given.

DEVS representation of the coupled atomic models

The representation is the following:

$$NM = \langle X, Y, D, \{M_d \mid d \in D\}, Select, EIC, EOC, IC \rangle$$

X and *Y* are empty sets (no inputs/outputs external to the model were defined), which implies that also *EOC* y *EIC* (External Input/Output Coupling) are empty sets.

The set of references *D* is composed by elements which are responsibly for *identifying* each component of the QBG. Each of the latter components is defined by a structure M_d , an atomic model as defined in the previous paragraph for capacitors, resistors, etc.

The set *IC* (Internal Coupling) is defined as follows:

Let *BN* be the set of bonds in the QBG. Then, $BN=\{(r_j,s_k)\}$, where the ordered pair (r_j,s_k) denotes the presence of a bond between the *j-th* port of element *r* and the *k*-th port of element *s* (*r* and *s* belonging to *D*). The convention for the bond causality is that the *r* element computes the effort. For each element of *BN* the following definitions correspond:

$ic1_{rj,sk}$ = ((*r*,"$out_{j,r}$"),(*s*,"$in_{k,s}$")
if *s* is not a source and *r* is not a *1*-junction
$ic1_{rj,sk}$ = ((*r*,"$outf_{,r}$"),(*s*,"$in_{k,s}$")
if *s* is not a source and *r* is a *1*-junction
If *s* is a flow source, the element $ic1_{rj,sk}$ does not exist.
$ic2_{rj,sk}$ = ((*s*,"$out_{k,s}$"),(*r*,"$in_{j,r}$")
if *r* is not a source and *s* is not a *0*-junction
$ic2_{rj,sk}$ = ((*r*,"$oute_{s}$"),(*s*,"$in_{j,r}$")
if *r* is not a source and *s* is a *0*-junction
If *r* is an effort source, the element $ic2_{rj,sk}$ does not exist.
Further, the internal coupling set is:
$IC = \{ic1_{rj,sk}\} \cup \{ic2_{rj,sk}\}$

Finally, the *Select* function is defined as to give priority to the structural elements when simultaneous events are scheduled. This is done in order to avoid the possibly lost of output events of the structural elements.

When in the Bond Graph there is no coupling among resistors (no algebraic loops) and among storage elements (no derivative causality) the above defined structure exactly represents a QBG.

COUPLED STORAGES IN QUANTIZED BOND GRAPHS

The presence of coupled storages in continuous Bond Graphs causes derivative causality in some storage elements. The direct simulation of this class of systems needs similar tools to that used to solve algebraic loops. This problem dissapears in QBG, as will be shown next.

Fig. 9 shows a simple *"physically quantized"* hydraulic system. With the assumption of no resistance between the tanks, both capacitors in the bond graph are causally coupled.

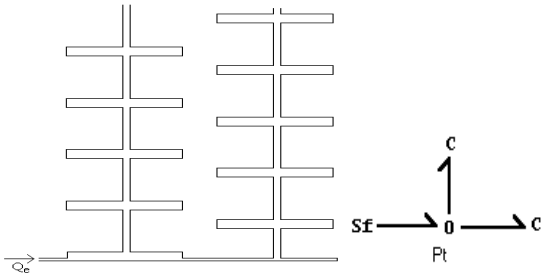

Figure 9. *"Physically Quantized"* System and its Bond Graph with causally coupled storages.

In a continuous system one capacitor should be in derivative causality, since it is impossible that both compute effort simultaneously. Nevertheless, the system of Fig. 9 works in a different way. Suppose feeding the tank system from zero initial conditions. The volume in the tank on the left begins to grow, without liquid flowing into the tank on the right (because the height of its bottom recipient as well as the diameter of its first column are both almost zero). The situation reverses after the bottom recipient of the tank on the left is full: the volume in the second compartment on the right begins to grow, without liquid flowing now into the tank on the left. The pressure at the bottom of the system is determined exclusively by the compartment being filled at each instant of time. The pressure changes discontinuously immediately after each compartment becomes full. This behavior indicates that integral causality alternates in time from one to the other capacitor, what in turn reminds the behavior of Switched Bond Graphs, where the Switch impresses either zero flow or zero effort. In this case (QBG), there are components forcing zero flow or a quantized effort value (in the case of the example, where the storages are capacitors).

The previous idea can be generalized to the case of having coupling of many storages, including both, inertias and capacitors. This situation would result in a model with several alternative causal configurations, in dependance of the values of the energy variables.

The presence of hysteresis (which is absolutely necessary in order to have a simulable scheme) does not modify the concepts underlying the previous considerations.

STABILITY PROPERTIES OF QUANTIZED BOND GRAPHS

When a simulation method is developed, it is important to guarantee that the resulting simulation model conserves some properties of the original system like equilibrium points and stability.

The following theorems give sufficient conditions to assure that such properties are conserved in QSS. Taking in account the fact that a QBG under the assumptions done in Theorem 4 defines a QSS, these properties deduced for a QSS will be conserved in a QBG satisfying the mentioned hypothesis.

Theorem 6: Let a continuous system without inputs (15) and its associated Quantized State System (16):

$$\begin{cases} \dot{x}_1 = f_1(x_1,...,x_n) \\ \quad \vdots \\ \dot{x}_n = f_n(x_1,...,x_n) \end{cases} \tag{15}$$

$$\begin{cases} \dot{x}_1 = f_1(q_1,...,q_n) \\ \quad \vdots \\ \dot{x}_n = f_n(q_1,...,q_n) \end{cases} \tag{16}$$

The point $[\bar{x}_1,...,\bar{x}_n]$ is an equilibrium point of (15) if and only if the point $[\bar{q}_1,...,\bar{q}_n]$ is an equilibrium point of (16), being $\bar{q}_j = \bar{x}_j \quad j = 1,...,n$.

The proof of this theorem is straightforward and it can be extended to system with constant inputs under the condition that quantized versions of the inputs are equal to the original inputs.

Theorem 6 imply that the quantized versions of the state variables in the quantized system have the same possible values in the equilibrium as the state variables of the original system. However, it does not imply that state variables in the quantized system will have such values.

Theorem 7: Consider a system as the one defined in (15) that has an equilibrium point in the origin with the functions f_i being continuous and having bounded partial derivatives. Assume that it is also possible to find a Lyapunov function, with its temporal derivative being negative defined and continuous in an open region *Z* that includes the origin. Let $Z^* \subset Z$ be a region limited by a level surface of function *V*. Then, given an arbitrary open region $Z_1 \subset Z^*$ including the origin it is always possible to find a quantization so that any trajectory of the resulting associated quantized state system starting into an arbitrary closed region Z_2 ($Z^* \supset Z_2 \supset Z_1$) converges to the interior of Z_1.

Proof: Let $V(\mathbf{X})$ be the mentioned Lyapunov function and $\dot{V}(\mathbf{X})$ its temporal derivative. Since it is negative defined in *Z*, in the points $\mathbf{X} / \mathbf{X} \in Z_3 = Z_2 \cap \overline{Z}_1$ (closed region that do not include the origin) it exists a number $m > 0 / \dot{V}(\mathbf{X}) < -m \; \forall \mathbf{X} \in Z_3$.

Let $\mathbf{X_1}$ be an arbitrary point in Z_3. Around this point the following function is defined:

$$a_{X_1}(\mathbf{X}) = \text{grad}[V(\mathbf{X_1})] \times \mathbf{f}(\mathbf{X_1} - \mathbf{X}) \tag{17}$$

This is a continuous function since it is the scalar product of a constant vector and a continuous function ($\mathbf{f} = [f_1,..., f_n]^T$). It is also verified::

$$a_{X_1}(\mathbf{0}) = \dot{V}(\mathbf{X_1}) < -m \tag{18}$$

Then, it is defined the following function:

$$a(\mathbf{X}) = \sup[a_{X_1}(\mathbf{X}) \mid \mathbf{X_1} \in Z_3] \tag{19}$$

It can be easily seen that this function is continuous and verify:

$$a(\mathbf{0}) < -m \tag{20}$$

Thus, a positive number r_2 can be found satisfying:

$$a(\mathbf{X}) < 0 \quad if \quad \|\mathbf{X}\| < r_2 \tag{21}$$

Then, any pair of points $\mathbf{X_1}, \mathbf{X} \in Z_3 / \|\mathbf{X_1} - \mathbf{X}\| < r_2$ verify:

$$\text{grad}[V(\mathbf{X_1})] \times \dot{\mathbf{X}} = a_{X_1}(\mathbf{X_1} - \mathbf{X}) \le a(\mathbf{X_1} - \mathbf{X}) < 0 \tag{22}$$

Inequation (22) shows that the trajectory direction defined in point **X** is to the interior of to the level surface of function *V* in the point $\mathbf{X_1}$.

In a quantized system as the defined in (16), the trajectory direction in the point $\mathbf{X_1}$ that has an associated quantized value given by $[q_1,..., q_n]^T$ can be calculated as the trajectory direction of the continuous system in the point **X**= $[q_1,..., q_n]^T$. (Figure 10)

Being **X** an internal point to the quantization interval that contains $\mathbf{X_1}$, it is possible to choice the quantization interval so that the distance between any point of the interval and the quantized point **X** is smaller than r_2. If it is done, trajectory directions of the quantized system will be to the interior of the level surfaces of *V*. Then, if all the quantization interval corresponding to all the points of the region Z_3 satisfy the mentioned condition, the convergence to the region Z_1 will be guaranteed, what completes the proof of the theorem.

A way of achieve that condition over the distance between two points of the same interval is defining:

$$\Delta q + e < \frac{r_2}{\sqrt{n}} \qquad (23)$$

being Δq the distance between consecutive quantization values of a state variable, e the hysteresis window and n the continuous model order (number of dimensions of the state space).

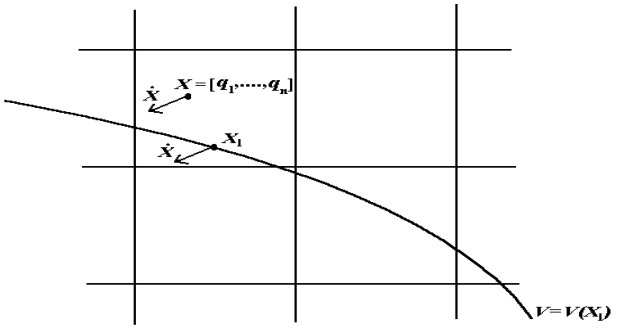

Figure 10. Trajectories and level surfaces in the QSS.

Theorem 7 can be easily extended to systems with constant inputs and with equilibrium points others than the origin. Theorems 6 and 7 show that the method can be implemented achieving a result with an arbitrarily small final error. They also show the way of doing the quantization in final error order to obtain a final error bounded to some arbitrary value (given by the choice of the region Z_1).

EXAMPLES AND RESULTS

The techniques here presented for discrete event simulation of continuous bond graphs have been implemented in a version of the software *Power-DynaMo* (Kofman and Junco, 1999), which allows edition and simulation using different forms of quantization.

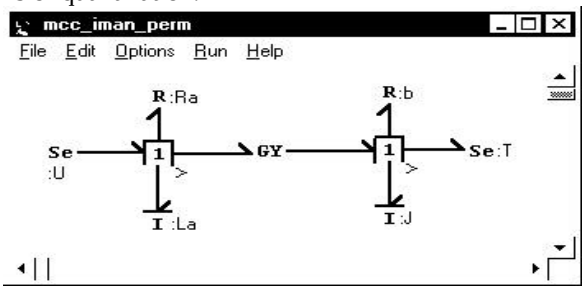

Figure 11. BG of a Permanent Magnet DC Motor

Some simulations on a model of a Permanent Magnet DC Motor (Fig. 8) are presented. A start-up at no-load, followed by the injection of a constant load at time t = 1 s is the simulated experiment. The physical parameters of the motor are: Ra = 0.1, La = 10 x 10^{-6} , b = 1, J = 1, U = 10, T = 10; with consistent units. The simulation parameters are uniform quantization intervals of 10×10^{-6} for the inductance, and of 0.1 for the mechanical inertia.

The simulation results are shown in Figs. 12, 13 and 14. The comparison of Figs. 13 and 14 shows that the system is strongly stiff.

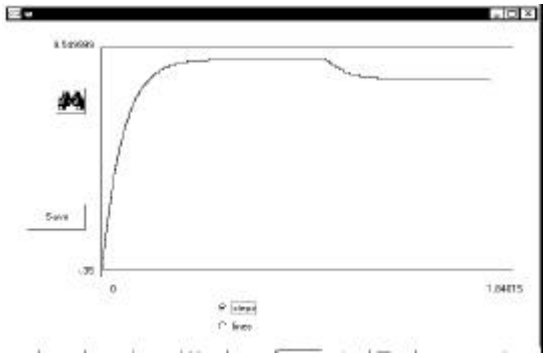

Figure 12. Speed Trajectory

The number of internal transitions performed by the inductance and inertia atomic DEVS models was 203 and 102 , respectively. It means that the total numbers of steps necessary in order to finish the simulation was 305. To obtain a similar precision using the method of Euler, more than 10,000 simulation steps are needed, while the Runge-Kutta algorithm needs more than 8,000 steps. A variable-step algorithm like Runge-Kutta 4-5 (Press, *et al*., 1986) (Matlab's ode45) needs more than 6,000 steps. Matlab's ode15s obtain similar results with only 70 steps, but it must be considered that this is a fifth order implicit method, in comparison to the simple, first order explicit DEVS-based simulation method. The previous comparisons only take into account the number of steps, but not the computational complexity at each step. In this regard, DEVS-based methods generally outperform discrete-time methods, then at each step the latter technique calculates the evolution of all the state variables of the model, while the former one *only computes the next value of the state performing the transition and of the affected variables*, which in high-order systems is of paramount importance.

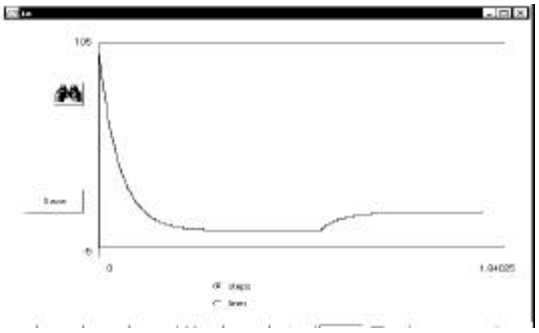

Figure 13. Armature Current Trajectory.

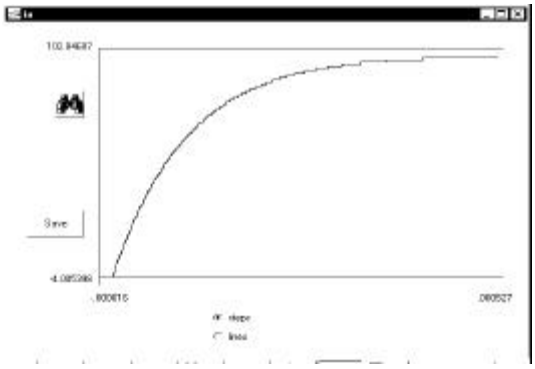

Figure 14. Zoom of the first 500μs of the current evolution.

CONCLUSIONS

In this paper, a new class of Bond Graphs, Quantized Bond Graphs, have been introduced. QBG are exactly representable by a discrete event specification, which has been demonstrated in the article via the definition of a generic hierarchical DEVS structure associated to any QBG. The quantization of an ordinary, continuous-variable Bond Graph, and its subsequent representation by the associated DEVS-structure, allow the discrete event simulation of continuous physical systems represented by Bond Graphs.

Some convergence properties have been pointed-out, and important advantages of this discrete event based representation and simulation technique vis-à-vis time discretization based techniques have been shown. Particularly interesting among them are: the conversion of the usually involved differential causality problems in continuous models into a simpler switching problem in QBG, and the reduced number of calculation steps involved in simulation.

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